Supplementary Material to “Inflation Persistence, Noisy Information, and the Phillips Curve”

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SM.1. Persistence in New Keynesian Models

In this section, I study the determinants of inflation persistence in a structural macro framework. I show that the empirical findings documented in the previous section present a puzzle in the NK model. I cover a wide range of NK frameworks and show that they cannot explain the fall in inflation persistence in an empirically consistent manner.

SM.1.1. Structural Shocks

In the benchmark NK model, in which agents form RE using complete information, the demand (output gap) and supply side (inflation) dynamics are modeled as two forward-looking stochastic equations, commonly referred to as the Dynamic IS (DIS) and New Keynesian Phillips (NKPC) curves.[1] Nominal interest rates are set by the CB following a reaction function that takes the form of a standard Taylor rule. The CB reacts to excess inflation and output gap and controls an exogenous component, v_t, which follows an independent AR(1) process which innovations are treated as serially uncorrelated monetary policy shocks.

Inserting the Taylor rule (9)-(10) into the DIS curve (8), one can write the model as a system of two first-order stochastic difference equations that can be solved analytically. In particular, inflation dynamics satisfy

$$\pi_t = -\psi_\pi v_t = \rho\pi_{t-1} - \psi_\pi \sigma_\varepsilon \varepsilon_t^\nu \tag{SM.1}$$

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[1]The model derivation is relegated to Online Appendix OA.4.

where ψ_π is given by (14), and output gap dynamics are given by $\widetilde{y}_t = -\psi_y v_t = \rho \widetilde{y}_{t-1} - \psi_y \sigma_\varepsilon \varepsilon_t^\nu$, ψ_y defined in Online Appendix SM.2. Notice that inflation is proportional to the exogenous shock. As a result inflation will inherit its dynamic properties from the exogenous driving force.[2] A final implication is that inflation is only *extrinsically* persistent: its persistence is determined by the v_t AR(1) process' persistence.

In order to explain the fall in the persistence of inflation I discuss each causal explanation separately. First, I explore whether there has been a change in the structural shocks affecting the economy. I show that these exogenous forces' dynamics have been remarkably stable since the beginning of the sample. Second, I investigate if a change in the monetary stance around 1985:Q1, for which Clarida et al. (2000); Lubik and Schorfheide (2004) provide empirical evidence, could have affected inflation dynamics. I show that the change in the monetary stance can explain the fall in volatility but has null or modest effects on persistence. Finally, I explore if changes in intrinsic persistence, generated via backward-looking assumptions on the firm side, have a sizeable effect on persistence. As in the previous case, I show that these have only marginal effects.

I documented in Section 2.1. that inflation persistence and volatility fell in recent decades. The NK model suggests that such a fall is inherited from a fall in the persistence of the monetary policy shock process. I now seek to find evidence on the time-varying properties of such persistence.

Persistence. The challenge that the econometrician faces is that she does not have an empirical proxy for v_t. The monetary policy shocks estimated by the literature are not serially correlated, and are therefore a better picture of the monetary policy innovation, ε_t^ν.[3,4] However, one can use the model properties and rewrite the Taylor rule (9) using the AR(1) properties of (10), as

$$i_t = \rho i_{t-1} + \left(\phi_\pi \pi_t + \phi_y y_t\right) - \rho\left(\phi_\pi \pi_{t-1} + \phi_y y_{t-1}\right) + \sigma_\varepsilon \varepsilon_t^\nu \tag{SM.2}$$

[2]One can also notice that the benchmark model predicts that output gap and inflation are equally persistent, and their dynamics will only differ due to the differential monetary policy shock impact effect, captured by ψ_y and ψ_π. Another implication is that the Pearson correlation coefficient between the output gap and inflation is equal to 1, an aspect rejected in the data.

[3]In fact, the process v_t is a model device engineered to produce inertia yet still allows us to obtain a tractable solution. If inertia is directly introduced in the nominal interest rate equation, I would not be able to obtain a tractable solution (SM.1) since the system would also feature a backward-looking term whose coefficients would depend on the roots of a quadratic polynomial.

[4]For example, Romer and Romer (2004) use the cumulative sum of their estimated monetary policy shocks to derive the IRFs.

	(1) Full Sample	(2) Structural Break
i_{t-1}	0.942^{***}	0.930^{***}
	(0.0155)	(0.0424)
$i_{t-1} \times \mathbb{1}_{\{t \geq t^*\}}$		0.0124
		(0.0539)
Constant	0.117	0.236
	(0.101)	(0.415)
Constant $i_{t-1} \times \mathbb{1}_{\{t \geq t^*\}}$		-0.122
		(0.381)
Observations	202	202

HAC robust standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

TABLE SM.1. Regression table.

where the error term is the monetary policy shock.[5] Hence, an estimate of the first-order autoregressive coefficient in (SM.2) identifies the persistence of the monetary policy shock process.[6] I test for a potential structural break in the persistence of the nominal interest rate process, described by (SM.2), around 1985:Q1. I use GMM and estimate $i_t = \alpha_i + \alpha_{i,*}\mathbb{1}_{\{t \geq t^*\}} + \rho_i i_{t-1} + \rho_{i,*} i_{t-1}\mathbb{1}_{\{t \geq t^*\}} + \boldsymbol{\gamma}\boldsymbol{X}_t + u_t$, where $\boldsymbol{X}_t$ is a set of control variables that includes current and lagged output gap and inflation.[7] I report the results in the first column of table SM.1. I then report its sytuctural break version in the second column. There is no evidence for a decrease in the persistence of the nominal interest rate (and thus, the persistence of the monetary shock process) over time.

This set of results is inconsistent with the NK model, since the model suggests that the empirically documented fall in inflation persistence can only be explained by an *identical* fall in nominal interest rates persistence.

[5]Using the lag operator, I can write the monetary policy shock process (10) as $\nu_t = (1 - \rho L)^{-1}\varepsilon_t^{\nu}$. Introducing this last expression into (9), multiplying by $(1 - \rho L)$ and rearranging terms, I obtain (SM.2).

[6]Our measure of the nominal rate will be the effective Fed Funds rate (EFFR), calculated as a volume-weighted median of overnight federal funds transactions, and is available at a daily frequency. I use the quarterly frequency series.

[7]The instrument set includes four lags of the Effective Fed Funds rate, GDP Deflator, CBO Output Gap, labor share, Commodity Price Inflation, Real M2 Growth, and the spread between the long-term bond rate and the three-month Treasury Bill rate.

Parameter	Description	Value	Source/Target
ρ_a	Technology shock persistence	0.9	Galí (2015)
ρ_u	Cost-push shock persistence	0.8	Galí (2015)
$\sigma_{\varepsilon a}$	Technology innovation pre-1985	1	Galí (2015)
$\sigma_{\varepsilon u}$	Cost-push innovation	1	Galí (2015)

TABLE SM.2. Persistence and Volatility Parameters

Additional Structural shocks. In the model studied above, I have only considered monetary policy shocks. It could be the case that other relevant shocks have lost persistence in recent decades, and could thus explain the fall in the persistence of inflation. I additionally consider demand (technology) and supply (cost-push) shocks. In this case inflation dynamics follow

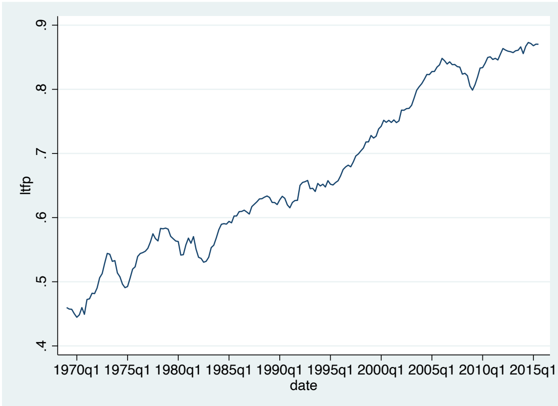
$$\pi_t = \psi_{\pi v} v_t + \psi_{\pi a} a_t + \psi_{\pi u} u_t \tag{SM.3}$$

where a_t is the technology shock, u_t is the cost-push shock, $\psi_{\pi x}$ for $x \in \{v, a, u\}$ are scalars that depend on model parameters, defined in Online Appendix SM.2, and shock processes follow respective AR(1) processes $x_t = \rho_x x_{t-1} + \varepsilon_t^x$. In this framework with multiple shocks, I consider inflation persistence as the first-order autocorrelation coefficient ρ_1 as

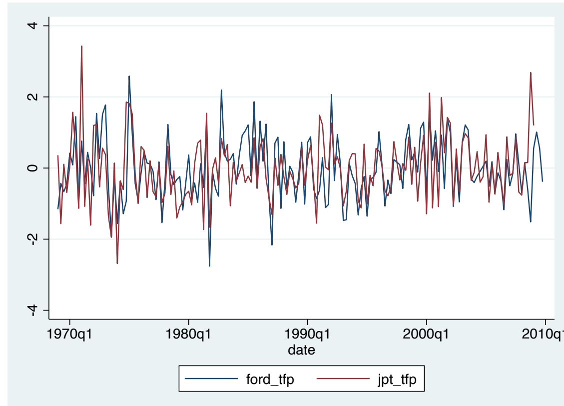
$$\rho_1 = \frac{\rho_v \frac{\psi_{\pi v}^2 \sigma_{\varepsilon v}^2}{1-\rho_v^2} + \rho_a \frac{\psi_{\pi a}^2 \sigma_{\varepsilon a}^2}{1-\rho_a^2} + \rho_u \frac{\psi_{\pi u}^2 \sigma_{\varepsilon u}^2}{1-\rho_u^2}}{\frac{\psi_{\pi v}^2 \sigma_{\varepsilon v}^2}{1-\rho_v^2} + \frac{\psi_{\pi a}^2 \sigma_{\varepsilon a}^2}{1-\rho_a^2} + \frac{\psi_{\pi u}^2 \sigma_{\varepsilon u}^2}{1-\rho_u^2}} \tag{SM.4}$$

Using different measures of technology shocks from Fernald (2014); Francis et al. (2014); Justiniano et al. (2011) and cost-push shocks from Nekarda and Ramey (2020), there is no empirical evidence supporting a fall in their persistence. Additionally, I find that an increase in ϕ_π from 1 to 2, as the one documented by Clarida et al. (2000), can only generate a fall of 0.003% in the first-order autocorrelation (SM.4). Finally, a fall in the volatility of cost-push shocks with respect to the volatility of the other two shocks can explain a fall in the first-order autocorrelation from 0.8 to 0.745 when $\sigma_{\varepsilon u} = 0$, insufficient to explain the documented fall. Therefore, I rule out these explanations.

Technology Shocks. In this section I rely on the vast literature on technology shocks, dating back to Solow (1957); Kydland and Prescott (1982). Early work in the literature generally assumed that a regression on the (log) production function reports residuals that can be interpreted as (log) TFP neutral shocks, as the one discussed in this section. Due to endo-



A. (Log) TFP process from Fernald (2014)



B. Technology shocks from Francis et al. (2014) and Justiniano et al. (2011)

FIGURE SM.1. TFP dynamics

geneity concerns between capital and TFP, the literature moved forward and estimated TFP shocks through different assumptions and methods. In this new wave, Galí (1999) used long-run restrictions to identify neutral technology shocks by assuming that technology shocks are the only that can have permanent effects on labor productivity. Following this idea, Francis et al. (2014) identify technology shocks as the shock that maximizes the forecast error variance share of labor productivity at some horizon. Basu et al. (2006) instead estimate TFP by adjusting the annual Solow residual for utilization (using hours per worker as a proxy), and Fernald (2014) extended the series to quarterly frequency. Finally, Justiniano et al. (2011) obtain technology shocks by estimating a NK model, incorporating other technology-related shocks such as investment-specific technology and marginal efficiency of investment shocks. Ramey (2016) compares the shocks, and shows that the IRFs of standard aggregate variables after the each shock series are similar. In particular, Francis et al. (2014) and Justiniano et al. (2011) produce remarkably similar IRFs of real GDP, hours and consumption.

I plot the different series in Figure SM.1. Notice the difference between the left and right panels: while Fernald (2014) estimates directly (log) technology a_t, Francis et al. (2014); Justiniano et al. (2011) estimate the technology shock ε_t^a. I overcome the difficulty with the estimation of technology persistence by estimating persistence in the natural real interest rate process. In the standard NK model, the natural real rate is given by (OA.39), which can be rewritten using the AR(1) properties of the technology process as

$$r_t^n = \rho_a r_{t-1}^n - \sigma\psi(1-\rho_a)\varepsilon_t^a \tag{SM.5}$$

I use the Federal Reserve estimate of the natural interest rate series, produced by Holston

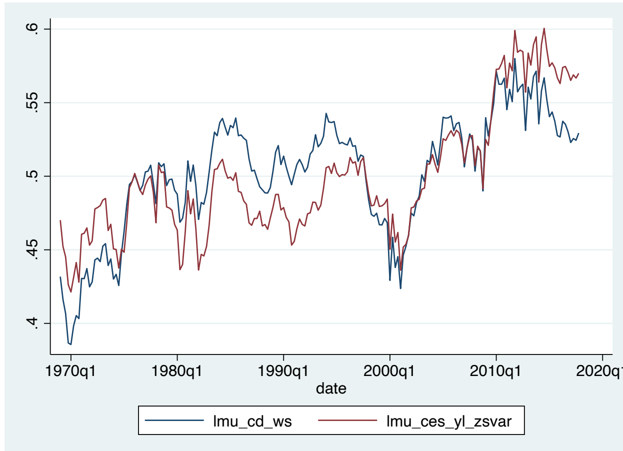


FIGURE SM.2. Markup series

et al. (2017), as the proxy for r_t^n. Table SM.3 reports the results. The first two columns report the (direct) estimate of the technology process (OA.31) persistence and its structural break around 1985:I, while columns three to six report the estimate of the natural real rate process (SM.5) using the technology series constructed by Francis et al. (2014); Justiniano et al. (2011), respectively. I do not find any evidence of a fall in technology persistence over time.

Cost-push Shocks. In the benchmark NK model with monopolistic competition among firms, cost-push shocks are interpreted as the deviation from the desired time-varying price-cost markup, which depends on the elasticity of substitution among good varieties. Nekarda and Ramey (2020) estimate the structural time-varying price-cost markup under a richer framework than the benchmark NK model. In particular, they consider both labor and capital as inputs in the production function. They argue that measured wages are a better indicator for marginal costs than labor compensation, and provide a range of markup measures depending on the elasticity of substitution between capital and labor. As a result, they obtain markup estimates either from labor side or the capital side. Since our model does not include capital, I will rely on the labor-side estimates. Figure SM.2 plots two different measures of the cost-push shock. In the first, the authors rely on a Cobb-Douglas production function in order to estimate the markup, while in the second the authors rely on a CES production function, estimating labor-augmented technology using long-run restrictions as in Galí (1999). I therefore estimate the first-order autocorrelation using these two measures. Our results are reported in table SM.4. Columns one and two report the estimates based on the Cobb-Douglas production function, while columns three to four report the estimates based on the (labor-side) CES production function. I find no evidence

	Technology	Structural Break	Natural rate	Structural Break	Natural rate	Structural Break
(Log) TFP$_{t-1}$	0.998***	0.989***				
	(0.00454)	(0.00853)				
(Log) TFP$_{t-1} \times \mathbb{1}_{\{t \geq t^*\}}$		0.00344				
		(0.00341)				
Natural rate$_{t-1}$			0.951***	0.948***	0.963***	0.961***
			(0.0317)	(0.0326)	(0.0367)	(0.0400)
Technology shock (Francis et al. 2014)			0.0511**	0.0514**		
			(0.0234)	(0.0237)		
Natural rate$_{t-1} \times \mathbb{1}_{\{t \geq t^*\}}$				-0.00645		-0.00345
				(0.0129)		(0.0141)
Technology shock (Justiniano et al. 2011)					0.0191	0.0193
					(0.0278)	(0.0280)
Constant	0.00360	0.00764*	0.128	0.149	0.0878	0.102
	(0.00327)	(0.00438)	(0.0968)	(0.108)	(0.114)	(0.139)
Observations	186	186	163	163	160	160

HAC robust standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

TABLE SM.3. Regression table.

	Cobb-Douglas	Structural Break	CES	Structural Break
Markup_{t-1}	0.945^{***}	0.936^{***}	0.963^{***}	0.947^{***}
	(0.0246)	(0.0309)	(0.0234)	(0.0253)
$\text{Markup}_{t-1} \times \mathbb{1}_{\{t \geq t^*\}}$		0.00246		0.00480
		(0.00444)		(0.00425)
Constant	0.0280^{**}	0.0316^{**}	0.0189	0.0252^{**}
	(0.0125)	(0.0147)	(0.0117)	(0.0121)
Observations	195	195	195	195

HAC robust standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

TABLE SM.4. Regression table

of a change in cost-push persistence over time

SM.1.2. Monetary Stance

I now consider exogenous changes in the reaction function of the monetary authority. Let us first consider the benchmark framework, with inflation dynamics described by (SM.1). I already argued that changes in the policy rule do not affect inflation persistence. Let us now consider extensions of the benchmark model that could explain the fall in inflation persistence.

Indeterminacy. I begin by considering a hypothetical change in monetary policy, conducted via the Taylor rule (9)-(10). The previous literature has considered the possibility of the Fed conducting a passive monetary policy before 1985, which in the lens of the theory would lead to a multiplicity of equilibria. For example, Clarida et al. (2000) document that the inflation coefficient in the Taylor rule was well below one, not satisfying the Taylor principle. Lubik and Schorfheide (2004) estimate an NK model under determinacy and indeterminacy and argue that monetary policy after 1982 is consistent with determinacy, whereas the pre-Volcker policy is not. I study if this change in the monetary stance could have affected inflation persistence.

In order to obtain the model dynamics, I set parameters to the values reported in table 4, with the exception of ϕ_π. For the indeterminate case I set $\phi_{\pi,\text{ind}} = 0.83$, the estimate reported by Clarida et al. (2000). I find that inflation dynamics are more persistent in the indeterminacy region, with an autocorrelation of 0.643, falling to 0.5 in the determinacy region after the mid-1980s.[8] This could explain more than 50% of the overall fall in inflation persistence. Another interesting result is that, even in the case of multiple equilibria arising from non-fundamental sunspot shocks, the first-order autocorrelation coefficient is unique.

Optimal Monetary Policy under Discretion. The second extension that I inspect is an optimal monetary policy under discretion. I show that an increase in ϕ_π can be micro-founded through a change in the monetary stance in which the CB follows a Taylor rule in the pre-1985 period, while it follows optimal monetary policy under discretion in the post-1985 period. In such case, inflation dynamics follow (SM.3) in the pre-1985 period, and $\pi_t = \rho_u \pi_{t-1} + \psi_d \varepsilon_t^u$ in the post-1985 period, where ψ_d is a positive scalar that depends on deep parameters and inflation persistence is inherited from the cost-push shock. Compared to the pre-1985 dynamics, described by (SM.3), there is no significant change in inflation persistence: in the pre-period, model persistence (measured by the first-order autocorrelation of (SM.3)) is around 0.799, while in the post-period persistence is around 0.800. Therefore, such change in the policy stance would have generated an *increase* in inflation persistence, which rules out this explanation.

Optimal Monetary Policy under Commitment. Consider the benchmark NK model with the optimal monetary policy under commitment. Under commitment, the monetary authority can credibly control households' and firms' expectations. In this framework, inflation dynamics are given by $\pi_t = \rho_c \pi_{t-1} + \psi_c \Delta u_t$, where ρ_c and ψ_c are positive scalars that depend on deep parameters, $\Delta u_t \equiv u_t - u_{t-1}$ is the exogenous cost-push shock process, with ρ_c governing inflation intrinsic persistence. Using a standard parameterization I find that $\rho_c = 0.310$, which suggests that this framework, although it produces an excessive fall in inflation persistence, could explain its fall. Its main drawback is that its *implied* Taylor rule in the post-1985 period would require an increase in ϕ_π from 1 to 4.5, as I show in Online Appendix SM.2, which is inconsistent with the documented evidence in table SM.1 Panel A.

Summary. I summarize in table SM.5 the findings in this section, concluding that changes on the monetary stance are either insufficient or inconsistent with empirical evidence.

[8] For the model derivation, I refer the reader to Online Appendix SM.2.

	Model	
Persistence	Pre 1985	Post 1985
Indeterminacy	0.643	0.5
Discretion	0.799	0.800
Commitment	0.799	0.400

TABLE SM.5. Summary

SM.1.3. Intrinsic Persistence

The main reason for the failure in explaining the change in the dynamics in the benchmark NK model is that the endogenous outcome variables, output gap, and inflation, are proportional to the monetary policy shock process and thus inherit its dynamics. This is a result of having a pure forward-looking model, which direct consequence is that endogenous variables are not intrinsically persistent, and their persistence is simply inherited from the exogenous driving force and unaffected by changes in the monetary stance. I, therefore, enlarge the standard NK model to accommodate a backward-looking dimension in the following discussed extensions, including a lagged term in the system of equations.

Price Indexation. I consider a backward-looking inflation framework, "micro-founded" through price indexation. In this framework, a restricted firm resets its price (partially) indexed to past inflation, which generates anchoring in aggregate inflation dynamics. In such a framework, inflation dynamics are given by $\pi_t = \rho_\omega \pi_{t-1} + \psi_\omega v_t$. In this framework inflation intrinsic persistence is increasing in the degree of price indexation ω, as I show in Online Appendix SM.2. A fall in the degree of indexation could explain the fall in inflation persistence. However, the parameterization of such a parameter is not a clear one. Price indexation implies that every price is changed every period, and therefore one could not identify the Calvo-restricted firms in the data and estimate ω. As a result, the parameter is usually estimated using aggregate data and trying to match the anchoring of the inflation dynamics, and its estimate will therefore depend on the additional model equations. Christiano et al. (2005) assume $\omega = 1$. Smets and Wouters (2007) estimate a value of $\omega = 0.21$ trying to match aggregate anchoring in inflation dynamics. It is hard to justify a particular micro estimate for ω since it is unobservable in the micro data.[9] A counterfactual prediction in this framework is that all prices are changed in every period, in contradiction with the empirical findings in Bils and Klenow (2004); Nakamura and Steinsson (2008). As a result, one cannot credibly claim that ω is the cause of the fall in

[9]One would need to identify the firms that were not hit by the Calvo fairy in a given period, yet they change their price.

	Model	
Persistence	Pre 1985	Post 1985
Price indexation	0.90	0.87
Trend inflation	0.91	0.84

TABLE SM.6. Summary

inflation persistence since it needs to be identified from the macro aggregate data, which makes it unfeasible to identify ω and the true inflation persistence separately. Finally, I find that a change in the monetary policy stance has now a significant effect on inflation persistence: a change of ϕ_π from 1 to 2 produces a fall in the first-order autocorrelation of inflation from around 0.895 to 0.865. However, is not enough to produce the effect that I observe in the data.

Trend Inflation. Our last extension is to include trend inflation, for which the literature has documented a fall from 4% in the 1947-1985 period to 2% afterward (see e.g., Ascari and Sbordone 2014; Stock and Watson 2007). Differently from the standard environment, I log-linearize the model equations around a steady state with positive trend inflation, which I assume is constant within eras. Augmenting the model with trend inflation creates intrinsic persistence in the inflation dynamics through relative price dispersion, which is a backward-looking variable that has no first-order effects in the benchmark NK model. Inflation dynamics are now given by $\pi_t = \rho_{\bar{\pi},1}\pi_{t-1} + \rho_{\bar{\pi},2}\pi_{t-2} + \psi_{\bar{\pi},1}\nu_t + \psi_{\bar{\pi},2}\nu_{t-1}$, where persistence is increasing in the level of trend inflation. I, therefore, investigate if the documented fall in trend inflation, coupled with the already discussed change in the monetary stance, can explain the fall in inflation persistence. Although in the correct direction, I find that the fall in trend inflation and the increase in the Taylor rule coefficients produce a small decrease in intrinsic persistence, from 0.91 to 0.84.

Summary. I summarize in table SM.6 the findings in this section, concluding that changes on the monetary stance are either insufficient or inconsistent with empirical evidence.

SM.2. Extensions to the Benchmark New Keynesian Model

SM.2.1. Forward-Looking Models

SM.2.1.1. Benchmark New Keynesian Model

Inserting the Taylor rule (9) into the DIS curve (8), one can write the model as a system of two first-order stochastic difference equations,

$$\boldsymbol{x}_t = \boldsymbol{\delta}\mathbb{E}_t\boldsymbol{x}_{t+1} + \boldsymbol{\varphi}v_t \tag{SM.6}$$

where $\boldsymbol{x} = [y_t \quad \pi_t]^\intercal$ is a 2×1 vector containing output and inflation, $\boldsymbol{\delta}$ is a 2×2 coefficient matrix and $\boldsymbol{\varphi}$ is a 2×1 vector satisfying

$$\boldsymbol{\delta} = \frac{1}{\sigma + \phi_y + \kappa\phi_\pi}\begin{bmatrix} \sigma & 1 - \beta\phi_\pi \\ \sigma\kappa & \kappa + \beta(\sigma + \phi_y) \end{bmatrix}, \qquad \boldsymbol{\varphi} = \frac{1}{\sigma + \phi_y + \kappa\phi_\pi}\begin{bmatrix} 1 \\ \kappa \end{bmatrix}$$

The system of first-order stochastic difference equations (SM.6) can be solved analytically, which is of help for our purpose. In particular, the solution to the above system of equations satisfies $\boldsymbol{x}_t = \Psi v_t$, where $\Psi = [\psi_y \quad \psi_\pi]^\intercal$ with ψ_π defined in (14) and $\psi_y = -\frac{1-\rho\beta}{(1-\rho\beta)[\sigma(1-\rho)+\phi_y]+\kappa(\phi_\pi-\rho)}$.

SM.2.1.2. Accommodating Technology and Cost-push Shocks

In this section, I extend the general model to accommodate cost-push shocks. The demand side is still described by (OA.40), which under the FIRE assumption collapses to

$$\widetilde{y}_t = -\frac{1}{\sigma}(i_t - \mathbb{E}_t\pi_{t+1}) - (1 - \rho_a)\psi a_t + \mathbb{E}_t\widetilde{y}_{t+1} \tag{SM.7}$$

To accommodate cost-push shocks, I allow the elasticity of substitution among goof varieties, ϵ, to vary over time according to some stationary process $\{\epsilon_t\}$. Assuming constant returns to scale in the production function (OA.30) ($\alpha = 0$) for simplicity, the PC becomes

$$\pi_t = \beta\mathbb{E}_t\pi_{t+1} - \lambda\widehat{\mu}_t + \lambda\widehat{\mu}_t^n = \beta\mathbb{E}_t\pi_{t+1} + \kappa\widetilde{y}_t + u_t \tag{SM.8}$$

where $\mu_t^n = \log\frac{\epsilon_t}{\epsilon_t - 1}$ is the time-varying desired markup and $\widehat{\mu}_t^n = \mu_t^n - \mu$. I assume that the exogenous process $u_t = \lambda\widehat{\mu}_t^n$ follows an AR(1) process with autoregressive coefficient ρ_u. Combining (SM.7), (SM.8), (9) and the respective shock processes, I can write the equilibrium conditions as a system of stochastic difference equations

$$\widetilde{A}\boldsymbol{x}_t = \widetilde{B}\mathbb{E}_t\boldsymbol{x}_{t+1} + \widetilde{C}\boldsymbol{w}_t \tag{SM.9}$$

where $\boldsymbol{x}_t = [y_t \quad \pi_t]^\top$ is a 2×1 vector containing output and inflation, $\boldsymbol{w}_t = [v_t \quad a_t \quad u_t]^\top$ is a 3×1 vector containing the monetary, technology and cost-push shocks, $\widetilde{A}$ is a 2×2 coefficient matrix, $\widetilde{B}$ is a 2×2 coefficient matrix and $\widetilde{C}$ is a 2×3 matrix satisfying

$$\widetilde{A} = \begin{bmatrix} \sigma + \phi_y & \phi_\pi \\ -\kappa & 1 \end{bmatrix}, \quad \widetilde{B} = \begin{bmatrix} \sigma & 1 \\ 0 & \beta \end{bmatrix}, \quad \text{and} \quad \widetilde{C} = \begin{bmatrix} -1 & -\sigma(1-\rho_a)\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Premultiplying the system by $\widetilde{A}^{-1}$ I obtain

$$\boldsymbol{x}_t = \boldsymbol{\delta}\mathbb{E}_t\boldsymbol{x}_{t+1} + \boldsymbol{\varphi}\boldsymbol{w}_t \tag{SM.10}$$

where $\boldsymbol{\delta} = \widetilde{A}^{-1}\widetilde{B}$ and $\boldsymbol{\varphi} = \widetilde{A}^{-1}\widetilde{C}$. Notice that $\boldsymbol{w}_t$ follows a VAR(1) process with autorregressive coefficient matrix $R = \operatorname{diag}(\rho_v, \rho_a, \rho_u)$. Using the method for undetermined coefficients, the solution to (SM.10) is conjectured to be of the form $\widetilde{y}_t = \Psi_y\boldsymbol{w}_t$ and $\widetilde{\pi}_t = \Psi_\pi\boldsymbol{w}_t$, where $\Psi_y = [\psi_{yv} \quad \psi_{ya} \quad \psi_{yu}]$ and $\Psi_\pi = [\psi_{\pi v} \quad \psi_{\pi a} \quad \psi_{\pi u}]$. Imposing the conjectured relations into (SM.10) allows one to solve for the undetermined coefficients ψ_{yv}, ψ_{ya}, ψ_{yu}, $\psi_{\pi v}$, $\psi_{\pi a}$ and $\psi_{\pi u}$, which satisfy the condition $\Psi = \boldsymbol{\delta}\Psi R + \boldsymbol{\varphi}$, where $\Psi = [\Psi_y \quad \Psi_\pi]^\top$ is a 2×3 vector containing all the unknown parameters. The solution to the above system of unknown parameters satisfied

$$\psi_{yv} = -\frac{1-\rho_v\beta}{(1-\rho_v\beta)[\sigma(1-\rho_v)+\phi_y]+\kappa(\phi_\pi-\rho_v)}, \quad \psi_{ya} = -\frac{\sigma\psi(1-\rho_a)(1-\rho_a\beta)}{(1-\rho_a\beta)[\sigma(1-\rho_a)+\phi_y]+\kappa(\phi_\pi-\rho_a)}$$
$$\psi_{yu} = -\frac{\phi_\pi-\rho_u}{(1-\rho_u\beta)[\sigma(1-\rho_u)+\phi_y]+\kappa(\phi_\pi-\rho_u)}, \quad \psi_{\pi v} = -\frac{\kappa}{(1-\rho_v\beta)[\sigma(1-\rho_v)+\phi_y]+\kappa(\phi_\pi-\rho_v)}$$
$$\psi_{\pi a} = -\frac{\kappa\sigma\psi(1-\rho_a)}{(1-\rho_a\beta)[\sigma(1-\rho_a)+\phi_y]+\kappa(\phi_\pi-\rho_a)}, \quad \psi_{\pi u} = \frac{\sigma(1-\rho_u)+\phi_y}{(1-\rho_u\beta)[\sigma(1-\rho_u)+\phi_y]+\kappa(\phi_\pi-\rho_u)}$$

and therefore equilibrium dynamics are given by (SM.3) and $\widetilde{y}_t = \psi_{yv}v_t + \psi_{ya}a_t + \psi_{yu}u_t$.

SM.2.1.3. Optimal Monetary Policy under Discretion

Following Galí (2015), the welfare losses experienced by a representative consumer, up to a second-order approximation, are proportional to

$$\mathbb{E}_0 \sum_{k=0}^{\infty} \beta^t \left(\pi_t^2 + \frac{\kappa}{\epsilon} x_t^2 \right) \tag{SM.11}$$

where $x_t \equiv y_t - y_t^e$ is the welfare-relevant output gap, with $y_t^e = \psi a_t$ denoting the (log) efficient level of output. Notice that κ/ϵ regulates the (optimal) relative weight that the social planner (or the monetary authority) assigns to the welfare-relevant output gap. In this case, the DIS can be written as

$$x_t = -\frac{1}{\sigma}(i_t - \mathbb{E}_t \pi_{t+1}) - (1 - \rho_a)\psi a_t + \mathbb{E}_t x_{t+1} \tag{SM.12}$$

I can also rewrite the PC as

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa x_t + u_t \tag{SM.13}$$

where $u_t \equiv \kappa(y_t^e - y_t^n)$. Again, I assume that the cost-push shock follows an AR(1) process with autoregressive coefficient ρ_u. Under discretion, the CB does not control future output gaps or inflation, but just the current measures. Therefore, the monetary authority minimizes $\pi^2 + \frac{\kappa}{\epsilon} x_t^2$ subject to the constraint $\pi_t = \kappa x_t + \xi_t$, where $\xi_t \equiv \beta \mathbb{E}_t \pi_{t+1} + u_t$ is treated as a non-policy shock (one can show that $\mathbb{E}_t \pi_{t+1}$ is a function of future output gaps). The optimality condition is

$$x_t = -\epsilon \pi_t \tag{SM.14}$$

In case of inflationary pressures, the CB will reduce output below its potential, "leaning against the wind". In this case, the welfare-relevant output gap and inflation follow

$$\widetilde{y}_t = -\frac{1 - \rho_u \beta + 2\epsilon\kappa}{\kappa(1 - \rho_u \beta + \epsilon\kappa)} u_t \tag{SM.15}$$

$$\pi_t = \frac{1}{1 - \rho_u \beta + \epsilon\kappa} u_t \tag{SM.16}$$

Using the DIS curve (8) and the optimality conditions (SM.15) and (SM.16), I can reverse-engineer the following Taylor rule, which replicates the optimal allocation under discretion

$$i_t = \frac{\rho_u + \epsilon\sigma(1-\rho_u)}{1-\beta\rho_u + \epsilon\kappa} u_t - (1-\rho_a)\psi a_t = \Psi_i u_t - (1-\rho_a)\psi a_t \tag{SM.17}$$

Unfortunately, such a rule yields multiple equilibria since it does not satisfy the Taylor Principle. However, adding a component $\phi_\pi \left(\pi_t - \frac{1}{1-\rho_u\beta+\epsilon\kappa} u_t\right) = 0$, I can write

$$i_t = \phi_\pi \pi_t + \frac{\epsilon\sigma(1-\rho_u) - (\phi_\pi - \rho_u)}{1-\beta\rho_u + \epsilon\kappa} u_t - (1-\rho_a)\psi a_t = \phi_\pi \pi_t + \Theta_i u_t - (1-\rho_a)\psi a_t \tag{SM.18}$$

Inserting condition (SM.16) to eliminate the cost-push shock yields

(SM.19)

$$i_t = \phi_\pi \pi_t + [\epsilon\sigma(1-\rho_u) - (\phi_\pi - \rho_u)]\pi_t - (1-\rho_a)\psi a_t = \phi_\pi \pi_t + \phi_{\pi,\mathbb{1}_{\{t\geq 1985:I\}}} \pi_t - (1-\rho_a)\psi a_t$$

As a result, one could understand the documented increase in the Taylor rule as a version of optimal discretionary policy. In our benchmark specification I find $\phi_{\pi,\mathbb{1}_{\{t\geq 1985:I\}}} = 0.95$, which aligns well with the data. I already discussed that an increase in ϕ_π does not affect inflation persistence. What if the change in the monetary stance was not a mere increase in the elasticity of nominal rates with respect to inflation, but an additional response to cost-push shocks in the Taylor rule? Recall that, under discretion, inflation dynamics are given by (SM.16), which I can write as

$$\pi_t = \rho_u \pi_{t-1} + \frac{1}{1-\rho_u\beta + \epsilon\kappa} \varepsilon_t^u \tag{SM.20}$$

Compared to the pre-1985 dynamics, described by (SM.3) and disregarding technology shocks for simplicity, inflation persistence would be even larger if $\rho_u > \rho_\nu$, which I have documented in tables SM.1 Panel A and SM.4. That is, the optimal discretionary policy would not explain the fall in inflation persistence, provided that cost-push persistence has been stable throughout the decades, and that cost-push shocks are more persistent than monetary policy shocks, which would have generated an *increase* in inflation persistence.[10]

[10] Including technology shocks in the comparison of (SM.3) and (SM.20) would alter the results, provided that $\rho_a > \rho_u > \rho_\nu$. However, since ρ_u is in between the two other highly persistent parameters and none of them have changed over time, the difference (if any) in reduced-form persistence in (SM.3) and (SM.20) would be small, and would not explain the documented large fall.

SM.2.1.4. Indeterminacy

Consider the standard framework in (SM.6). I have explored inflation dynamics under determinacy. In this section, I uncover the (multiple) stable solutions under indeterminacy, where $\phi_\pi < 1 - \frac{1-\beta}{\kappa}\phi_y$. Following Lubik and Schorfheide (2003), I rewrite the model as $\Gamma_0\xi_t = \Gamma_1\xi_{t-1} + \Psi\varepsilon_t^\nu + \Pi\eta_t$, where $\xi_t = [\xi_t^y \quad \xi_t^\pi \quad \nu_t]^\intercal$, $\eta_t = [\eta_t^y \quad \eta_t^\pi]^\intercal$ and I denote the conditional forecast $\xi_t^x = \mathbb{E}_t x_{t+1}$ and the forecast error $\eta_t^x = x_t - \xi_{t-1}^x$, with

$$\Gamma_0 = \begin{bmatrix} 1 & \frac{1}{\sigma} & -\frac{1}{\sigma} \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad \Gamma_1 = \begin{bmatrix} 1+\frac{\phi_y}{\sigma} & \frac{\phi_\pi}{\sigma} & 0 \\ -\kappa & 1 & 0 \\ 0 & 0 & \rho \end{bmatrix}, \qquad \Psi = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \qquad \Pi = \begin{bmatrix} 1+\frac{\phi_y}{\sigma} & \frac{\phi_\pi}{\sigma} \\ -\kappa & 1 \\ 0 & 0 \end{bmatrix}$$

Premultiplying the system by Γ_0^{-1} I obtain the reduced-form dynamics $\xi_t = \Gamma_1^*\xi_{t-1} + \Psi^*\varepsilon_t^\nu + \Pi^*\eta_t$. Using the Jordan decomposition of $\Gamma_1^* = J\Lambda J^{-1}$, and denoting $w_t = J^{-1}\xi_t$, I can write $w_t = \Lambda w_{t-1} + J^{-1}\Psi^*\varepsilon_t^\nu + J^{-1}\Pi^*\eta_t$. Let the $\boldsymbol{w}_{it}$ denote ith element of w_t, $[J^{-1}\Psi^*]_i$ denote the ith row of $J^{-1}\Psi^*$ and $[J^{-1}\Pi^*]_i$ denote the ith row of $J^{-1}\Pi^*$. Since Λ is a diagonal matrix, the dynamic process can be decomposed into 3 uncoupled AR(1) processes. Define $\mathcal{I}_x$ denote the set of unstable AR(1) processes, and let Ψ_x^J and Π_x^J be the matrices composed of the row vectors $[J^{-1}\Psi^*]_i$ and $[J^{-1}\Pi^*]_i$ such that $i \in \mathcal{I}_x$. Finally, I proceed with a singular value decomposition of the matrix Π_x^J,

$$\Pi_x^J = \begin{bmatrix} U_1 & U_2 \end{bmatrix} \begin{bmatrix} D_{11} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_1^\intercal \\ V_2^\intercal \end{bmatrix} = U_1 D_1 1 V_1^\intercal$$

Lubik and Schorfheide (2003) prove that if there exists a solution in the indeterminacy region, it is of the form $\xi_t = \Gamma_1^*\xi_{t-1} + [\Psi^* - \Pi^* V_1 D_{11}^{-1} U_1^\intercal \Psi_x^J]\varepsilon_t^\nu + \Pi^* V_2(\widetilde{M}\varepsilon_t^\nu + M_\zeta\zeta_t)$. Two aspects deserve a discussion. First, matrices $\widetilde{M}$ and M_ζ do not depend on model parameters, which yields the multiplicity of equilibria. Following Lubik and Schorfheide (2003), I select the equilibrium that produces the same dynamics as the determinate framework on impact.[11] Second, the model features i.i.d sunspot shocks ζ_t that affect equilibrium dynamics.

SM.2.2. Backward-looking New Keynesian Models

The main reason for the failure in explaining the change in the dynamics in the benchmark NK model is that endogenous outcome variables, output gap, and inflation, are proportional to the monetary policy shock process and thus inherit its dynamics. This is a result of

[11] I set $\widetilde{M}$ such that $-V_1 D_{11}^{-1} U_1^\intercal \Psi_x^J + V_2\widetilde{M} = -\psi_\pi$, and M_ζ such that $V_{2,2}\zeta_0 = \psi_\pi\varepsilon_0^\nu$.

having a pure forward-looking model. A direct consequence is that endogenous outcome variables are not intrinsically persistent, and therefore its persistence is simply inherited from the exogenous driving force. In this section, I enlarge the standard NK model to accommodate a backward-looking dimension, including a lagged term $\boldsymbol{x}_{t-1}$ in the system of equations (SM.6).

I do so in two different ways: in the first extension, discussed in section SM.2.2.1, I explore a change in the monetary stance from a passive Taylor rule towards optimal policy under commitment. In the second extension, discussed in section SM.2.2.2, I include price-indexing firms, which introduces anchoring on the supply side. In the third extension, I introduce log-linearize the standard model around a steady state with trend inflation, which endogenously creates anchoring in the demand and supply sides.

SM.2.2.1. Optimal Monetary Policy under Commitment

My first backward-looking framework is the benchmark NK model with the optimal monetary policy under commitment. Under commitment, the monetary authority can credibly control households' and firms' expectations. As a result, the CB program is to minimize (SM.11) subject to the sequence of constraints (SM.13). The optimality conditions from this program yield the following conditions relating to the welfare-relevant output gap and inflation

$$x_0 = -\epsilon\pi_0, \quad x_t = x_{t-1} - \epsilon\pi_t \tag{SM.21}$$

for $t \geq 1$. Notice that these two conditions can be jointly represented as an implicit price-level target

$$x_t = -\epsilon\,\widehat{p}_t \tag{SM.22}$$

where $\widehat{p}_t \equiv p_t - p_{-1}$ is the (log) deviation of the price level from an initial target. Combining the PC (SM.13) and the optimal price level target (SM.22) I obtain a second-order stochastic difference equation $\widehat{p}_t = \gamma\widehat{p}_{t-1} + \gamma\beta\mathbb{E}_t\widehat{p}_{t+1} + \gamma u_t$, where $\gamma = (1+\beta+\epsilon\kappa)^{-1}$. The stationary solution to the above condition satisfies

$$\widehat{p}_t = \delta\,\widehat{p}_{t-1} + \frac{\delta}{1-\beta\delta\rho_u}u_t \tag{SM.23}$$

where $\delta = \frac{1-\sqrt{1-4\beta\gamma^2}}{2\gamma\beta} \in (0,1)$ is the inside root of the following lag polynomial $\mathcal{F}(x) = \gamma\beta x^2 - x + \gamma$. Inserting the price level target (SM.22) into (SM.23), I can write the welfare-relevant output gap in terms of the cost-push shock

$$x_0 = -\frac{\epsilon\delta}{1-\delta\beta\rho_u}u_0, \quad x_t = \delta x_{t-1} - \frac{\epsilon\delta}{1-\delta\beta\rho_u}u_t \tag{SM.24}$$

Notice that (SM.26) can be written in terms of the lag polynomial as $\Delta x_t = -\frac{\epsilon\delta}{1-\delta\beta\rho_u}\frac{1}{1-\delta L}\Delta u_t$, which I can insert back into (SM.21) to obtain inflation dynamics

$$\pi_0 = \frac{\delta}{1-\delta\beta\rho_u}u_0 \quad \pi_t = \delta\pi_{t-1} + \frac{\delta}{1-\delta\beta\rho_u}\Delta u_t \tag{SM.25}$$

Rewriting the output gap dynamics,

$$\widetilde{y}_t = \delta\widetilde{y}_{t-1} - \frac{1-\delta(\beta\rho_u - \kappa\epsilon)}{1-\delta\beta\rho_u}u_t + \frac{\delta}{\kappa}u_{t-1} \tag{SM.26}$$

Just as in the case under discretion, the monetary authority can engineer a Taylor rule that produces the optimal dynamics. Inserting (SM.22), (SM.23) and (SM.26) into the DIS curve (SM.12) I can specify the following Taylor rule,

$$i_t = (1-\delta)(\sigma\epsilon - 1)\widehat{p_t} - \sigma\psi(1-\rho_a)a_t = \phi_p\widehat{p}_t + \xi_t \tag{SM.27}$$

which produces the same allocation as the optimal policy. Inserting (SM.25) in the Taylor rule, I can write

$$\begin{aligned}
i_t &= (1-\delta)(\sigma\epsilon - 1)\widehat{p_t} - \sigma\psi(1-\rho_a)a_t + \phi_\pi\left(\pi_t - \delta\pi_{t-1} - \frac{\delta}{1-\delta\beta\rho_u}\Delta u_t\right) \\
&= \phi_\pi\pi_t + (1-\delta)(\sigma\epsilon - 1)\widehat{p_t} - \phi_\pi\delta\pi_{t-1} - \frac{\phi_\pi\delta}{1-\delta\beta\rho_u}\Delta u_t - \sigma\psi(1-\rho_a)a_t \\
&= \phi_\pi\pi_t + (1-\delta)(\sigma\epsilon - 1)(\pi_t + \widehat{p}_{t-1}) - \phi_\pi\delta\pi_{t-1} - \frac{\phi_\pi\delta}{1-\delta\beta\rho_u}\Delta u_t - \sigma\psi(1-\rho_a)a_t \\
&= \phi_\pi\pi_t + \phi_{\pi,\mathbb{1}_{\{t\geq 1985:I\}}}\pi_t + \phi_{\pi,\mathbb{1}_{\{t\geq 1985:I\}}}\widehat{p}_{t-1} - \phi_\pi\delta\pi_{t-1} - \frac{\phi_\pi\delta}{1-\delta\beta\rho_u}\Delta u_t - \sigma\psi(1-\rho_a)a_t \\
&= \phi_\pi\pi_t + \phi_{\pi,\mathbb{1}_{\{t\geq 1985:I\}}}\pi_t + \phi_{\pi,\mathbb{1}_{\{t\geq 1985:I\}}}\widehat{p}_{t-1} - \phi_\pi\delta\pi_{t-1} - \frac{\phi_\pi\delta}{1-\delta\beta\rho_u}\Delta u_t + \xi_t
\end{aligned}$$

where ξ_t is an AR(1) process. My standard parameterization, reported in table 4, suggests $\phi_{\pi,\mathbb{1}_{\{t\geq 1985:I\}}} = 3.56$, which is excessive considering my previous empirical findings. To confirm this, I estimate the above Taylor rule.

Table SM.7 reports our results. Columns one and two repeat our previous exercise but assume no response to output gap deviations. Columns three to four report the estimates

	(1) Taylor rule	(2) SB	(3) Optimal MP (CD)	(4) Optimal MP (CES)
π_t	1.389***	1.247***	1.173***	1.169***
	(0.0659)	(0.0730)	(0.0724)	(0.0727)
$\pi_t \times \mathbb{1}_{\{t \geq t^*\}}$		0.553***	2.065**	2.018**
		(0.152)	(0.944)	(0.986)
$\pi_{t-1} \times \mathbb{1}_{\{t \geq t^*\}}$			0.581	0.598
			(0.763)	(0.752)
$p_t \times \mathbb{1}_{\{t \geq t^*\}}$			-0.00252***	-0.00243***
			(0.000794)	(0.000830)
$u_t \times \mathbb{1}_{\{t \geq t^*\}}$			-1.148*	-1.057
			(0.629)	(0.688)
Observations	203	203	192	192

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

TABLE SM.7. Regression table

of the optimal Taylor rule under commitment, using Nekarda and Ramey (2020) estimates of markups. Our results support the notion that the Fed included the price level and the cost-push shock in its Taylor rule. However, the results are inconsistent with the theory, since the increase in the inflation coefficient and the increase in the price level coefficients are of opposite signs. Additionally, the change in the inflation coefficient is still far from the model-implied change that supports a commitment rule.

SM.2.2.2. Price Indexation

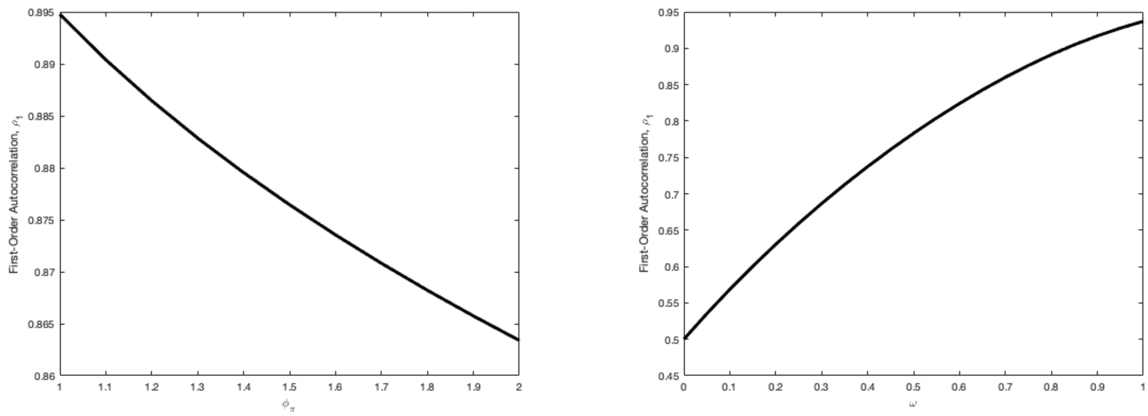
Consider a backward-looking version of the PC, micro-founded through price indexation at the firm level and governed by ω

$$\pi_t = \frac{\omega}{1+\beta\omega}\pi_{t-1} + \frac{\kappa}{1+\beta\omega}\widetilde{y}_t + \frac{\beta}{1+\beta\omega}\mathbb{E}_t\pi_{t+1} \tag{SM.28}$$

The rest of the model equations are the same as in the benchmark model, (8), (9), and (10). The model derivation is relegated to Online Appendix SM.3, and the parameterization is identical to that of table 4, with the model enlarged by the price-indexation parameter ω. The parameterization of such a parameter is not a clear one. As I show below, price

Parameter	Description	Value	Source/Target
ω	Price indexation	0.75	Range literature

TABLE SM.8. Model Parameters



A. Change in ϕ_π B. Change in ω

FIGURE SM.3. Inflation first-order autocorrelation in the backward-looking NK model

indexation implies that every price is changed every period, and therefore one could not identify the Calvo-restricted firms in the data and estimate ω. As a result, the parameter is usually estimated using aggregate data and trying to match the anchoring of the inflation dynamics, and its estimate will therefore depend on the additional model equations. I set $\omega = 0.75$, which is in the range of the literature (0.21 in Smets and Wouters (2007), 1 in Christiano et al. (2005)).

The model can be collapsed to a system of three second-order stochastic difference equations $\boldsymbol{x}_t = \boldsymbol{\Gamma_b}\boldsymbol{x}_{t-1} + \boldsymbol{\Gamma_f}\mathbb{E}_t\boldsymbol{x}_{t+1} + \boldsymbol{\Lambda}\nu_t$, where $\boldsymbol{x}_t = [y_t \quad \pi_t]^\intercal$. The solution of the above system satisfies $\boldsymbol{x}_t = A\boldsymbol{x}_{t-1} + \Psi\nu_t$, where both matrices $A(\phi_\pi, \Phi)$ and $\Psi(\phi_\pi, \Phi)$ depend now on ϕ_π and the rest of the model parameters Φ. Notice that a key difference between the benchmark model and this backward-looking version is that a change in ϕ_π will affect inflation persistence, and could therefore explain the fall in inflation persistence.

In Figure SM.3A I show that a change in the monetary policy stance has now a significant effect on inflation persistence: a change of ϕ_π from 1 to 2, as I have documented in table SM.1 Panel A, produces a fall in the first-order autocorrelation of inflation from around 0.895 to 0.865. However, is not enough to produce the effect that I observe in the data. The target now is to find a candidate parameter that can explain the observed loss in inflation persistence. The ideal candidate is ω, since this term produces anchoring in the PC (SM.28). As I show in Figure SM.3B, as ω decreases so does inflation persistence.

I show in figure SM.3B that the decrease in ω from 1 (full indexation) to 0 (no indexation) produces a factual fall in inflation persistence, and I would be back to the standard model with no indexation. The model is indeed successful in reducing persistence. The natural question is then: what is ω? Does a fall from 1 to 0 makes sense? In the backward-looking NK model, a firm i that is unable to reset its (log) price gets to reset its price to

$$p_{it} = p_{it-1} + \omega\pi_{t-1} \tag{SM.29}$$

The presence of the term $\omega\pi_{t-1}$ is what gives anchoring. What is the value of ω in the literature? Christiano et al. (2005) assume $\omega = 1$. Smets and Wouters (2007) estimate a value of $\omega = 0.21$ trying to match aggregate anchoring in inflation dynamics. The main problem here is that it is hard to justify a particular micro estimate for ω, since it is unobservable in the microdata. One would need to identify the firms that were not hit by the Calvo fairy in a given period and then regress (SM.29). However, the price indexation suggests that all prices are changed in every period, which makes it unfeasible to identify the Calvo-restricted firms. Another aspect in which $\omega > 0$ is inconsistent with the micro-data is that it implies that all prices change every period, in contradiction with Bils and Klenow (2004); Nakamura and Steinsson (2008). As a result, one cannot claim that ω is the cause of the fall in inflation persistence, since it needs to be identified from the macro aggregate data, which makes it unfeasible to separately identify ω and the true inflation persistence.

I, therefore, conclude that extending the benchmark framework to price indexation does not have the quantitative bite to explain the fall in inflation persistence, although the estimates move in the correct direction.

SM.2.2.3. Trend Inflation

Although it is well known that CBs' objective is to have a stable inflation rate around 2%, most New Keynesian models are log-linearized around a zero inflation steady state since the optimal steady-state level of inflation is 0%. Ascari and Sbordone (2014) extend the benchmark model to account for trend inflation. The non-linear model is identical to the one presented in the previous section. Differently from the standard environment, they log-linearize the model around a steady with a certain level of trend inflation $\bar{\pi}$, which is constant over time. Price dispersion, a backward-looking variable that has no first-order effects in the benchmark NK model, is now relevant for the trend NK model. Augmenting the model with trend inflation creates intrinsic persistence in the inflation dynamics through relative price dispersion. The model, similar to the one in Ascari and Sbordone (2014), is derived in Online Appendix SM.3. The model can now be summarized

Parameter	Description	Value	Source/Target
$\bar{\pi}$	Trend inflation	$1.02^{1/4}$ – $1.04^{1/4}$	Ascari and Sbordone (2014)

TABLE SM.9. Model Parameters

as a system of six equations, including (8), (9) and (10), with the additional inclusion of the price dispersion dynamics (SM.30)

$$s_t = \frac{\epsilon}{1-\alpha}\frac{\delta-\chi}{1-\chi}\pi_t - \frac{\omega\epsilon}{1-\alpha}\frac{\delta-\chi}{1-\chi}\pi_{t-1} + \delta s_{t-1} \tag{SM.30}$$

and the PC, which is now given by the system

$$\begin{aligned}\pi_t &= \kappa_\pi \pi_{t-1} + \kappa_\psi \psi_t + \kappa_y y_t + \beta_\psi \mathbb{E}_t \psi_{t+1} + \beta_\pi \mathbb{E}_t \pi_{t+1} \\ \psi_t &= (1-\beta\delta)\varphi s_t + \frac{1+\varphi}{1-\alpha}(1-\beta\delta)\, y_t - \frac{\omega\epsilon}{1-\alpha}\beta\delta\pi_t + \beta\delta\mathbb{E}_t\psi_{t+1} + \frac{\epsilon}{1-\alpha}\beta\delta\mathbb{E}_t\pi_{t+1}\end{aligned} \tag{SM.31}$$

where $\Theta = \frac{1-\alpha}{1-\alpha+\epsilon\alpha}$, $\delta(\bar{\pi}) = \theta\bar{\pi}^{\frac{\epsilon(1-\omega)}{1-\alpha}}$ and $\chi(\bar{\pi}) = \theta\bar{\pi}^{(\epsilon-1)(1-\omega)}$, $\kappa_\pi = \frac{\omega}{1-\omega[\Theta(\epsilon-1)\beta(1-\chi)-\beta\chi]}$, $\kappa_\psi = \frac{\Theta(1-\chi)}{\chi\{1-\omega[\Theta(\epsilon-1)\beta(1-\chi)-\beta\chi]\}}$, $\kappa_y = -\frac{\Theta(1-\sigma)(1-\chi)(1-\beta\chi)}{\chi\{1-\omega[\Theta(\epsilon-1)\beta(1-\chi)-\beta\chi]\}}$, $\beta_\psi = -\frac{\Theta\beta(1-\chi)}{1-\omega[\Theta(\epsilon-1)\beta(1-\chi)-\beta\chi]}$ and $\beta_\pi = -\frac{\Theta(\epsilon-1)\beta(1-\chi)-\beta\chi}{1-\omega[\Theta(\epsilon-1)\beta(1-\chi)-\beta\chi]}$. The parameterization is identical to that of tables 4 and SM.9, extended to trend inflation between 0%-6%, except for the value of $\phi_y = 0$ which is bounded from above by the determinacy conditions. The model can be collapsed to a system of four second-order stochastic difference equations $\boldsymbol{x}_t = \boldsymbol{\Gamma_b}\boldsymbol{x}_{t-1} + \boldsymbol{\Gamma_f}\mathbb{E}_t\boldsymbol{x}_{t+1} + \boldsymbol{\Lambda}\nu_t$, where $\boldsymbol{x}_t = [y_t \quad \pi_t \quad \psi_t \quad s_t]^\intercal$. The solution of the above system satisfies $\boldsymbol{x}_t = A\boldsymbol{x}_{t-1} + \Psi\nu_t$, where both matrices $A(\phi_\pi, \bar{\pi}, \Phi)$ and $\Psi(\phi_\pi, \bar{\pi}, \Phi)$ depend now on ϕ_π, trend inflation $\bar{\pi}$, and the rest of the model parameters Φ. In this framework, I define s_t as (log) price dispersion at time t, and ψ_t as the present discounted value of future marginal costs. Notice that I have extended an otherwise standard trend-inflation NK model with price indexation (governed by ω) as in (SM.29). Even in the zero-indexation case, there will be anchoring coming from the price dispersion equation, which is the only backward-looking equation in the system. To see this, under zero-indexation, inflation dynamics are given by

$$\pi_t = a_s s_{t-1} + b_\pi \nu_t = \left(a_s \frac{\epsilon}{1-\alpha}\frac{\delta-\chi}{1-\chi} + \delta\right)\pi_{t-1} + b_\pi(\nu_t - \delta\nu_{t-1})$$

In the price-indexation case, inflation dynamics are given by

$$\pi_t = a_\pi\pi_{t-1} + a_s s_{t-1} + b_\pi\nu_t = \left(a_\pi + \delta + a_s\frac{\epsilon}{1-\alpha}\frac{\delta-\chi}{1-\chi}\right)\pi_{t-1} - \left(a_\pi\delta + a_s\frac{\omega\epsilon}{1-\alpha}\frac{\delta-\chi}{1-\chi}\right)\pi_{t-2} + b_\pi(\nu_t - \delta\nu_{t-1})$$

Most importantly, one can see that the parameter that governs anchoring (and persistence) in the system, δ in (SM.30), is increasing in the level of trend inflation $\bar{\pi}$. This framework, therefore, has the potential of explaining the fall in inflation persistence *if* trend inflation had fallen. Stock and Watson (2007) and Ascari and Sbordone (2014) provide evidence of a fall of trend inflation from 4% in the 1969-1985 period to 2% afterwards. They estimate trend inflation using a Bayesian VAR with time-varying coefficients (figure 3 in Ascari and Sbordone 2014). Importantly, they find that their estimated trend inflation is correlated (0.96) with the 10-year inflation expectations reported in the Survey of Professional Forecasters (after 1981).

As I argued before, a fall in the trend inflation $\bar{\pi}$ would decrease $\delta(\bar{\pi})$ and thus reduce aggregate anchoring in the system. I, therefore, investigate if such a fall, together with the already discussed change in ϕ_π, can explain the documented fall in inflation persistence.

I compute the first-order autocorrelation of inflation for values of $(\phi_\pi, \bar{\pi}) \in [1.2, 2] \times [0\%, 6\%]$ in the trend inflation model with price indexation. I plot our results in figure SM.4. As I previewed above, the decrease in trend inflation documented by Ascari and Sbordone (2014) can explain (part of) the fall in persistence. In particular, a fall in trend inflation from 6% to 2% (holding $\phi_\pi = 1.5$ constant) produces a fall in inflation persistence from 0.887 to 0.851. Similarly, an increase in the aggressiveness towards inflation from 1 to 2 (Clarida et al. 2000), holding $\bar{\pi} = 2\%$ constant, produces a fall in inflation persistence from 0.879 to 0.845. Jointly, they produce a fall from 0.912 to 0.845. Although in the correct direction, the trend inflation model lacks enough quantitative bite to produce the large fall documented in table 2, panel A. I, therefore, conclude that extending the benchmark framework to trend inflation and price indexation does not explain the fall in inflation persistence, although the estimates move in the correct direction.

SM.3. Derivation of the FIRE Trend-Inflation New Keynesian Model

SM.3.1. Firms

As in the benchmark NK model, price rigidities take the form of Calvo-lottery friction. At every period, each firm is able to reset its price with probability $(1 - \theta)$, independent of the time of the last price change. However, a firm that is unable to re-optimize gets to reset its price to a partial indexation on past inflation. Formally, $P_{jt} = P_{j,t-1}\Pi_{t-1}^{\omega}$, where ω is the elasticity of prices with respect to past inflation. As a result, a firm that last reset its price in period t will face a nominal price in period $t + k$ of $P_t^* \chi_{t,t+k}$, where

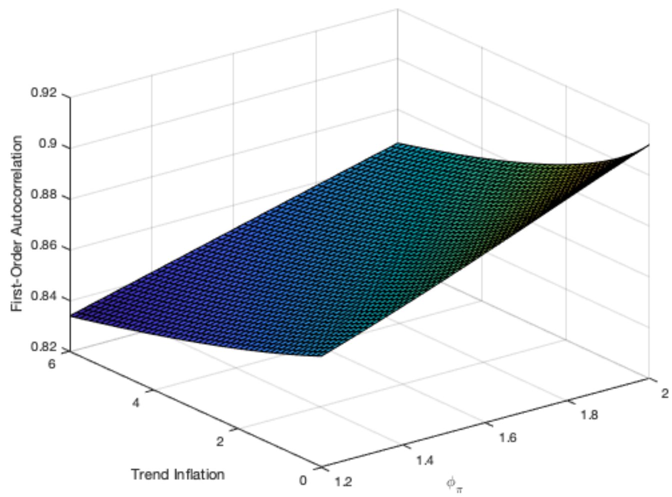


FIGURE SM.4. First-order autocorrelation for values $(\phi_\pi, \bar{\pi}) \in [1.2, 2] \times [0\%, 6\%]$

$chi_{t,t+k} = \begin{cases} \Pi_t^{\omega}\Pi_{t+1}^{\omega}\Pi_{t+2}^{\omega}\cdots\Pi_{t+k-1}^{\omega} & \text{if } k \geq 1 \\ 1 & \text{if } k = 0 \end{cases}$. Such an environment implies that aggregate price level dynamics are given by $P_t = \left[\theta\Pi_{t-1}^{(1-\epsilon)\omega}P_{t-1}^{1-\varepsilon} + (1-\theta)(P_{jt}^*)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}$. Dividing by P_t and rearranging terms, I can write $\frac{P_{jt}}{P_t} = \left[\frac{1-\theta\Pi_{t-1}^{(1-\epsilon)\omega}\Pi_t^{\epsilon-1}}{1-\theta}\right]^{\frac{1}{1-\epsilon}}$. Log-linearizing the above expression around a steady state with trend inflation I obtain

$$p_{jt}^* - p_t = \frac{\theta\bar{\pi}^{(\epsilon-1)(1-\omega)}}{1-\theta\bar{\pi}^{(\epsilon-1)(1-\omega)}}(\pi_t - \omega\pi_{t-1}) \tag{SM.32}$$

Optimal Price Setting. A firm re-optimizing in period t will choose the price P_{jt}^* that maximizes the current market value of the profits generated while the price remains effective. Formally, $P_{jt}^* = \arg\max_{P_{jt}} \sum_{k=0}^{\infty} \theta^k \mathbb{E}_{jt}\left\{\Lambda_{t,t+k}\frac{1}{P_{t+k}}\left[P_{jt}\chi_{t,t+k}Y_{j,t+k|t} - \mathcal{C}_{t+k}(Y_{j,t+j|t})\right]\right\}$ subject to the sequence of demand schedules $Y_{j,t+k|t} = \left(\frac{P_{jt}\chi_{t,t+k}}{P_{t+k}}\right)^{-\epsilon} C_{t+k}$, where $\Lambda_{t,t+k} \equiv \beta^k\left(\frac{C_{t+k}}{C_t}\right)^{-\sigma}$ is the stochastic discount factor, $\mathcal{C}_t(\cdot)$ is the (nominal) cost function where $\mathcal{C}_{t+k} = W_{t+k}N_{j,t+k|t} = W_{t+k}\left(\frac{Y_{j,t+k|t}}{A_{t+k}}\right)^{\frac{1}{1-\alpha}}$ and $Y_{j,t+k|t}$ denotes output in period $t+k$ for a firm j that last reset its price in period t. The First-Order Condition is

$$\sum_{k=0}^{\infty}\theta^k\mathbb{E}_{jt}\left\{\Lambda_{t,t+k}\left[(1-\epsilon)(P_{jt}^*)^{-\epsilon}\left(\frac{\chi_{t,t+k}}{P_{t+k}}\right)^{1-\epsilon}Y_{j,t+k|t} + \frac{\epsilon}{1-\alpha}(P_{jt}^*)^{\frac{\alpha-1-\epsilon}{1-\alpha}}\frac{W_{t+k}}{P_{t+k}}\left(\frac{Y_{j,t+k|t}}{A_{t+k}}\right)^{\frac{1}{1-\alpha}}\left(\frac{\chi_{t,t+k}}{P_{t+k}}\right)^{-\frac{\epsilon}{1-\alpha}}\right]\right\} = 0$$

where $\Psi_{j,t+k|t} \equiv \mathcal{C}^{\intercal}_{t+k}(Y_{j,t+j|t})$ denotes the (nominal) marginal cost for firm j, $\Psi_{j,t+k|t} = \frac{1}{1-\alpha}A_{t+k}^{-\frac{1}{1-\alpha}}W_{t+k}Y_{j,t+k|t}^{\frac{\alpha}{1-\alpha}}$. The FOC can be rewritten as

$$(P^*_{it})^{\frac{1-\alpha+\epsilon\alpha}{1-\alpha}} = \mathcal{M}\frac{1}{1-\alpha}\frac{\mathbb{E}_t\sum_{k=0}^{\infty}\theta^k\Lambda_{t,t+k}\frac{W_{t+k}}{P_{t+k}}\left(\frac{Y_{j,t+k|t}}{A_{t+k}}\right)^{\frac{1}{1-\alpha}}\left(\frac{\chi_{t,t+k}}{P_{t+k}}\right)^{-\frac{\epsilon}{1-\alpha}}}{\mathbb{E}_t\sum_{k=0}^{\infty}\theta^k\Lambda_{t,t+k}\left(\frac{\chi_{t,t+k}}{P_{t+k}}\right)^{1-\epsilon}Y_{j,t+k|t}}$$

where $\mathcal{M} = \frac{\epsilon}{\epsilon-1}$. Diving the above expression by $P_t^{\frac{1-\alpha+\epsilon\alpha}{1-\alpha}} = P_t^{1-\epsilon+\frac{\epsilon}{1-\alpha}} = P_t^{1-\epsilon}P_t^{\frac{\epsilon}{1-\alpha}}$,

(SM.33)

$$\left(\frac{P^*_{it}}{P_t}\right)^{\frac{1-\alpha+\epsilon\alpha}{1-\alpha}} = \mathcal{M}\frac{1}{1-\alpha}\frac{\mathbb{E}_t\sum_{k=0}^{\infty}\theta^k\Lambda_{t,t+k}\frac{W_{t+k}}{P_{t+k}}\left(\frac{Y_{j,t+k|t}}{A_{t+k}}\right)^{\frac{1}{1-\alpha}}\left(\chi_{t,t+k}^{-\frac{1-\omega}{\omega}}\Pi_t\right)^{-\frac{\epsilon}{1-\alpha}}}{\mathbb{E}_t\sum_{k=0}^{\infty}\theta^k\Lambda_{t,t+k}\left(\chi_{t,t+k}^{-\frac{1-\omega}{\omega}}\Pi_t\right)^{1-\epsilon}Y_{j,t+k|t}} = \frac{\mathcal{M}}{1-\alpha}\frac{\Psi_t}{\Phi_t}$$

where the auxiliary variables are defined, recursively, as

(SM.34)

$$\Psi_t \equiv \mathbb{E}_t\sum_{j=0}^{\infty}(\beta\theta)^k Y_{j,t+k|t}^{\frac{1-\sigma(1-\alpha)}{1-\alpha}}A_{t+k}^{-\frac{1}{1-\alpha}}\frac{W_{t+k}}{P_{t+k}}\left(\chi_{t,t+k}^{-\frac{1-\omega}{\omega}}\Pi_t\right)^{-\frac{\epsilon}{1-\alpha}} = \frac{W_t}{P_t}A_t^{-\frac{1}{1-\alpha}}Y_{jt|t}^{\frac{1-\sigma(1-\alpha)}{1-\alpha}} + \beta\theta\Pi_t^{-\frac{\epsilon\omega}{1-\alpha}}\mathbb{E}_t\left[\Pi_{t+1}^{\frac{\epsilon}{1-\alpha}}\Psi_{t+1}\right]$$

(SM.35)

$$\Phi_t \equiv \mathbb{E}_t\sum_{j=0}^{\infty}(\beta\theta)^k Y_{j,t+k|t}^{1-\sigma}\left(\chi_{t,t+k}^{-\frac{1-\omega}{\omega}}\Pi_t\right)^{1-\epsilon} = Y_{jt|t}^{1-\sigma} + \beta\theta\Pi_t^{\omega(1-\epsilon)}\mathbb{E}_t\left[\Pi_{t+1}^{\epsilon-1}\Phi_{t+1}\right]$$

Log-linearizing (SM.33), (SM.34) and around a steady state with trend inflation yields, respectively

(SM.36)

$$\psi_t - \phi_t = \frac{1-\alpha+\epsilon\alpha}{1-\alpha}(p^*_{jt} - p_t)$$

(SM.37)

$$\psi_t = \left[1-\theta\beta\bar{\pi}^{\frac{\epsilon(1-\rho)}{1-\alpha}}\right]\left(w^r_t - \frac{1}{1-\alpha}a_t + \frac{1-\sigma(1-\alpha)}{1-\alpha}y_t\right) + \theta\beta\bar{\pi}^{\frac{\epsilon(1-\omega)}{1-\alpha}}\left(\mathbb{E}_t\psi_{t+1} + \frac{\epsilon}{1-\alpha}\mathbb{E}_t\pi_{t+1} - \frac{\omega\epsilon}{1-\alpha}\pi_t\right)$$

(SM.38)

$$\phi_t = \left[1-\theta\beta\bar{\pi}^{(\epsilon-1)(1-\omega)}\right](1-\sigma)y_t + \theta\beta\bar{\pi}^{(\epsilon-1)(1-\omega)}\left[\omega(1-\epsilon)\pi_t + \mathbb{E}_t\phi_{t+1} + (\epsilon-1)\mathbb{E}_t\pi_{t+1}\right]$$

SM.3.2. Equilibrium

Market clearing in the goods market implies that $Y_{jt} = C_{jt} = \int_{\mathcal{I}_h} C_{ijt}\, di$ for each j good/firm. Aggregating across firms, I obtain the aggregate market clearing condition: since assets are in zero net supply and there is no capital, investment, government consumption nor net exports, production equals consumption: $\int_{\mathcal{I}_f} Y_{jt}\, dj = \int_{\mathcal{I}_h}\int_{\mathcal{I}_f} C_{ijt}\, dj\, di \implies Y_t = C_t$. Aggregate employment is given by the sum of employment across firms, and must meet aggregate labor supply $N_t = \int_{\mathcal{I}_h} N_{it}\, di = \int_{\mathcal{I}_f} N_{jt}\, dj$. Using the production function and optimal good consumption, together with goods market clearing

$$\text{(SM.39)} \qquad N_t = \int_{\mathcal{I}_f} \left(\frac{Y_{jt}}{A_t}\right)^{\frac{1}{1-\alpha}} dj = \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} \int_{\mathcal{I}_f} \left(\frac{P_{jt}}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}} dj = \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} S_t$$

where S_t is a measure of price dispersion and is bounded below one (see Schmitt-Grohe and Uribe (2005)). Price dispersion can be understood as the resource costs coming from price dispersion: the smaller S_t, the larger labor amount is necessary to achieve a particular level of production. In the benchmark model with no trend inflation, $\Pi = \bar{\pi} = 1$ and S_t does not affect real variables up to the first order. Schmitt-Grohe and Uribe (2005) show that relative price dispersion can be written as

$$\text{(SM.40)} \qquad S_t = (1-\theta)\left(\frac{P^*_{jt}}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}} + \theta \Pi_{t-1}^{-\frac{\epsilon\omega}{1-\alpha}} \Pi_t^{\frac{\epsilon}{1-\alpha}} S_{t-1}$$

Log-linearizing (SM.39) and (SM.40) around a steady state with trend inflation I can write, respectively

$$\text{(SM.41)} \qquad n_t = s_t + \frac{1}{1-\alpha}(y_t - a_t)$$

$$\text{(SM.42)} \qquad s_t = -\frac{\epsilon}{1-\alpha}\left(1-\theta\bar{\pi}^{\frac{\epsilon(1-\omega)}{1-\alpha}}\right)(p^*_{jt} - p_t) + \theta\bar{\pi}^{\frac{\epsilon(1-\omega)}{1-\alpha}}\left(-\frac{\epsilon\omega}{1-\alpha}\pi_{t-1} + \frac{\epsilon}{1-\alpha}\pi_t + s_{t-1}\right)$$

Aggregate DIS and Phillips Curves. Combining the intratemporal labor supply condition and the production function (SM.41), I can write real wages as

$$\text{(SM.43)} \qquad w^r_t = \varphi s_t + \frac{\varphi + \sigma(1-\alpha)}{1-\alpha} y_t - \frac{\varphi}{1-\alpha} a_t$$

Combining the optimal price setting rule (SM.36) and the aggregate price level dynamics condition (SM.32), denoting $\Delta_t = \pi_t - \omega\pi_{t-1}$, I can write ϕ_t in terms of Δ_t,

$$\phi_t = \psi_t - \frac{1-\alpha+\epsilon\alpha}{1-\alpha}\frac{\theta\bar{\pi}^{(\epsilon-1)(1-\omega)}}{1-\theta\bar{\pi}^{(\epsilon-1)(1-\omega)}}\Delta_t \tag{SM.44}$$

Combining the price dispersion dynamics (SM.42) and the aggregate price level dynamics condition (SM.32), I can write current price dispersion as a backward-looking equation in inflation and price dispersion. This equation, which does not affect real variables in the benchmark model, will be key in order to generate anchoring,

$$\begin{aligned} s_t &= -\frac{\epsilon}{1-\alpha}\left(1-\theta\bar{\pi}^{\frac{\epsilon(1-\omega)}{1-\alpha}}\right)\frac{\theta\bar{\pi}^{(\epsilon-1)(1-\omega)}}{1-\theta\bar{\pi}^{(\epsilon-1)(1-\omega)}}\Delta_t + \theta\bar{\pi}^{\frac{\epsilon(1-\omega)}{1-\alpha}}\left(\frac{\epsilon}{1-\alpha}\Delta_t + s_{t-1}\right) \\ &= -\frac{\epsilon}{1-\alpha}\left[\left(1-\theta\bar{\pi}^{\frac{\epsilon(1-\omega)}{1-\alpha}}\right)\frac{\theta\bar{\pi}^{(\epsilon-1)(1-\omega)}}{1-\theta\bar{\pi}^{(\epsilon-1)(1-\omega)}} - \theta\bar{\pi}^{\frac{\epsilon(1-\omega)}{1-\alpha}}\right]\Delta_t + \theta\bar{\pi}^{\frac{\epsilon(1-\omega)}{1-\alpha}}s_{t-1} = \frac{\epsilon}{1-\alpha}\frac{\delta-\chi}{1-\chi}\Delta_t + \delta s_{t-1} \end{aligned}$$

where $\delta(\bar{\pi}) = \theta\bar{\pi}^{\frac{\varepsilon(1-\omega)}{1-\alpha}}$, $\chi(\bar{\pi}) = \theta\bar{\pi}^{(\varepsilon-1)(1-\omega)}$. Inserting the real wage equation (SM.43) into the net present value of marginal costs (SM.37)

$$\begin{aligned} \psi_t &= \left[1-\theta\beta\bar{\pi}^{\frac{\epsilon(1-\rho)}{1-\alpha}}\right]\left[\varphi s_t + \frac{1+\varphi}{1-\alpha}(y_t - a_t)\right] + \theta\beta\bar{\pi}^{\frac{\epsilon(1-\omega)}{1-\alpha}}\left(\mathbb{E}_t\psi_{t+1} + \frac{\epsilon}{1-\alpha}\mathbb{E}_t\Delta_{t+1}\right) \\ &= (1-\beta\delta)\left[\varphi s_t + \frac{1+\varphi}{1-\alpha}(y_t - a_t)\right] + \beta\delta\left(\mathbb{E}_t\psi_{t+1} + \frac{\epsilon}{1-\alpha}\mathbb{E}_t\Delta_{t+1}\right) \end{aligned}$$

Finally, introducing (SM.44) into (SM.38), I can write the New Keynesian PC,

$$\Delta_t = \Theta\frac{1-\chi}{\chi}\psi_t - \Theta(1-\sigma)\frac{(1-\chi)(1-\beta\chi)}{\chi}y_t - \Theta\beta(1-\chi)\mathbb{E}_t\psi_{t+1} - \left[\Theta(\epsilon-1)\beta(1-\chi) - \beta\chi\right]\mathbb{E}_t\Delta_{t+1}.$$

Monetary Authority. The model is closed through a CB reaction function. Following Taylor (1993, 1999) I model the reaction function in terms of elasticities. The CB reacts to excess inflation and output gap through a set of parameters $\{\phi_\pi, \phi_y\}$. On top of that, the monetary authority controls an exogenous component, the monetary policy shock $\varepsilon_t^\nu \sim \mathcal{N}(0, \sigma_\nu^2)$ that are serially uncorrelated. Formally, I can write the Taylor rule as

$$i_t = \rho_i i_{t-1} + (1-\rho_i)(\phi_\pi\pi_t + \phi_y y_t) + \varepsilon_t^\nu \tag{SM.45}$$

Steady State. In steady state, the model exhibits trend inflation. The model consists of 5 equations and 5 variables, which can be written in steady-state as

$$
\begin{aligned}
Y &= \left[\frac{(\epsilon-1)(1-\alpha)A^{\frac{1+\varphi}{1-\alpha}}}{\epsilon S^{\varphi}}\right]^{\frac{1-\alpha}{\varphi+\sigma+\alpha(1-\sigma)}} = \left[\frac{(\epsilon-1)(1-\alpha)}{\epsilon S^{\varphi}}\right]^{\frac{1-\alpha}{\varphi+\sigma+\alpha(1-\sigma)}} \\
\Pi &= \bar{\pi} \\
1+i &= \frac{\bar{\pi}}{\beta} \\
\Psi &= \frac{S^{\varphi}A^{-\frac{1+\varphi}{1-\alpha}}Y^{\frac{1+\varphi}{1-\alpha}}}{1-\theta\beta\bar{\pi}^{\frac{\epsilon(1-\omega)}{1-\alpha}}} = \frac{S^{\varphi}Y^{\frac{1+\varphi}{1-\alpha}}}{1-\theta\beta\bar{\pi}^{\frac{\epsilon(1-\omega)}{1-\alpha}}} = \frac{S^{\varphi}Y^{\frac{1+\varphi}{1-\alpha}}}{1-\beta\delta} \\
S &= \frac{1-\theta}{1-\theta\bar{\pi}^{\frac{\epsilon(1-\omega)}{1-\alpha}}}\left[\frac{1-\theta\bar{\pi}^{(\epsilon-1)(1-\omega)}}{1-\theta}\right]^{\frac{\epsilon}{(\epsilon-1)(1-\alpha)}} = \frac{1-\theta}{1-\delta}\left(\frac{1-\chi}{1-\theta}\right)^{\frac{\epsilon}{(\epsilon-1)(1-\alpha)}}
\end{aligned}
$$

hence, I can write

$$
\begin{aligned}
y &= \frac{1-\alpha}{\varphi+\sigma+\alpha(1-\sigma)}\left[\log\frac{(\epsilon-1)(1-\alpha)}{\epsilon} - \varphi s\right] \\
\pi &= \log\bar{\pi} \\
i &= \log\bar{\pi} - \log\beta = \pi - \log\beta \\
\psi &= \frac{1+\varphi}{1-\alpha}y + \varphi s - \log(1-\beta\delta) \\
s &= \log\frac{1-\theta}{1-\delta} + \frac{\epsilon}{(\epsilon-1)(1-\alpha)}\log\frac{1-\chi}{1-\theta}
\end{aligned}
$$

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