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Version A

Microeconomics II

SDPE, Stockholm School of Economics

Fourth Assignment

"On Perfect Bayesian Equilibrium"

Section 1. The Court

1. There are two players, a plaintiff and a defendant in a civil suit. The plaintiff knows whether or not he will win the case if it goes to trial, but the defendant does not have this information. The defendant has prior beliefs that there is a probability of 1/3 that the plaintiff will win. If the plaintiff wins, his payoff is 3 and the defendant's payoff is -4; if the plaintiff loses, his payoff is -1 and the defendant's is 0 (this corresponds to the defendant paying cash damages of 3 if the plaintiff wins, and the loser of the case paying court costs of 1). The plaintiff has two possible actions: ask for a low settlement of $m = 1$ or a high settlement of $m = 2$. If the defendant accepts a settlement offer of m, the plaintiff's payoff is m and the defendant's is $-m$. If the defendant rejects the settlement offer, the case goes to court. List all pure strategy PBE strategy profiles. For each such profile, specify the posterior beliefs of the defendant as a function of m, and verify that the combination of this beliefs and the profile is in fact a PBE.

   Hint: Similar to "Quiche or Beer?"

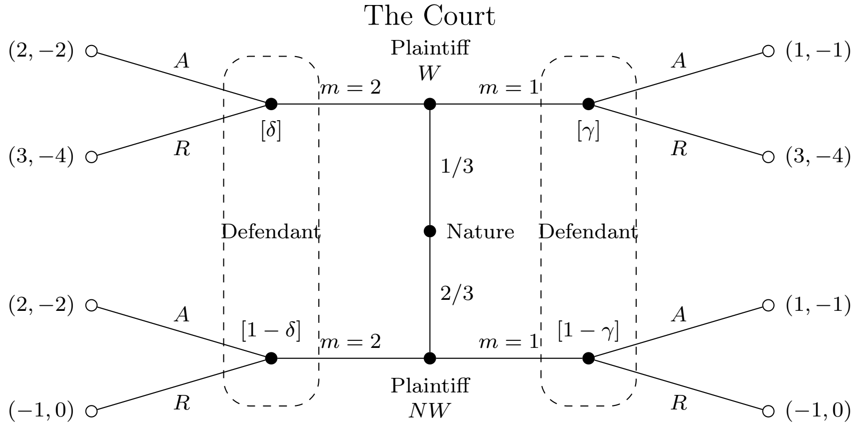
Answer Key for Exam A

Section 1. The Court

1. There are two players, a plaintiff and a defendant in a civil suit. The plaintiff knows whether or not he will win the case if it goes to trial, but the defendant does not have this information. The defendant has prior beliefs that there is a probability of $1/3$ that the plaintiff will win. If the plaintiff wins, his payoff is 3 and the defendant's payoff is -4; if the plaintiff loses, his payoff is -1 and the defendant's is 0 (this corresponds to the defendant paying cash damages of 3 if the plaintiff wins, and the loser of the case paying court costs of 1). The plaintiff has two possible actions: ask for a low settlement of $m = 1$ or a high settlement of $m = 2$. If the defendant accepts a settlement offer of m, the plaintiff's payoff is m and the defendant's is $-m$. If the defendant rejects the settlement offer, the case goes to court. List all pure strategy PBE strategy profiles. For each such profile, specify the posterior beliefs of the defendant as a function of m, and verify that the combination of this beliefs and the profile is in fact a PBE.

   Hint: Similar to "Quiche or Beer?"

Figure 1: *
The Court



Answer: Let us first consider the separating equilibria (SE):

- $(m = 2, m = 1)$: in this case Defendant can form priors about Plaintiff. Defendant knows that when Plaintiff is W will play $m = 2$. Hence Defendant will choose A. On the other hand, when Plaintiff is NW will play $m = 1$. In that case Defendant will play R. Therefore, the equilibrium is

$$\text{“}(m = 2 \text{ if } W,\ m = 1 \text{ if } NW;\ A \text{ if } m = 2,\ R \text{ if } m = 1)\text{"}$$

Is there any incentive to deviate for Plaintiff? If Plaintiff as W plays $m = 1$ instead, he/she will be mistakenly thought as NW, and Defendant will play R. By this, Plaintiff as W would get a higher payoff ($2 < 3$), thus it is *not* an equilibrium!

- $(m = 1, m = 2)$: in this case Defendant can form priors about Plaintiff. Defendant knows that when Plaintiff is W will play $m = 1$. Hence Defendant will choose A. On the other hand, when Plaintiff is NW will play $m = 2$. In that case Defendant will play R. Therefore, the equilibrium would be

$$\text{“}(m = 1 \text{ if } W,\ m = 2 \text{ if } NW;\ R \text{ if } m = 2,\ A \text{ if } m = 1)\text{"}$$

Is there any incentive to deviate for Plaintiff? If Plaintiff as W plays $m = 2$ instead, he/she will be mistakenly thought as NW, and Defendant will play R. By this, Plaintiff as W would get a higher payoff ($1 < 3$), thus it is *not* an equilibrium!

Let us now consider pooling equilibria (PE):

- $(m = 1, m = 1)$: in this case Defendant cannot form any prior on Plaintiff. Defendant's expected payoffs are

$$\mathbb{E}(A) = 1/3 \cdot (-1) + 2/3 \cdot (-1) = -1$$
$$\mathbb{E}(R) = 1/3 \cdot (-4) + 2/3 \cdot 0 = -4/3$$

Hence, Defendant will play A. But what will Defendant do if he observes $m = 2$? Consider δ his prior on Plaintiff playing $m = 2$ as W (and $1-\delta$ on Plaintiff playing $m = 2$ while being NW). Payoffs for Defendant would be

$$\mathbb{E}(A) = \delta(-2) + (1 - \delta)(-2) = -2$$
$$\mathbb{E}(R) = \delta(-4) + (1 - \delta)0 = -4\delta$$

Hence, Defendant plays R if $\delta \leq \frac{1}{2}$, and A otherwise. The goal now is to define δ such that there are no possible deviations (i.e., find an equilibrium). Plaintiff is now obtaining payoffs of 1 (W) and 1 (NW). Notice that whatever Defendant plays, Plaintiff would like to deviate when W $(2, 3 > 1)$. Hence, it is *not* an equilibrium!

- $(m = 2, m = 2)$: in this case Defendant cannot form any prior on Plaintiff. Defendant's expected payoffs are

$$\mathbb{E}(A) = 1/3 \cdot (-2) + 2/3 \cdot (-2) = -2$$
$$\mathbb{E}(R) = 1/3 \cdot (-4) + 2/3 \cdot 0 = -4/3$$

Hence, Defendant will play R. But what will Defendant do if he observes $m = 1$? Consider γ his prior on Plaintiff playing $m = 1$ as W (and $1 - \gamma$ on Plaintiff playing $m = 1$ while being NW). Payoffs for Defendant would be

$$\mathbb{E}(A) = \gamma(-1) + (1 - \gamma)(-1) = -1$$
$$\mathbb{E}(R) = \gamma(-4) + (1 - \gamma)0 = -4\gamma$$

Hence, Defendant plays A if $\gamma \geq \frac{1}{4}$, and R otherwise. The goal now is to define γ such that there are no possible deviations (i.e., find an equilibrium). Plaintiff is now obtaining payoffs of 3 (W) and -1 (NW). Notice that if Defendant plays R, there would be no incentive to deviate (same payoffs, remember that a profitable deviation only happens when there are strictly greater benefits!) Hence, we need $\gamma \geq \frac{1}{4}$. Therefore, the equilibrium is

$$\text{“}(m = 2 \text{ always; } R \text{ always; } \gamma \in [0, \frac{1}{4}])\text{"}$$