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Version A

Microeconomics II

SDPE, Stockholm School of Economics

Third Assignment

“On Repeated Games"

Section 1. Repeated Games

1. Consider an infinitely repeated Prisoner's Dilemma in which the payoffs of the component game are given below

		Player 2	
		C	D
Player 1	C	$5, 5$	$0, 6$
	D	$6, 0$	$1, 1$

A Tit-for-Two-Tat (Tf2T) strategy is defined as

P1: C_0^{P1}, (D_t^{P1} if D_{t-1}^{P2} and D_{t-2}^{P2} $\forall t$, C_t^{P1} otherwise)
P2: C_0^{P2}, (D_t^{P2} if D_{t-1}^{P1} and D_{t-2}^{P1} $\forall t$, C_t^{P2} otherwise))

Is $Tf2T$ a NE?

2. Show two subgames in which $Tf2T$ is not NE.

3. In one of the two subgames show the best response for a player that gains by deviating.

Answer Key for Exam A

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P2: C_0^{P2}, (D_t^{P2} if D_{t-1}^{P1} and D_{t-2}^{P1} $\forall t$, C_t^{P2} otherwise))

Is $Tf2T$ a NE?

Answer: If both players follow the strategy they will obtain payoffs

$$u_i(Tf2T = \{C, C, C, ...\}) = 5 + \delta 5 + \delta^2 5 + ... = \frac{5}{1-\delta} \quad (1)$$

Consider the case of a one-shot deviation in the first round, say, by P2. Then payoffs are

$$u_2(dev = \{D, C, C, ...\}) = 6 + \delta 5 + \delta^2 5 + ... = 6 + \frac{\delta 5}{1-\delta} \quad (2)$$

Hence, there is not such $\delta \in (0,1)$ such that $u_2(Tf2T) \geq u_2(dev)$. Therefore, $Tf2T$ is not a NE.

2. Show two subgames in which $Tf2T$ is not NE.

Answer: *a*) Consider that (C, D) has been played in round 0. Then, the payoffs if following $Tf2T$ are

$$u_1(Tf2T = \{C, C, C, ...\}) = 0 + \delta 5 + \delta^2 5 + ... = \frac{\delta 5}{1-\delta}$$
$$u_2(Tf2T = \{D, C, C, ...\}) = 6 + \delta 5 + \delta^2 5 + ... = 6 + \frac{\delta 5}{1-\delta}$$

However, if P2 deviates in round 2, P1 will still play C. Hence,

$$u_2(dev = \{D, C, D, C, C, C...\}) = 6 + \delta 5 + \delta^2 6 + \delta^3 5 + \delta^4 5... = 6 + \delta 5 + \delta^2 6 + \frac{\delta^3 5}{1-\delta} \tag{3}$$

Hence, there is not such $\delta \in (0, 1)$ such that $u_2(Tf2T) \geq u_2(dev)$. Therefore, this subgame is not a NE.

b) Consider that (D, D) has been played in round 0. Then, the payoffs if following $Tf2T$ are

$$u_i(Tf2T = \{D, C, C, ...\}) = 1 + \delta 5 + \delta^2 5 + ... = 1 + \frac{\delta 5}{1-\delta}$$

However, if P2 deviates in round 2, P1 will still play C. Hence,

$$u_2(dev = \{D, C, D, C, C, C...\}) = 1 + \delta 5 + \delta^2 6 + \delta^3 5 + \delta^4 5... = 1 + \delta 5 + \delta^2 6 + \frac{\delta^3 5}{1-\delta} \tag{4}$$

Hence, there is not such $\delta \in (0, 1)$ such that $u_2(Tf2T) \geq u_2(dev)$. Therefore, this subgame is not a NE.

3. In one of the two subgames show the best response for a player that gains by deviating.

Answer: In the first subgame, the BR is to deviate in even rounds, such that

$$u_2(dev = \{D, C, D, C, D, C...\}) = 6 + \delta 5 + \delta^2 6 + \delta^3 5 + \delta^4 6... = 6 + \frac{\delta 5 + \delta^2 6}{1-\delta^2} \tag{5}$$

There are no series of payoffs greater than this, since P1 would defect.