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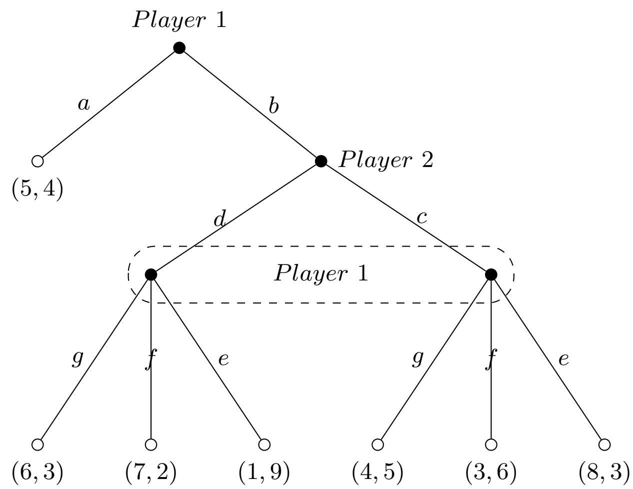
**Version A**

# Microeconomics II

**SDPE, Stockholm School of Economics**
**Second Assignment**
**“On Subgame Perfection"**

## Section 1. Subgame Perfect Equilibrium

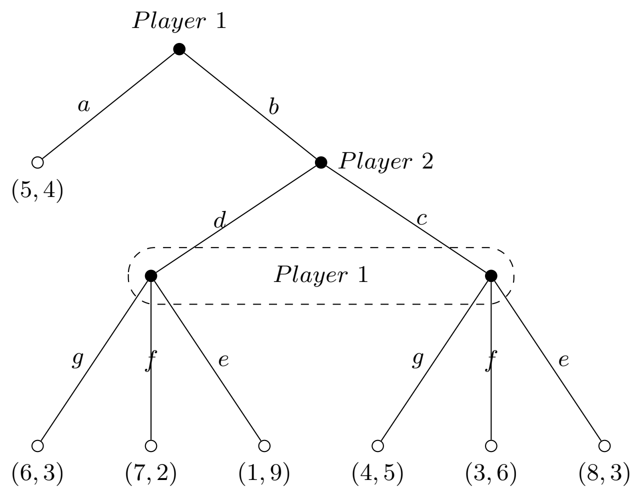
1. Find the unique Subgame Perfect equilibrium (SPE) of this game.



# Answer Key for Exam A

Section 1. Subgame Perfect Equilibrium

1. Find the unique Subgame Perfect equilibrium (SPE) of this game.




**Answer:** There are only 2 possible subgames:

- Starting from P2, the game in normal form is

Player 2

| | | $c$ | $d$ |
|---|---|---|---|
| Player 1 | $e$ | $8,3$ | $1,9$ |
| | $f$ | $3,6$ | $7,2$ |
| | $g$ | $4,5$ | $6,3$ |

One can verify that there is no PNE. Let us check all possible MNE:

- $\{e, f\} \times \{c, d\}$: let $p_1$ denote the probability that P1 plays $e$. For P2 to be willing to randomize, we need

$$\mathbb{E}(d) = p_1 9 + (1 - p_1)2$$
$$\mathbb{E}(c) = p_1 3 + (1 - p_1)6$$
$$\mathbb{E}(d) = \mathbb{E}(c) \implies p_1 = \frac{2}{5}$$

Let $p_2$ denote the probability that P2 plays $c$. For P1 to be willing to randomize, we need

$$\mathbb{E}(e) = p_2 8 + (1 - p_2)1$$
$$\mathbb{E}(f) = p_2 3 + (1 - p_2)7$$
$$\mathbb{E}(e) = \mathbb{E}(f) \implies p_2 = \frac{6}{11}$$

Such situation would report utility for P1,

$$u_1(p_1 e + (1 - p_1)f) = p_1(8p_2 + 1(1 - p_2)) + (1 - p_1)(3p_2 + 7(1 - p_2)) \simeq 4.82$$

Is there any profitable deviation? If P1 instead plays $g$,

$$u_1(g) = 4p_2 + 6(1 - p_2) \simeq 4.91$$

Hence, there is no MNE in this case.

– $\{e, g\} \times \{c, d\}$: let $p_1$ denote the probability that P1 plays $e$. For P2 to be willing to randomize, we need

$$\mathbb{E}(d) = p_1 9 + (1 - p_1)3$$
$$\mathbb{E}(c) = p_1 3 + (1 - p_1)5$$
$$\mathbb{E}(d) = \mathbb{E}(c) \implies p_1 = \frac{1}{4}$$

Let $p_2$ denote the probability that P2 plays $c$. For P1 to be willing to randomize, we need

$$\mathbb{E}(e) = p_2 8 + (1 - p_2)1$$
$$\mathbb{E}(g) = p_2 4 + (1 - p_2)6$$
$$\mathbb{E}(e) = \mathbb{E}(g) \implies p_2 = \frac{5}{9}$$

Such situation would report utility for P1,

$$u_1(p_1 e + (1 - p_1)g) = p_1(8p_2 + 1(1 - p_2)) + (1 - p_1)(4p_2 + 6(1 - p_2)) \simeq 4.89$$

Is there any profitable deviation? If P1 instead plays $f$,

$$u_1(f) = 3p_2 + 7(1-p_2) \simeq 4.78$$

Hence, P1 would prefer to randomize if P2 randomizes. Therefore, $(\frac{1}{4}e + \frac{3}{4}g, \frac{5}{9}c + \frac{4}{9}d)$ is a NE.

- $\{f, g\} \times \{c, d\}$: note that $d$ is a strictly dominated strategy, so there is no equilibrium.
- $\{e, f, g\} \times \{c, d\}$: let $p_2$ denote the probability that P2 plays $c$. For P2 to be willing to randomize, we need

$$\begin{aligned}
\mathbb{E}(e) &= p_2 8 + (1-p_2)1 \\
\mathbb{E}(f) &= p_2 3 + (1-p_2)7 \\
\mathbb{E}(g) &= p_2 4 + (1-p_2)6
\end{aligned}$$

which can never be satisfied simultaneously. Hence, the unique NE of the subgame is $(\frac{1}{4}e + \frac{3}{4}g, \frac{5}{9}c + \frac{4}{9}d)$.

- The payoffs from playing that strategy are

$$\begin{aligned}
u_1(\frac{1}{4}e + \frac{3}{4}g, \frac{5}{9}c + \frac{4}{9}d) &= p_2(p_1 8 + (1-p_1)4) + (1-p_2)(p_1 1 + (1-p_1)6) = \frac{44}{9} \\
u_2(\frac{1}{4}e + \frac{3}{4}g, \frac{5}{9}c + \frac{4}{9}d) &= p_2(p_1 3 + (1-p_1)5) + (1-p_2)(p_1 9 + (1-p_1)3) = \frac{27}{6}
\end{aligned}$$

Hence, P1 must decide between $a$ (payoff 5) or $b$ (payoff 4.89). Hence, P1 will choose $a$. The unique SPE is $(a, \frac{1}{4}e + \frac{3}{4}g; \frac{5}{9}c + \frac{4}{9}d)$.