Macroeconomics II
Problem Set 5

Jose E. Gallegos

April 30, 2020

Science: This last problem set contains two exercises. The first considers a small open economy model with complete asset markets. The second exercise considers a quantitative model of sovereign default à la Eaton & Gersovitz. You can solve this model by using value function iteration as in the Matlab scripts grid.m, solveEGVFI.m and simulateEG.m uploaded on Mondo.

Miscellaneous: Deadline is *Wed 20 May at 9:00.* Submission via email: jose.elias.gallegos@iies.su.se. By that same time, I will upload solutions to my webpage, `https://www.joseeliasgallegos.com/macroeconomics-ii-phd.html`. This time you will not have 24 hours to go through the solutions. I did not want to you to solve the Problem Set before the material was being taught in the lecture. But we will discuss them in class in detail, and I encourage you to go through my solutions in the hour in between. Only one solution set is allowed per student.

I cannot stress enough how much I encourage working in groups. I learnt more in those discussions than in the lectures. As usual, your solution set should be genuinely unique.

I do not require typed solutions. Actually, I only recommend it if you have the objective of learning LaTeX. If you are already proficient, do not use your time on that. I do appreciate legible hand-writting.

And, please, stay safe.

Programming: Exercise 2 is particularly heavy for a MATLAB first-time user. Do not worry about getting it to work perfectly, or even getting it to work at all. What I want you is to stare at it, try to understand what is being done, and come to class with questions. Rest assured, I will go throughout the MATLAB files slowly, so that we all leave this course with the same programming skills.

Exercise 1: Complete Asset Markets and a Discrete Endowment Process

The economy is populated by a large number of identical agents (countries) with preferences described by the utility function

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t\right)$$

where u is a strictly increasing, concave and differentiable function. Each agent has initial financial wealth bo that is exogenous and measured in terms of the consumption good c. In each period, there are two possible states of nature, H and L, with transition probability matrix

$$\pi = \begin{bmatrix} \pi(H,H) & \pi(H,L) \\ \pi(L,H) & \pi(L,L) \end{bmatrix}$$

where $\pi(i,j)$ denotes the probability that the state of nature in period $t+1$ is $j \in \{H, L\}$ conditional on the state of nature in t being i $\in \{H, L\}$. Each agent is endowed with $y_t = \{y_H, y_L\}$ units of consumption goods in t where $y_H > y_L$. They also have access to the world financial market which offers a complete set of state-contingent claims. Let $p_{t+1}^t(i,j)$ be the period t price of a statecontingent claim that pays a unit of consumption in $t+1$ if the state of nature in $t+1$ is $j \in \{H, L\}$ conditional on the state of nature in t being $i \in \{H, L\}$. Suppose that the state of nature in period 0 is H and that $p_{t+1}^t(i,j) = \beta\pi(i,j) \forall t$

(a) State the agent's optimization problem.

(b) Characterize the equilibrium process of consumption and the trade balance.

(c) Derive an expression for the rate of return on a risk-free bond r_t

(d) In the standard SOE-RBC model agents can only trade in a single asset, namely a riskfree bond. In that model, the marginal utility of consumption follows a random walk when $\left(1 + r_t\right)\beta = 1 \forall t$ which in turn leads to a random walk in optimal consumption and the trade balance. In the light of this result, explain why complete asset markets induce a stationary solution.

Exercise 2: The Eaton-Gersovitz Model

Consider the standard Eaton-Gersovitz model where preferences are captured by the instantaneous utility function

$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}$$

and countries receive a stochastic and exogenous endowment $y \in Y = [y_{\min}, y_{\max}]$. In each period a country that defaults or that has defaulted previously is in bad financial standing, and a country that so far never defaulted is in good financial standing. A country that is in good financial standing faces a budget constraint $c + d = y + q(d')\, d'$ and a country that is in bad financial standing consumes its endowment ($c = y$). The default set ($D(d)$) contains all endowment levels at which a country chooses to default given a particular level of debt (d)

$$D(d) = \left\{ y \in Y : v^b(y) > v^c(d, y) \right\}$$

and provided that this is not an empty set, $D(d) = [y_{\min}, y^*(d)]$ where $y^*(d)$ is increasing in d if $y^*(d) < y_{\max}$. Foreign lenders are risk-neutral and perfectly competitive and the net world interest rate is $r^* > 0$

(a) State the value functions associated with bad financial standing, continuing to participate in capital markets and good financial standing. Also state the trade balance and the current account.

(b) What is the relation between the gross country interest rate premium and the probability of repayment (or an approximation of the probability of default)? Explain how a positively serially correlated endowment process affects this relationship.

Suppose now that the endowment process is serially correlated according to $\ln y_t = \rho \ln y_{t-1} + \sigma_\varepsilon \varepsilon_t, \varepsilon_t \sim N(0,1)$ and assume that a country that is in bad financial standing faces the probability θ of transiting to good financial standing in each period. Moreover, let countries in bad financial standing incur the loss $L(y_t) = \max\left\{0, a_0 + a_1 y_t + a_2 y_t^2\right\}$ in each period t, i.e. so that the net endowment is $\tilde{y}_t = y_t - L(y_t)$

(c) Use the the Matlab script grid.m that generates a grid for $\ln y$ to get a grid for y when $a_0 = 0, a_1 = -0.35$ and $a_2 = \frac{1-a_1}{2} \frac{1}{y_{\max}}$ and solve the model with value function iteration by using solveEGVFI.m. Simulate the model by using simulateEG.m. What is the average per period output loss of being in bad financial standing as share of total output, conditional on being in bad financial standing? Present the following quantitative predictions: The default frequency (per century), $\mathbb{E}[d/y], \mathbb{E}[r - r^*], \sigma(r - r^*), \operatorname{corr}(r - r^*, y)$ and $\operatorname{corr}(r - r^*, tb/y)$.[1] Show a graph of the distribution of debt in this economy and discuss how the output loss function affects the results.

[1]Note that these are quarterly values, except for the annual country premium $r - r^*$.

(d) Set $a_0 = a_2 = 0$ and a_1 to match the conditional average output loss of being in bad financial standing calculated in (c). Discuss how the quantitative predictions change. In particular, how does the modification of the loss function affect the model's ability to predict that countries default in bad times?