Mathematics III
Problem Set 1: Differential Equations

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Deadline is *Mon 27 November at 17:00.* Submission via email: jose.elias.gallegos@iies.su.se or in class. By that same time, I will upload solutions to my webpage, joseeliasgallegos.com.

Exercise 1: First-Order Differential Equations

Solve these linear equations in the form $y = y_n + y_p$ with $y_n = y(0)e^{at}$.
(a) $\dot{y} - 4y = -5$
(b) $\dot{y} + 4y = 6$
(c) $\dot{y} - y = 5e^{3t}$, with $y(0) = 2$.
(d) $\dot{y} + y = 8e^{-2t}$, with $y(0) = 2$.

Exercise 2: Separable Differential Equations

Solve the following differential equations:
(a) $\dot{K} = An_0^{\alpha}a^bK^{b-c}e^{(\alpha\nu+\varepsilon)t}$, with $b - c \neq 1$, $\alpha\nu + \varepsilon \neq 0$.
(b) $\dot{x} = \frac{(\beta-\alpha x)(x-a)}{x}$ with $\alpha, \beta, a > 0$, $\alpha a \neq \beta$. *Hint:* $\frac{x}{(\beta-\alpha x)(x-a)} = \frac{1}{\beta-\alpha a}\left(\frac{\beta}{\beta-\alpha x} + \frac{a}{x-a}\right)$.

Exercise 3: The General Case

Given $x(T) = x_T$, show that the solution of $\dot{x} + a(t)x = b(t)$ is

$$x(t) = x_T e^{\int_t^T a(\xi)d\xi} - \int_t^T b(s)e^{\int_t^s a(\xi)d\xi}ds$$

Exercise 4: Second-Order Linear Equations

Find a real solution to the following second-order equation

$$\ddot{y} + 2\dot{y} + 10y = 0$$

with $y(0) = 2$ and $\dot{y}(0) = 1$.

Exercise 5: Sinusoidal functions

The differential equation $\dot{x} + 2x = 2\cos(2t)$ has complete solution

$$x = Ce^{-2t} + \frac{\cos(2t) + \sin(2t)}{2}$$

(a) Plot this function for different C's. How is the behavior of the function affected by changes in C?

(b) Show analytically that the complete solution is equivalent to

$$x = Ce^{-2t} + \frac{1}{\sqrt{2}}\cos\left(2t - \frac{\pi}{4}\right)$$

(c) Confirm this graphically by plotting both versions of the complete solution.