Name: ______________________________

TA: __________José Gallegos__________

PhD Group: ______________________________

Version A

Microeconomics II

SDPE, Stockholm School of Economics

First Assignment

"On the Nash Equilibrium"

Instructions

1. Assignments will be posted in the course Portal.
2. Each Assignment must be submitted at 9am the day before the TA session. By that same time, I will upload solutions to my webpage, joseeliasgallegos.com. I suggest you to have a look at them before coming to class.
3. Submission via email: jose.elias.gallegos@iies.su.se
4. Grading: Pass (P) or Fail (F).
5. I do not strictly demand Latex-written solutions. However, I think it would be useful for those of you who are not experts on Latex. What I will not accept are *pictures* of the hand-written solutions. They are usually unreadable. Please, make sure to scan them in a proper way such that I can easily understand.

Section 1. Pure Nash Equilibrium

Candidates A and B are competing in an election whose outcome is determined by simple plurality[1]. Of the n voters, k support candidate A and $m = n - k$ support candidate B. Each voter simultaneously makes a choice, $v_i \in \{0, 1\}$, with $v_i = 0$ if he/she abstains and $v_i = 1$ if he/she votes. Voting is costly: a voter incurs a cost of c if he/she votes where $c \in (0, 1)$.

A voter who abstains receives the payoff of 2 if the candidate he/she supports wins, 1 if this candidate ties for first place, and 0 if this candidate loses.

A voter who votes receives the payoffs $2 - c$, $1 - c$ or $-c$ in the three former cases.

1. Find the set of Nash equilibria (NE) in pure strategies for the case that $k = m$.

2. Find the set of Nash equilibria (NE) in pure strategies for the case that $k < m$.

[1] A plurality voting system is a voting system in which each voter is allowed to vote for only one candidate, and the candidate who polls more votes than any other candidate (a plurality) is elected.

Answer Key for Exam A

Section 1. Pure Nash Equilibrium

Candidates A and B are competing in an election whose outcome is determined by simple plurality[2]. Of the n voters, k support candidate A and $m = n - k$ support candidate B. Each voter simultaneously makes a choice, $v_i \in \{0, 1\}$, with $v_i = 0$ if he/she abstains and $v_i = 1$ if he/she votes. Voting is costly: a voter incurs a cost of c if he/she votes where $c \in (0, 1)$.

A voter who abstains receives the payoff of 2 if the candidate he/she supports wins, 1 if this candidate ties for first place, and 0 if this candidate loses.

A voter who votes receives the payoffs $2-c$, $1-c$ or $-c$ in the three former cases.

1. Find the set of Nash equilibria (NE) in pure strategies for the case that $k = m$.

   Answer: Assume $i \in k$ (the argument is symmetric for any $j \in m$). Let us check the two most intuitive (possible) NE's:

   - Suppose everyone votes. Then $i \in k$ obtains payoff $1 - c$. Has he/she any profitable deviation? If he/she does not vote, he/she does not incur in cost c, but candidate A loses and therefore $i's$ payoff is 0. Since $c \in (0, 1)$, $1 - c > 0$ and there is no incentive to deviate. $\boxed{\text{NE: every } i \in k \text{ and } j \in m \text{ vote}}$
   - Suppose no one votes. Then $i \in k$ obtains payoff 1. If he/she votes, candidate A would win and therefore $i's$ would be $2 - c$. But then, some $j \in m$ would vote, since he/she was obtaining 0 and by voting he/she makes $v_k = v_m$ (sloppy...), candidates tie and $j's$ payoff would be $1-c$. Then A would have an incentive to vote and undo the tie...the argument continues until everyone votes, which is the above NE.

2. Find the set of Nash equilibria (NE) in pure strategies for the case that $k < m$.

   Answer: Again, let me focus on the most intuitive NE.

   - Suppose everyone votes. Any $i \in k$ obtains payoff $-c$, and any $j \in m$ obtains $2-c$. Is there any profitable deviation? Any $i \in k$ would prefer not to vote, and thus obtain payoff 0. So no $i \in k$ has an incentive to vote.
     In this case, any $j \in m$ would prefer not to vote, and obtain payoff 2. The argument continues until one $j \in m$ votes. But then any $i \in k$ would vote and obtain payoff $1 - c$. Then, another $j \in m$ would vote, another $i \in k$...the argument goes until the marginal $i \in k$ would have no incentive to vote. Realizing this, no $i \in k$ should vote. But then, any $j \in m$ would prefer not to vote...as one can check, the argument starts to cycle and a NE is never reached. Hence, there is no pure NE.

[2] A plurality voting system is a voting system in which each voter is allowed to vote for only one candidate, and the candidate who polls more votes than any other candidate (a plurality) is elected.