Online Appendix to "Inflation Persistence, Noisy Information, and the Phillips Curve"

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OA.1. Robustness

OA.1.1. Inflation Persistence

Structural Break. I begin the robustness analysis by considering alternative inflation measures. I repeat the structural break analysis discussed in the main body for CPI and PCE inflation, and I find similar results in Table OA.1, with the structural change in dynamics being less evident in the core series.

Autocorrelation Function. I start with the most agnostic analysis of inflation persistence. Figure OA.1 plots the autocorrelation function for the three main inflation measures across subsamples. Focusing on the second and third columns, I find evidence for a fall in the first-order autocorrelation for the three measures. For instance, the first-order autocorrelation for all inflation measures in the pre-1985 sample is around 0.75, while the same statistic for the second period ranges from 0.5 to 0.3 depending on the measure.

Rolling Sample. I compute rolling-sample estimates of an independent AR(1) process using a 14-year window for the different inflation measures. Figure OA.2 plots the time-varying persistence parameter ρ_t with 95% confidence bands. The results suggest that there is time variation in the persistence of inflation.

E-mail: jose.elias.gallegos@bde.es. In this online appendix, I refer to sections, equations, tables, and figures in the main paper. The views expressed are those of the author and do not necessarily represent the views of the Banco de España and the Eurosystem.

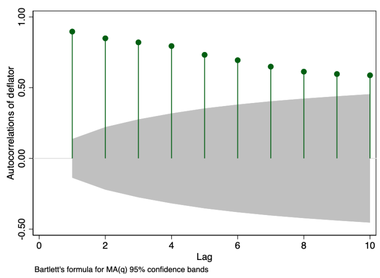
	(1) CPI	(2) PCE
π_{t-1}	0.793***	0.837***
	(0.0827)	(0.0672)
$\pi_{t-1} \times \mathbb{1}_{\{t \geq t^*\}}$	-0.497***	-0.434***
	(0.143)	(0.117)
Constant	1.396**	0.990**
	(0.542)	(0.431)
Constant$\times \mathbb{1}_{\{t \geq t^*\}}$	0.370	0.283
	(0.607)	(0.477)
Observations	206	206

HAC robust standard errors in parentheses.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

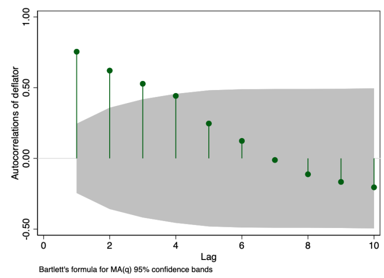
TABLE OA.1. Regression table

Unit Root Tests. Inspecting Figure OA.2, one could hypothesize that inflation was characterized by a unit root process in the pre-1985 sample and not afterward. To investigate this, I proceed via a cross-sample unit root analysis using both the Augmented Dickie-Fuller and the Phillips-Perron tests. I report our results in Table OA.2, including the p-values of both unit root tests under the null of a unit root. Focusing on the last two rows I find that, consistent with our previous evidence on the first-order autocorrelation, the null hypothesis of a unit root series cannot be rejected by any of the unit root tests conducted in the different inflation measures in the pre-1985 period. When I repeat a similar analysis in the post-1985 period, I find a strong rejection of the null hypothesis, suggesting that inflation can no longer be described as a unit root process. Having understood the close relationship between the roots of the inflation dynamic process and its persistence, I can conclude that inflation persistence fell in the post-1985 period.

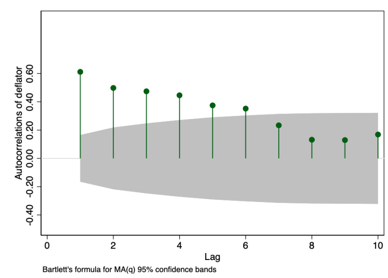
Dominant Root. A further procedure of studying persistence that relies on the roots of the dynamic process of inflation is the dominant root analysis. Consider the AR(p) process $\pi_t = \rho_1 \pi_{t-1} + \rho_2 \pi_{t-2} + \ldots + \rho_p \pi_{t-p} + \varepsilon_t^{\pi}$, with companion matrix $R(p)$. The root of the characteristic polynomial of $R(p)$ with the largest magnitude is the dominant root of interest. Notice that in the case of an AR(p) where $p > 1$, the dominant root will depend not only on the first lag coefficient but on all of them. An AR(p) is considered to be stable if all the roots of the characteristic polynomial of matrix $R(p)$ have an absolute value lower than



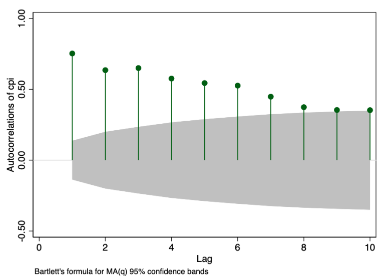
A. GDP Deflator, 1947-1985



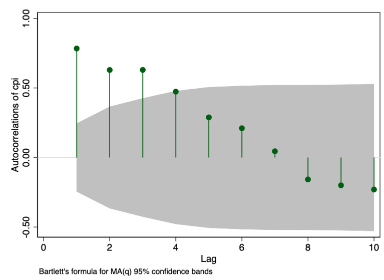
B. GDP Deflator, 1969-1985



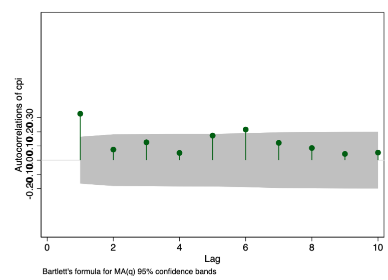
C. GDP Deflator, 1985-2020



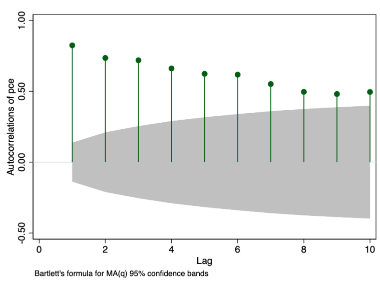
D. CPI, 1947-1985



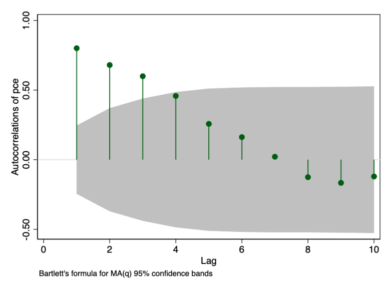
E. CPI, 1969-1985



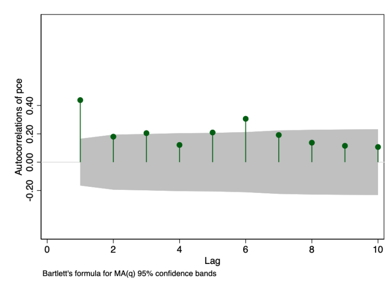
F. CPI, 1985-2020



G. PCE, 1947-1985



H. PCE, 1969-1985



I. PCE, 1985-2020

FIGURE OA.1. Autocorrelation function of GDP Deflator (first row), CPI (second row) and PCE (last row).

1. One can therefore proceed as in the unit root case, and study the dominant root of the underlying inflation process over the different subsamples. I find that the dominant root in the 1968:Q4-1984:Q4 period is 0.870 and 0.841 in the 1985:Q1-2020:Q2 period, suggesting a moderate fall in persistence.

OA.1.2. Empirical Evidence on Information Frictions

Rolling Sample Regression. I obtain a rolling-sample estimate version of (2). Figure OA.3 plots the rolling estimate $\beta_{CG,t}$ over time. The figure suggests that information frictions were reduced after the 1980s, with a smaller local peak in the late 2000s, which coincides with the local peak in inflation persistence in Figure OA.2.

A. GDP Deflator B. CPI C. PCE

FIGURE OA.2. First-order autocorrelation of GDP Deflator, CPI, and PCE, rolling sample (14y window).

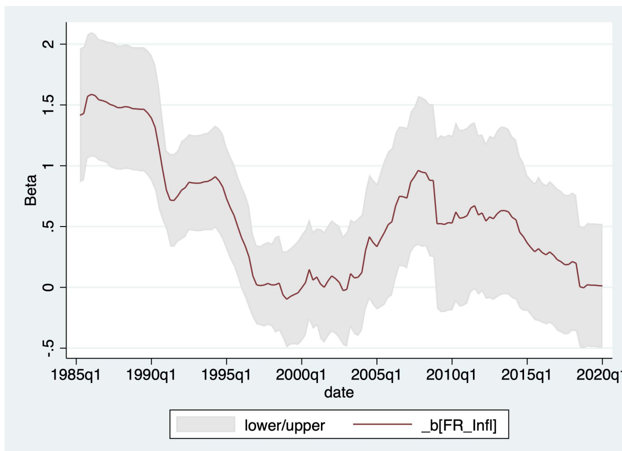


FIGURE OA.3. Time-varying $\beta_{CG,t}$ in the CG regression (2) using a 14y window.

Forecast Error response to Monetary Policy Shocks. Under FIRE, ex-ante average forecast errors should be unpredictable by ex-ante available information. Therefore, the IRF of forecast errors to monetary policy shocks should be insignificant. Coibion and Gorodnichenko (2012) show that forecast errors react to several exogenous shocks to the economy. To study if the sensitivity of ex-post forecast errors has changed after the 1985:Q1 structural break, I produce the local projection of Romer and Romer (2004) monetary policy shocks on the average forecast error, forecast error$_{t+h} = \beta_h \varepsilon_t^{\nu} + \beta_{h*} \varepsilon_t^{\nu} \times \mathbb{1}_{t \geq t^*} + \gamma X_t + u_t$, where h denotes the horizon and X_t includes four lags of Romer and Romer (2004) shocks and four lags of forecast errors. I report the implied impulse responses in Figure OA.4. I find that the IRF is positive in the pre-1985 period, suggesting that forecasts react less to monetary shocks than the forecasted variable (see Figure OA.4A). After 1985, forecast errors do not react to monetary shocks, suggesting that information frictions lessened (see Figure OA.4B). I show in Figure OA.4C that the difference between the IRFs under the two regimes is significant.

Variable	Dickey-Fuller	Phillips-Perron
	1968:Q4-2020:Q2	
GDP Deflator	0.3444	0.1104
CPI	0.1598	0.0001
PCE	0.2149	0.0038
	1968:Q4-1984:Q4	
GDP Deflator	0.1543	0.673
CPI	0.2109	0.0875
PCE	0.0584	0.0938
	1985:Q1-2020:Q2	
GDP Deflator	0.1237	0.0000
CPI	0.0081	0.0000
PCE	0.0151	0.0000

MacKinnon approximate p-values.

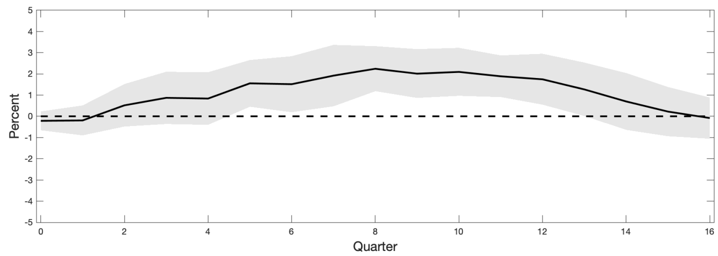
TABLE OA.2. Unit Root Tests for Inflation Measures.

Accounting for Unbalancedness. The number of respondents of the SPF has steadily decreased, from around 90 respondents in the 1960s to around 40 nowadays. Using the quarterly average response would overweight the recent period. To correct this, I

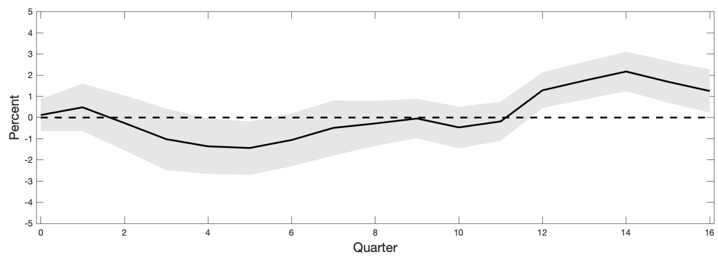
$$\text{forecast error}_{it} = \alpha_{\text{rev}} + \beta_{\text{rev}}\ \text{revision}_t + u_t, \tag{OA.1}$$

where forecast $\text{error}_{it} \equiv \pi_{t+3,t} - \mathbb{F}_{it}\pi_{t+3,t}$ is the *individual* ex-ante forecast error. I reproduce columns 1-5 of table 2, panel B by using this alternative specification and find similar results, reported in table OA.3.

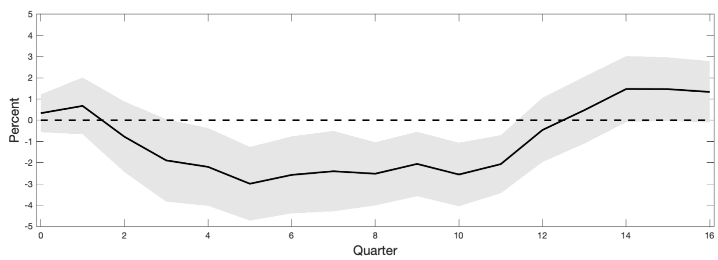
Disagreement. I define a measure of "disagreement" as the cross-sectional standard deviation of forecasts at each time, $disagreement_t = \sigma_i(\mathbb{F}_{it}\pi_{t+3,t})$. Under the assumption of common complete information, disagreement should be zero since all agents would have observed the same past, their information set would therefore be the same, and their expectation around a future variable should coincide, provided that agents are ex-ante identical. As I observe in Figure OA.5, disagreement was large around the 1980s, coinciding with the beginning of the Volcker activism and the lack of public disclosure of the Federal Reserve decisions, and fell dramatically until the 1990s, stabilizing at that level after the



A. Pre-1985 period.



B. Post-1985 period.



C. Change.

FIGURE OA.4. Impulse response function of average forecasts to monetary policy shocks.

1990s.

Under the assumption of sticky information, disagreement should react to monetary policy shocks, since a share of agents has observed the shock. Again using local projections, I test this theoretical prediction, $\text{disagreement}_{t+h} = \beta_h \varepsilon_t^{\nu} + \beta_{h*} \varepsilon_t^{\nu} \times \mathbb{1}_{t \geq t^*} + \gamma X_t + u_t$, where h denotes the horizon and X_t includes four lags of Romer and Romer (2004) shocks and four lags of disagreement. I report the implied impulse responses in Figure OA.6. I find do not find any evidence of a reaction of disagreement to monetary policy shocks, consistent with noisy information and FI.

Livingston Survey. Using the Livingston survey on firms, I test for a structural break in belief formation around 1985:I. Since the survey is conducted semiannually, I estimate the following structural-break variant of (3)

$$\pi_{t+2,t} - \mathbb{E}_t \pi_{t+2,t} = \alpha_{CG} + \left(\beta_{CG} + \beta_{CG*} \mathbb{1}_{\{t \geq t^*\}}\right) \left(\mathbb{E}_t \pi_{t+2,t} - \mathbb{E}_{t-2} \pi_{t+2,t}\right) + u_t \tag{OA.2}$$

	(1) CG Regression	(2) 1968:Q4-1984:Q4	(3) 1985:Q1-2020:Q2	(4) Structural Break	(5)
Revision	1.703***	1.131***	-0.0854	1.850***	1.131***
	(0.153)	(0.200)	(0.138)	(0.188)	(0.199)
Revision $\times \mathbb{1}_{\{t \geq t^*\}}$				-0.833***	-1.216***
				(0.264)	(0.243)
Constant	-0.0392**	0.438***	-0.329***	-0.0719***	0.438***
	(0.0183)	(0.0554)	(0.0138)	(0.0213)	(0.0554)
Constant $\times \mathbb{1}_{\{t \geq t^*\}}$					-0.767***
					(0.0571)
Observations	6688	2294	4394	6688	6688

HAC robust standard errors in parentheses.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

TABLE OA.3. Estimates of regression (OA.1).

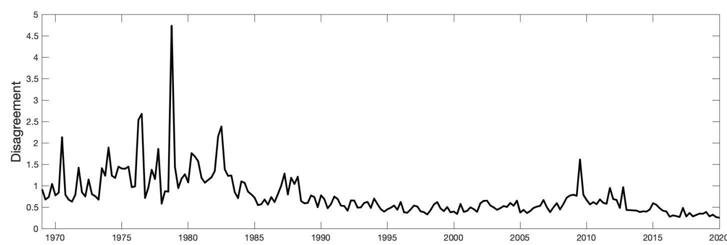


FIGURE OA.5. Cross-sectional volatility of (annual) inflation forecasts at each period.

Our results, reported in the first column in Table OA.4, suggest a strong violation of the FIRE assumption: the measure of information frictions, β_{CG}, is significantly different from zero. Secondly, a significant estimate of β_{CG*} would suggest a break in the information frictions faced by agents. Our results in the second column in Table 2, panel B suggest that there is a structural break around the period in which the Fed changed its monetary stance. Our result $\beta_{CG*} < 0$ suggests that agents became *more* more informed about inflation, with individual forecasts relying less on priors and more on news. A t-test under the null that $\beta_{CG} + \beta_{CG,*} = 0$ has an associated p-value of 0.2156. I can therefore conclude that information frictions on the CPI vanish, consistent with our findings on CPI persistence in Figure OA.2B.

As a second exercise, I estimate (19) using the Livingston Survey data. Since the survey is only conducted semiannually and only asks for 6m and 12m ahead forecasts I only consider the cases $k = 2$ and $k = 4$. Our results suggest no evidence of a structural break in

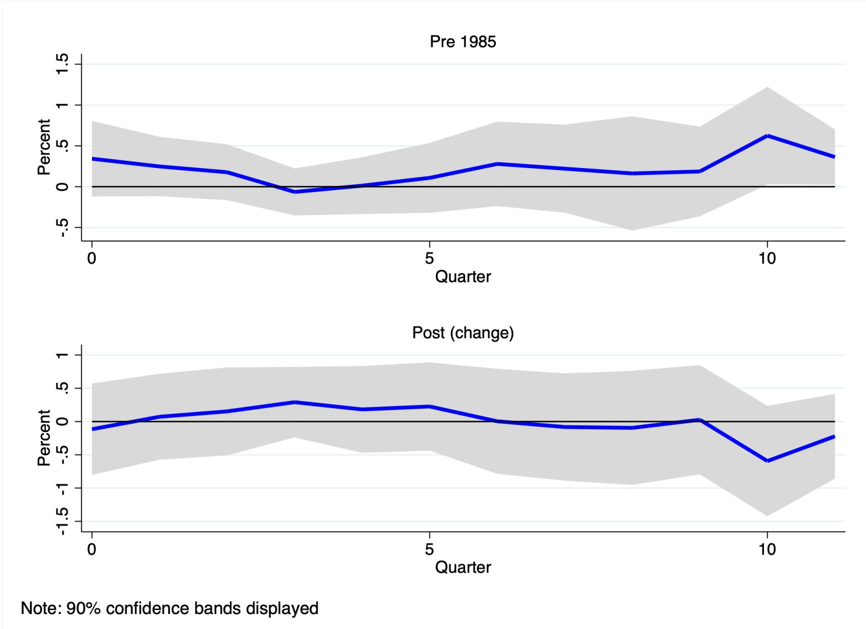


FIGURE OA.6. Impulse response function of forecast disagreement to monetary policy shocks.

κ once I control for non-standard expectations.

OA.2. Extending Information Frictions to Households

In this section, I relax the FIRE assumption on households. I show in Online Appendix OA.4 that in such case, the individual household policy function is given by

$$c_{it} = -\frac{\beta}{\sigma}\mathbb{E}_{it}r_t + (1-\beta)\mathbb{E}_{it}\widetilde{y}_t + \beta\mathbb{E}_{it}c_{i,t+1}, \quad \text{with } \widetilde{y}_t = \int c_{it}\, di \tag{OA.3}$$

I still maintain the FIRE assumption on the monetary authority, which is not subject to information frictions. In this case, the model equations are (OA.3), (6), (9) and (10).

Information Structure. To generate heterogeneous beliefs and sticky forecasts, I assume that the information is incomplete and dispersed. Each agent l in the group $g \in$ {household, firm} observes a noisy signal x_{lgt} that contains information on the monetary shock ν_t, and takes the standard functional form of "outcome plus noise". Formally, signal x_{lgt} is described as

$$x_{lgt} = \nu_t + \sigma_{gu}u_{lgt}, \quad \text{with } u_{lgt} \sim \mathcal{N}(0,1) \tag{OA.4}$$

	Full Sample	Structural Break
Revision	0.359*	0.384*
	(0.210)	(0.213)
Revision$\times\mathbb{1}_{\{t\geq t^{*}\}}$		-0.960**
		(0.479)
Constant	-0.173**	-0.0826
	(0.0829)	(0.106)
Observations	148	148

HAC robust standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

TABLE OA.4. Regression table.

where signals are agent-specific. This implies that each agent's information set is different, and therefore generates heterogeneous information sets across the population of households and firms. Notice that I allow for heterogeneity in the variance that each of the groups (households and firms) face.

An equilibrium must therefore satisfy the individual-level optimal pricing policy functions (6), the individual DIS curve (OA.3), the Taylor rule (9), and rational expectation formation should be consistent with the exogenous monetary shock process (10) and the signal process (OA.4).

The following proposition outlines inflation and output gap dynamics.

PROPOSITION 1. *Under noisy information the output gap, price level and inflation dynamics are given by*

$$\boldsymbol{a}_t = A(\vartheta_1, \vartheta_2, \vartheta_3)\boldsymbol{a}_{t-1} + B(\vartheta_1, \vartheta_2, \vartheta_3)\nu_t \tag{OA.5}$$

where $\boldsymbol{a}_t = \begin{bmatrix} \widetilde{y}_t & p_t & \pi_t \end{bmatrix}^\intercal$ *is a vector containing output, price level and inflation,* $A(\vartheta_1, \vartheta_2, \vartheta_3)$ *is a* 3×3 *matrix and* $B(\vartheta_1, \vartheta_2, \vartheta_3)$ *is a* 3×1 *vector, where* $(\vartheta_1, \vartheta_2, \vartheta_3)$ *are three scalars that are given by the reciprocal of three of the four outside roots of the characteristic polynomial of the following matrix*[1]

$$\boldsymbol{C}(z) = \begin{bmatrix} C_{11}(z) & C_{12}(z) \\ C_{21}(z) & C_{22}(z) \end{bmatrix} \tag{OA.6}$$

[1]The other outside root is always equal to θ and is canceled out.

	GDP Growth	Structural Break
$\widetilde{y}_t^e$	0.830***	0.851***
	(0.0475)	(0.0444)
$\widetilde{y}_t^e \times \mathbb{1}_{\{t \geq t^*\}}$		-0.113
		(0.0741)
π_t^e	-0.116**	-0.0599
	(0.0536)	(0.0611)
Observations	95	95

HAC (1 lag) robust standard errors in parentheses.
Instrument set: four lags of forecasts of annual
real GDP growth and annual GDP Deflator growth.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

TABLE OA.5. Regression table.

where

$$C_{11}(z) = \left[(z-\beta)(z-\lambda_1)(1-\lambda_1 z) - \left(1-\frac{\lambda_1}{\rho}\right)(1-\rho\lambda_1)\left(1-\beta\left(1+\frac{\phi_y}{\sigma}\right)\right)z^2\right](1-\theta z)$$

$$C_{12}(z) = -(1-\theta)\left(1-\frac{\lambda_1}{\rho}\right)(1-\rho\lambda_1)z\frac{\beta}{\sigma}(1-z)(1-\phi_\pi z)$$

$$C_{21}(z) = -\left(1-\frac{\lambda_2}{\rho}\right)(1-\rho\lambda_2)(1-\theta z)\frac{\kappa\theta}{1-\theta}z^2$$

$$C_{22}(z) = (z-\beta\theta)(z-\lambda_2)(1-\lambda_2 z)(1-\theta z) - (1-\theta)\left(1-\frac{\lambda_2}{\rho}\right)(1-\rho\lambda_2)(1-\beta\theta)z^2$$

with λ_g, $g \in \{1, 2\}$ *being the inside root of the polynomial* $\mathcal{D}(z) \equiv z^2 - \left(\frac{1}{\rho} + \rho + \frac{\sigma_\epsilon^2}{\rho\sigma_{gu}^2}\right)z + 1.$

PROOF. By the end of this section. □

In the noisy information framework, inflation is intrinsically persistent and its persistence is governed by the new information-related parameters ϑ_1, ϑ_2 and ϑ_3, as opposed to the benchmark framework in which it is only extrinsically persistent, $A(0, 0, 0) = \mathbf{0}$. The intuition for this result is simple: inflation is partially determined by expectations (see condition (7) under noisy information, or (4) under complete information). Under noisy information, expectations are anchored and follow an autoregressive process (see (12)), which creates the additional source of anchoring in inflation dynamics, measured by ϑ_1, ϑ_2 and ϑ_3.

Empirical Evidence on Household's Information Frictions. There are now two different information parameters to calibrate, since I allow for heterogeneity in information precision by group. To calibrate the additional one, I use the Michigan Survey of Consumers' annual forecasts of inflation.[2] Consider the average forecast of annual inflation at time t, $\overline{\mathbb{E}}^c_t \pi_{t+3,t}$, where $\pi_{t+3,t}$ is the inflation between periods $t+3$ and $t-1$. I can think of this object as the action that the average consumer makes. A drawback of this source of expectations data is that it is only available at a forecasting horizon of one year and therefore revisions in forecasts over identical horizons are not available. Thus, I follow Coibion and Gorodnichenko (2015) and replace the forecast revision with the change in the year-ahead forecast, yielding the following quasi-revision: $\text{revision}_t \equiv \overline{\mathbb{E}}^c_t \pi_{t+3,t} - \overline{\mathbb{E}}^c_{t-1} \pi_{t+2,t-1}$. The average forecast revision provides information about the average agent's annual forecast after the inflow of information between periods t and $t-1$. Recent research (Coibion and Gorodnichenko 2012, 2015) has documented a positive co-movement between ex-ante average forecast errors and average forecast revisions.[3] Formally, the regression design is

$$\text{forecast error}_t = \alpha_{\text{rev}} + \beta_{\text{rev}} \, \text{revision}_t + u_t \tag{OA.7}$$

The error term now consists of the RE forecast error and $\beta_{\text{rev}}(\overline{\mathbb{E}}^c_{t-1}\pi_{t-1} - \overline{\mathbb{E}}^c_t \pi_{t+3})$ because forecasts horizons do not overlap. I therefore rely on an IV estimator, using as an instrument the (log) change in the oil price.[4]

Notice that a positive co-movement ($\widehat{\beta}_{\text{rev}} > 0$) suggests that positive revisions predict positive forecast errors. That is, after a positive revision of annual inflation forecasts, consumers consistently under-predict inflation. The results, reported in the first column in Table OA.6, suggest a strong violation of the FIRE assumption: the measure of information frictions, β_{rev}, is significantly different from zero. Agents underrevise their forecasts: a positive β_{rev} coefficient suggests that positive revisions predict positive (and larger) forecast errors. In particular, a 1 percentage point revision predicts a 1.012 percentage point forecast error. The average forecast is thus smaller than the realized outcome, which suggests that the forecast revision was too small, or that forecasts react sluggishly.

Following the previous analyses on inflation persistence, I assume that the break date

[2] Each quarter, the University of Michigan surveys 500–1,500 households and asks them about their expectation of price changes over the next year.

[3] I use the first-release value of annual inflation to construct the forecast error since forecasters did not have access to future revisions of the data.

[4] Coibion and Gorodnichenko (2015) argue that oil prices have significant effects on CPI inflation, and therefore are statistically significant predictors of contemporaneous changes in inflation forecasts and can account for an importantshare of their volatility.

	(1) All Sample	(2) Structural Break
Revision	1.012***	1.706*
	(0.299)	(1.018)
Revision $\times \mathbb{1}_{\{t \geq t^*\}}$		-1.083
		(1.066)
Constant	-0.571***	-0.571***
	(0.181)	(0.180)
Observations	182	182

Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

TABLE OA.6. Regression table

is 1985:Q1. I test for the null of no structural break in inflation dynamics around 1985:Q1.[5] I cannot the null of no break (p-value = 0.60). Following a similar structural break analysis as in Section 2.1, I study if there is a change in expectation formation (stickiness) around the same break date. Formally, I test for a structural break in belief formation around 1985:Q1 by estimating the following structural-break version of (OA.7),

$$\text{forecast error}_t = \alpha_{\text{rev}} + \left(\beta_{\text{rev}} + \beta_{\text{rev}*} \mathbb{1}_{\{t \geq t^*\}}\right) \text{revision}_t + u_t \tag{OA.8}$$

A significant estimate of $\beta_{\text{rev}*}$ suggests a break in the information frictions. The results in the second column in Table OA.6 suggest that there is no structural break around 1985:Q1.

Results. I calibrate the two information volatilities σ_{1u} and σ_{2u} to match jointly the empirical evidence on forecast sluggishness in Tables 2, panel B and OA.6. This results in $\sigma_{1u} = 13.535$ and $\sigma_{2u} = 12.041$ in the pre-1985 sample, and $\sigma_{1u} = 12.041$ and $\sigma_{2u} = 0.018$. In the pre-1985 period, the model-implied inflation first-order autocorrelation is $\rho_{\pi 1} = 0.808$. In the post-1985 period, inflation persistence falls to 0.709. The fall is smaller because the output gap, which is still intrinsically persistent because of households' information frictions, reduces the overall effect of the fall in firm information frictions. Comparing our model results to the empirical analysis in Tables 1 and 2 (panel A), I find that the noisy information framework can explain around 1/3 of the point estimate fall.

[5] If I instead are agnostic about the break date(s), the test suggests that there is no such break.

Proof of Proposition 1. Recall the policy functions

(OA.9)
$$c_{it} = \frac{\beta\phi_\pi}{\sigma}\mathbb{E}_{it}p_{t-1} + \left(1 - \beta - \frac{\beta\phi_y}{\sigma}\right)\mathbb{E}_{it}\widetilde{y}_t - \frac{\beta(1+\phi_\pi)}{\sigma}\mathbb{E}_{it}p_t + \frac{\beta}{\sigma}\mathbb{E}_{it}p_{t+1} - \frac{\beta}{\sigma}\mathbb{E}_{it}v_t + \beta\mathbb{E}_{it}c_{i,t+1}$$

(OA.10)
$$p^*_{jt} = (1-\beta\theta)\mathbb{E}_{jt}p_t + \frac{\kappa\theta}{1-\theta}\mathbb{E}_{jt}\widetilde{y}_t + \beta\theta\mathbb{E}_{jt}p^*_{j,t+1}$$

I now turn to solving the expectation terms. I can write the fundamental representation of the signal process as a system containing (10) and (11), which admits the following state-space representation

$$\begin{aligned} \boldsymbol{Z}_t &= \boldsymbol{F}\boldsymbol{Z}_{t-1} + \boldsymbol{\Phi}\boldsymbol{s}_{lgt} \\ x_{lgt} &= \boldsymbol{H}\boldsymbol{Z}_t + \boldsymbol{\Psi}_g\boldsymbol{s}_{lgt} \end{aligned} \tag{OA.11}$$

with $\boldsymbol{F} = \rho$, $\boldsymbol{\Phi} = \begin{bmatrix}\sigma_\varepsilon & 0\end{bmatrix}$, $\boldsymbol{Z}_t = v_t$, $\boldsymbol{s}_{lgt} = \begin{bmatrix}\varepsilon^v_t \\ u_{lgt}\end{bmatrix}$, $\boldsymbol{H} = 1$, and $\boldsymbol{\Psi}_g = \begin{bmatrix}0 & \sigma_{gu}\end{bmatrix}$. It is convenient to rewrite the uncertainty parameters in terms of precision: define $\tau_\varepsilon \equiv \frac{1}{\sigma^2_\varepsilon}$, $\tau_{gu} \equiv \frac{1}{\sigma^2_{gu}}$, and $\tau_g = \frac{\tau_{gu}}{\tau_\varepsilon}$. The signal system can be written as

$$x_{igt} = \frac{\sigma_\varepsilon}{1-\rho L}\varepsilon^v_t + \sigma_{gu}u_{lgt} = \begin{bmatrix}\frac{\tau_\varepsilon^{-\frac{1}{2}}}{1-\rho L} & \tau_{gu}^{-\frac{1}{2}}\end{bmatrix}\begin{bmatrix}\varepsilon^v_t \\ u_{lgt}\end{bmatrix} = \boldsymbol{M}_g(L)\boldsymbol{s}_{lgt}, \qquad \boldsymbol{s}_{lgt} \sim \mathcal{N}(0, I) \tag{OA.12}$$

The Wold theorem states that there exists another representation of the signal process (OA.12), $x_{lgt} = \boldsymbol{B}_g(L)\boldsymbol{w}_{lgt}$ such that $\boldsymbol{B}_g(z)$ is invertible and $\boldsymbol{w}_{lgt} \sim (0, \boldsymbol{V}_g)$ is white noise. Hence, I can write the following equivalence:

$$x_{lgt} = \boldsymbol{M}_g(L)\boldsymbol{s}_{lgt} = \boldsymbol{B}_g(L)\boldsymbol{w}_{lgt} \tag{OA.13}$$

In the Wold representation of x_{lgt}, observing $\{x_{lgt}\}$ is equivalent to observing $\{\boldsymbol{w}_{lgt}\}$, and $\{x^t_{lg}\}$ and $\{\boldsymbol{w}^t_{lg}\}$ contain the same information. Furthermore, note that the Wold representation has the property that both processes share the autocovariance generating function, $\rho^g_{xx}(z) = \boldsymbol{M}_g(z)\boldsymbol{M}^\top_g(z^{-1}) = \boldsymbol{B}_g(z)\boldsymbol{V}_g\boldsymbol{B}^\top_g(z^{-1})$. Given the state-space representation of the signal process (OA.11), optimal expectations of the exogenous fundamental take the form of a Kalman filter $\mathbb{E}_{lgt}v_t = \lambda_g\mathbb{E}_{it-1}v_{t-1} + \boldsymbol{K}_g x_{lgt}$, where $\lambda_g = (I - \boldsymbol{K}_g\boldsymbol{H})\boldsymbol{F}$, and $\boldsymbol{K}_g$ is given by

$$\boldsymbol{K}_g = \boldsymbol{P}_g\boldsymbol{H}^\top\boldsymbol{V}_g^{-1} \tag{OA.14}$$

$$\boldsymbol{P}_g = \boldsymbol{F}[\boldsymbol{P}_g - \boldsymbol{P}_g\boldsymbol{H}^\top \boldsymbol{V}_g^{-1}\boldsymbol{H}\boldsymbol{P}_g]\boldsymbol{F} + \boldsymbol{\Phi}\boldsymbol{\Phi}^\top \tag{OA.15}$$

I still need to find the unknowns $\boldsymbol{B}_g(z)$ and $\boldsymbol{V}_g$. Propositions 13.1-13.4 in Hamilton (1994) provide us with these objects. Unknowns $\boldsymbol{B}_g(z)$ and $\boldsymbol{V}_g$ satisfy $\boldsymbol{B}_g(z) = I + \boldsymbol{H}(I - \boldsymbol{F}z)^{-1}\boldsymbol{F}\boldsymbol{K}_g$ and $\boldsymbol{V}_g = \boldsymbol{H}\boldsymbol{P}_g\boldsymbol{H}^\top + \boldsymbol{\Psi}_g\boldsymbol{\Psi}_g^\top$. I can write (OA.15) as

$$\boldsymbol{P}_g^2 + \boldsymbol{P}_g[(1-\rho^2)\sigma_{gu}^2 - \sigma_\varepsilon^2] - \sigma_\varepsilon^2\sigma_{gu}^2 = 0 \tag{OA.16}$$

from which I can infer that $\boldsymbol{P}_g$ is a scalar. Denote $k_g = \boldsymbol{P}_g^{-1}$ and rewrite (OA.16) as $k_g = \frac{\tau_\varepsilon}{2}\left\{1-\rho^2-\tau_g \pm \sqrt{\left[\tau_g - (1-\rho^2)\right]^2 + 4\tau_g}\right\}$.

I also need to find $\boldsymbol{K}_g$. Now that I have found $\boldsymbol{P}_g$ in terms of model primitives, I can obtain $\boldsymbol{K}_g$ using condition (OA.14), $\boldsymbol{K}_g = \frac{1}{1+k_g\sigma_{gu}^2}$. I can finally write λ_g as

$$\lambda_g = \frac{k_g\sigma_{gu}^2\rho}{1+k_g\sigma_{gu}^2} = \frac{1}{2}\left[\frac{1}{\rho} + \rho + \frac{\tau_g}{\rho} \pm \sqrt{\left(\frac{1}{\rho} + \rho + \frac{\tau_g}{\rho}\right)^2 - 4}\right] \tag{OA.17}$$

One can show that one of the roots $\lambda_{g,[1,2]}$ lies inside the unit circle and the other lies outside as long as $\rho \in (0,1)$, which guarantees that the Kalman expectation process is stationary and unique. I set λ_g to the root that lies inside the unit circle (the one with the '–' sign). Notice that I can also write $\boldsymbol{V}_g$ in terms of λ_g, $\boldsymbol{V}_g = k^{-1} + \sigma_{gu}^2 = \frac{\rho}{\lambda_g\tau_{gu}}$, where I have used the identity $k_g = \frac{\lambda_g\tau_{gu}}{\rho-\lambda_g}$. Finally, I can obtain $\boldsymbol{B}_g(z) = 1 + \frac{\rho z}{(1-\rho z)(1+k\sigma_{gu}^2)} = \frac{1-\lambda_g z}{1-\rho z}$ and therefore one can verify that $\boldsymbol{B}_g(z)\boldsymbol{V}_g\boldsymbol{B}_g^\top(z^{-1}) = \boldsymbol{M}_g(z)\boldsymbol{M}_g^\top(z^{-1}) \implies (\rho-\lambda_g)(1-\rho\lambda_g) = \lambda_g\tau_g$.

Let us now move to the forecast of *endogenous* variables. Consider a variable $f_t = A(L)\boldsymbol{s}_{lgt}$. Applying the Wiener-Hopf prediction filter, I can obtain the forecast as $\mathbb{E}_{lgt}f_t = \left[A(L)\boldsymbol{M}^\top(L^{-1})\boldsymbol{B}(L^{-1})^{-1}\right]_+ \boldsymbol{V}^{-1}\boldsymbol{B}(L)^{-1}x_{lgt}$, where $[\cdot]_+$ denotes the annihilator operator.

Recall from conditions (OA.9)-(OA.10) that I am interested in obtaining $\mathbb{E}_{lgt}v_t$, $\mathbb{E}_{lgt}\,p_{t-k}$, $\mathbb{E}_{lgt}\tilde{y}_{t-k}$, $k = \{-1,0,1\}$, $\mathbb{E}_{lgt}c_{i,t+1}$ and $\mathbb{E}_{lgt}\,p^*_{i,t+1}$. Just as I did in the example above, I need to find the $A(z)$ polynomial for each of the forecasted variables. Let us start from the exogenous fundamental v_t to verify that the Kalman and Wiener-Hopf filters result in the same forecast. I can write the fundamental as $v_t = \begin{bmatrix} \frac{\tau_\varepsilon^{-\frac{1}{2}}}{1-\rho L} & 0 \end{bmatrix} \boldsymbol{s}_{it} = A_v(L)\boldsymbol{s}_{it}$. Let me now move to the endogenous variables. I start from the household side. I need to guess (and verify) that each firm j's policy function takes the following form: $c_{it} = h_1(L)x_{l1t}$. Aggregate output can then be expressed as $\tilde{y}_t = \int h_1(L)x_{l1t}\,dj = h_1(L)\frac{\tau_\varepsilon^{-\frac{1}{2}}}{1-\rho L}\varepsilon_t^v$. Using the guesses, I have $\tilde{y}_{t-k} = \begin{bmatrix} h_1(L)L^k\frac{\tau_\varepsilon^{-\frac{1}{2}}}{1-\rho L} & 0 \end{bmatrix}\boldsymbol{s}_{l1t} = A_{yk}(L)\boldsymbol{s}_{l1t}$ and $c^*_{i,t+1} = \frac{h_1(L)}{L}\boldsymbol{M}_1(L)\boldsymbol{s}_{l1t} =$

$\left[h_1(L)\frac{\tau_\varepsilon^{-\frac{1}{2}}}{L(1-\rho L)} \quad \tau_{1u}^{-\frac{1}{2}}\frac{h_1(L)}{L}\right]\boldsymbol{s}_{l1t} = A_{i1}(L)\boldsymbol{s}_{l1t}$. Let me now move to firms. In this case I need to guess (and verify) that each firm j's policy function takes the following form: $p_{jt}^* = h_2(L)x_{l2t}$. The aggregate price level can then be expressed as $p_t = (1-\theta)h_2(L)\frac{\tau_\varepsilon^{-\frac{1}{2}}}{(1-\rho L)(1-\theta L)}\varepsilon_t^\nu$. Using the guesses, I have $p_{t-k} = \left[(1-\theta)\tau_\varepsilon^{-\frac{1}{2}}\frac{h_2(L)L^k}{(1-\rho L)(1-\theta L)} \quad 0\right]\boldsymbol{s}_{l2t} = A_{pk}(L)\boldsymbol{s}_{l2t}$ and $p_{j,t+1}^* = \frac{h_2(L)}{L}\boldsymbol{M}_2(L)\boldsymbol{s}_{l2t} = \left[\tau_\varepsilon^{-\frac{1}{2}}\frac{h_2(L)}{L(1-\rho L)} \quad \tau_2^{-\frac{1}{2}}\frac{h_2(L)}{L}\right]\boldsymbol{s}_{l2t} = A_{i2}(L)\boldsymbol{s}_{l2t}$. I am now armed with the necessary objects in order to obtain the five different forecasts,

$$\mathbb{E}_{lgt}\nu_t = \left[A_\nu(L)\boldsymbol{M}_g^\intercal(L^{-1})\boldsymbol{B}_g(L^{-1})^{-1}\right]_+\boldsymbol{V}_g^{-1}\boldsymbol{B}_g(L)^{-1}x_{lgt} = \left[\frac{L}{(1-\rho L)(L-\lambda_g)}\right]_+\frac{\lambda\tau_g}{\rho}\frac{1-\rho L}{1-\lambda_g L}x_{lgt}$$

$$= \left[\frac{\phi_\nu(L)}{L-\lambda_g}\right]_+\frac{\lambda_g\tau_g}{\rho}\frac{1-\rho L}{1-\lambda_g L}x_{lgt} = \frac{\phi_\nu(L)-\phi_\nu(\lambda_g)}{L-\lambda_g}\frac{\lambda_g\tau_g}{\rho}\frac{1-\rho L}{1-\lambda_g L}x_{lgt}, \quad \phi_\nu(z) = \frac{z}{1-\rho z}$$

(OA.18)

$$= \frac{\lambda_g\tau_g}{\rho(1-\rho\lambda_g)}\frac{1}{1-\lambda_g L}x_{lgt} = \left(1-\frac{\lambda_g}{\rho}\right)\frac{1}{1-\lambda_g L}x_{lgt} = G_{1g}(L)x_{lgt}$$

$$\mathbb{E}_{lgt}\widetilde{y}_{t-k} = \left[A_{yk}(L)\boldsymbol{M}_g^\intercal(L^{-1})\boldsymbol{B}_g(L^{-1})^{-1}\right]_+\boldsymbol{V}_g^{-1}\boldsymbol{B}_g(L)^{-1}x_{lgt} = \left[\frac{h_1(L)L^{k+1}}{(1-\rho L)(L-\lambda_g)}\right]_+\frac{\lambda_g\tau_g}{\rho}\frac{1-\rho L}{1-\lambda_g L}x_{lgt}$$

$$= \left[\frac{\phi_y(L)}{L-\lambda_g}\right]_+\frac{\lambda_g\tau_g}{\rho}\frac{1-\rho L}{1-\lambda_g L}x_{lgt} = \frac{\phi_y(L)-\phi_y(\lambda_g)}{L-\lambda_g}\frac{\lambda_g\tau_{gu}}{\rho\tau_\varepsilon}\frac{1-\rho L}{1-\lambda_g L}x_{lgt}, \quad \phi_y(z) = \frac{h_1(z)z^{k+1}}{1-\rho z}$$

(OA.19)

$$= \frac{\lambda_g\tau_g}{\rho}\left[h_1(L)L^{k+1} - h_1(\lambda_g)\lambda_g^{k+1}\frac{1-\rho L}{1-\rho\lambda_g}\right]\frac{1}{(1-\lambda_g L)(L-\lambda_g)}x_{lgt} = G_{2gk}(L)x_{lgt}$$

$$\mathbb{E}_{lgt}p_{t-k} = \left[A_{pk}(L)\boldsymbol{M}_g^\intercal(L^{-1})\boldsymbol{B}_g(L^{-1})^{-1}\right]_+\boldsymbol{V}_g^{-1}\boldsymbol{B}_g(L)^{-1}x_{lgt}$$

$$= \left[\frac{h_2(L)L^{k+1}}{(1-\rho L)(L-\lambda_g)(1-\theta L)}\right]_+\frac{(1-\theta)\lambda_g\tau_g}{\rho}\frac{1-\rho L}{1-\lambda_g L}x_{lgt}$$

$$= \left[\frac{\phi_\pi(L)}{L-\lambda_g}\right]_+\frac{(1-\theta)\lambda_g\tau_g}{\rho}\frac{1-\rho L}{1-\lambda_g L}x_{lgt}, \quad \phi_\pi(z) = \frac{h_2(z)z^{k+1}}{(1-\rho z)(1-\theta z)}$$

$$= \frac{\phi_\pi(L)-\phi_\pi(\lambda_g)}{L-\lambda_g}\frac{(1-\theta)\lambda_g\tau_g}{\rho}\frac{1-\rho L}{1-\lambda_g L}x_{lgt}$$

(OA.20)

$$= (1-\theta)\frac{\lambda_g\tau_g}{\rho}\left[\frac{h_2(L)L^{k+1}}{1-\theta L} - h_2(\lambda_g)\lambda_g^{k+1}\frac{1-\rho L}{(1-\rho\lambda_g)(1-\theta\lambda_g)}\right]\frac{1}{(1-\lambda_g L)(L-\lambda_g)}x_{lgt} = G_{3gk}(L)x_{lgt}$$

$$\mathbb{E}_{lgt}a_{lg,t+1} = \left[A_{ig}(L)\boldsymbol{M}_g^\intercal(L^{-1})\boldsymbol{B}_g(L^{-1})^{-1}\right]_+\boldsymbol{V}_g^{-1}\boldsymbol{B}_g(L)^{-1}x_{lgt}$$

$$
\begin{aligned}
&= \left[\frac{h_g(L)}{\tau_\varepsilon(1-\rho L)(L-\lambda_g)} + \frac{h_g(L)(L-\rho)}{\tau_{gu}L(L-\lambda_g)}\right]_+ \frac{\lambda_g\tau_{gu}}{\rho}\frac{1-\rho L}{1-\lambda_g L}x_{lgt} \\
&= \left\{\left[\frac{h_g(L)}{\tau_\varepsilon(1-\rho L)(L-\lambda_g)}\right]_+ + \left[\frac{h_g(L)(L-\rho)}{\tau_{gu}L(L-\lambda_g)}\right]_+\right\} \frac{\lambda_g\tau_{gu}}{\rho}\frac{1-\rho L}{1-\lambda_g L}x_{lgt} \\
&= \left\{\left[\frac{\phi_{ig,1}(L)}{L-\lambda_g}\right]_+ + \left[\frac{\phi_{ig,2}(L)}{L(L-\lambda_g)}\right]_+\right\} \frac{\lambda_g\tau_{gu}}{\rho}\frac{1-\rho L}{1-\lambda_g L}x_{lgt} \\
&= \left\{\frac{\phi_{ig,1}(L)-\phi_{ig,1}(\lambda_g)}{L-\lambda_g} + \frac{\phi_{ig,2}(L)-\phi_{ig,2}(\lambda_g)}{\lambda_g(L-\lambda_g)} - \frac{\phi_{ig,2}(L)-\phi_{ig,2}(0)}{\lambda_g L}\right\} \frac{\lambda_g\tau_{gu}}{\rho}\frac{1-\rho L}{1-\lambda_g L}x_{lgt} \\
&= \frac{\lambda_g}{\rho}\left\{\frac{h_g(L)}{L-\lambda_g}\left[\frac{\tau_{gu}}{\tau_\varepsilon(1-\rho L)} + \frac{L-\rho}{L}\right] - \frac{h_g(\lambda_g)}{L-\lambda_g}\left[\frac{\tau_{gu}}{\tau_\varepsilon(1-\rho\lambda_g)} + \frac{\lambda_g-\rho}{\lambda_g}\right] - \frac{\rho h_g(0)}{\lambda_g L}\right\}\frac{1-\rho L}{1-\lambda_g L}x_{lgt}
\end{aligned}
$$

(OA.21)

$$
= G_{4g}(L)x_{lgt}, \quad \phi_{ig,1}(z) = \frac{h_g(z)}{\tau_\varepsilon(1-\rho z)}, \quad \phi_{ig,2}(z) = \frac{h_g(z)(z-\rho)}{\tau_{gu}}
$$

where $\mathbb{E}_{l1t}a_{l1,t+1} = \mathbb{E}_{it}c_{i,t+1}$ and $\mathbb{E}_{l2t}a_{l2,t+1} = \mathbb{E}_{jt}p^*_{j,t+1}$. Rearranging terms, expectations satisfy

$$
\begin{aligned}
\mathbb{E}_{lgt}v_t &= \left(1-\frac{\lambda_g}{\rho}\right)\frac{1}{1-\lambda_g z}x_{lgt} = G_{1g}(z)x_{lgt} \\
\mathbb{E}_{lgt}a_{k,t-1} &= (1-\theta_k)\left(1-\frac{\lambda_g}{\rho}\right)\left[\frac{h_k(z)z^2(1-\rho\lambda_g)}{1-\theta_k z} - \frac{h_k(\lambda_g)\lambda_g^2(1-\rho z)}{1-\theta_k\lambda_g}\right]\frac{1}{(1-\lambda_g z)(z-\lambda_g)}x_{lgt} = G_{2k}(z)x_{lgt} \\
\mathbb{E}_{lgt}a_{k,t} &= (1-\theta_k)\left(1-\frac{\lambda_g}{\rho}\right)\left[\frac{h_k(z)z(1-\rho\lambda_g)}{1-\theta_k z} - \frac{h_k(\lambda_g)\lambda_g(1-\rho z)}{1-\theta_k\lambda_g}\right]\frac{1}{(1-\lambda_g z)(z-\lambda_g)}x_{lgt} = G_{3k}(z)x_{lgt} \\
\mathbb{E}_{lgt}a_{k,t+1} &= (1-\theta_k)\left(1-\frac{\lambda_g}{\rho}\right)\left[\frac{h_k(z)(1-\rho\lambda_g)}{1-\theta_k z} - \frac{h_k(\lambda_g)(1-\rho z)}{1-\theta_k\lambda_g}\right]\frac{1}{(1-\lambda_g z)(z-\lambda_g)}x_{lgt} = G_{4k}(z)x_{lgt} \\
\mathbb{E}_{lgt}a_{lg,t+1} &= \left\{\frac{h_g(z)}{z-\lambda_g}\left[\left(1-\frac{\lambda_g}{\rho}\right)\frac{1-\rho\lambda_g}{1-\rho z} + \frac{\lambda_g(z-\rho)}{\rho z}\right] - \frac{h_g(0)}{z}\right\}\frac{1-\rho z}{1-\lambda_g z}x_{lgt} = G_{5g}(z)x_{lgt}
\end{aligned}
$$

Recall the best response for household i and firm j, conditions (OA.9)-(OA.10). In order to be consistent with agent optimization, the policy functions $h_g(z)$ must satisfy (OA.9)-(OA.10) at all times and signals. Plugging the obtained expressions, I can write

$$
a_{lgt} = \varphi_g\mathbb{E}_{lgt}v_t + \beta_g\mathbb{E}_{lgt}a_{lg,t+1} + \sum_{j=1}^{2}\mu_{gj}\mathbb{E}_{lgt}a_{j,t-1} + \sum_{j=1}^{2}\gamma_{gj}\mathbb{E}_{lgt}a_{j,t} + \sum_{j=1}^{2}\alpha_{gj}\mathbb{E}_{lgt}a_{j,t+1}
$$

$$
h_g(L)x_{lgt} = \varphi_g G_{1g}(L)x_{lgt} + \beta_g G_{5g}(L)x_{lgt} + \sum_{j=1}^{2}\mu_{gj}G_{2j}(L)x_{lgt} + \sum_{j=1}^{2}\gamma_{gj}G_{3j}(L)x_{lgt} + \sum_{j=1}^{2}\alpha_{gj}G_{4j}(L)x_{lgt}
$$

$$
\begin{aligned}
h_g(z) &= \varphi_g G_{1g}(z) + \beta_g G_{5g}(z) + \sum_{j=1}^{2} \mu_{gj} G_{2j}(z) + \sum_{j=1}^{2} \gamma_{gj} G_{3j}(z) + \sum_{j=1}^{2} \alpha_{gj} G_{4j}(z) \\
&= \varphi_g \left(1 - \frac{\lambda_g}{\rho}\right) \frac{1}{1-\lambda_g z} + \beta_g \left\{ \frac{h_g(z)}{z-\lambda_g} \left[\left(1-\frac{\lambda_g}{\rho}\right) \frac{1-\rho\lambda_g}{1-\rho z} + \frac{\lambda_g(z-\rho)}{\rho z} \right] - \frac{h_g(0)}{z} \right\} \frac{1-\rho z}{1-\lambda_g z} \\
&+ \sum_{j=1}^{2} \mu_{gj}(1-\theta_j) \left(1-\frac{\lambda_g}{\rho}\right) \left[\frac{h_j(z) z^2 (1-\rho\lambda_g)}{1-\theta_j z} - \frac{h_j(\lambda_g)\lambda_g^2(1-\rho z)}{1-\theta_j\lambda_g} \right] \frac{1}{(1-\lambda_g z)(z-\lambda_g)} \\
&+ \sum_{j=1}^{2} \gamma_{gj}(1-\theta_j) \left(1-\frac{\lambda_g}{\rho}\right) \left[\frac{h_j(z) z (1-\rho\lambda_g)}{1-\theta_j z} - \frac{h_j(\lambda_g)\lambda_g(1-\rho z)}{1-\theta_j\lambda_g} \right] \frac{1}{(1-\lambda_g z)(z-\lambda_g)} \\
&+ \sum_{j=1}^{2} \alpha_{gj}(1-\theta_j) \left(1-\frac{\lambda_g}{\rho}\right) \left[\frac{h_j(z)(1-\rho\lambda_g)}{1-\theta_j z} - \frac{h_j(\lambda_g)(1-\rho z)}{1-\theta_j\lambda_g} \right] \frac{1}{(1-\lambda_g z)(z-\lambda_g)}
\end{aligned}
$$

where $\varphi_1 = -\frac{\beta}{\sigma}$, $\beta_1 = \beta$, $\mu_{11} = 0$, $\mu_{12} = \frac{\beta\phi_\pi}{\sigma}$, $\gamma_{11} = 1 - \beta\left(1 + \frac{\phi_y}{\sigma}\right)$, $\gamma_{12} = -\frac{\beta(1+\phi_\pi)}{\sigma}$, $\alpha_{11} = 0$, $\alpha_{12} = \frac{\beta}{\sigma}$, $\theta_1 = 0$, $\varphi_2 = 0$, $\beta_2 = \beta\theta$, $\mu_{21} = 0$, $\mu_{22} = 0$, $\gamma_{21} = \frac{\kappa\theta}{1-\theta}$, $\gamma_{22} = 1 - \beta\theta$, $\alpha_{21} = 0$, $\alpha_{22} = 0$ and $\theta_2 = \theta$. Multiplying both sides by $z(z-\lambda_g)(1-\lambda_g z)(1-\theta_1 z)(1-\theta_2 z)$ I obtain

$$
\begin{aligned}
&h_g(z) z(z-\lambda_g)(1-\lambda_g z)(1-\theta_1 z)(1-\theta_2 z) = \varphi_g \left(1-\frac{\lambda_g}{\rho}\right) z(z-\lambda_g)(1-\theta_1 z)(1-\theta_2 z) \\
&+ \beta_g \left\{ h_g(z) \left[\left(1-\frac{\lambda_g}{\rho}\right)(1-\rho\lambda_g) z + \frac{\lambda_g}{\rho z}(z-\rho)(1-\rho z) \right] - h_g(0)(z-\lambda_g)(1-\rho z) \right\} (1-\theta_1 z)(1-\theta_2 z) \\
&+ \sum_{j=1}^{2} \mu_{gj}(1-\theta_j)\left(1-\frac{\lambda_g}{\rho}\right) \left[h_j(z) z^3 (1-\rho\lambda_g)(1-\theta_{\neg j} z) - \frac{h_j(\lambda_g)\lambda_g^2 z(1-\rho z)(1-\theta_1 z)(1-\theta_2 z)}{1-\theta_j\lambda_g} \right] \\
&+ \sum_{j=1}^{2} \gamma_{gj}(1-\theta_j)\left(1-\frac{\lambda_g}{\rho}\right) \left[h_j(z) z^2 (1-\rho\lambda_g)(1-\theta_{\neg j} z) - \frac{h_j(\lambda_g)\lambda_g z(1-\rho z)(1-\theta_1 z)(1-\theta_2 z)}{1-\theta_j\lambda_g} \right] \\
&+ \sum_{j=1}^{2} \alpha_{gj}(1-\theta_j)\left(1-\frac{\lambda_g}{\rho}\right) \left[h_j(z) z (1-\rho\lambda_g)(1-\theta_{\neg j} z) - \frac{h_j(\lambda_g) z(1-\rho z)(1-\theta_1 z)(1-\theta_2 z)}{1-\theta_j\lambda_g} \right]
\end{aligned}
$$

Rearranging the LHS by $h_g(z)$,

$$
\begin{aligned}
&h_g(z) \left\{ z(z-\lambda_g)(1-\lambda_g z)(1-\theta_1 z)(1-\theta_2 z) - \beta_g \left[\left(1-\frac{\lambda_g}{\rho}\right)(1-\rho\lambda_g) z + \frac{\lambda_g}{\rho z}(z-\rho)(1-\rho z) \right] (1-\theta_1 z)(1-\theta_2 z) \right\} \\
&\quad - \sum_{j=1}^{2} \mu_{gj}(1-\theta_j)\left(1-\frac{\lambda_g}{\rho}\right) z^3 (1-\rho\lambda_g)(1-\theta_{\neg j} z) h_j(z)
\end{aligned}
$$

$$
\begin{aligned}
&- \sum_{j=1}^{2} \gamma_{gj}(1-\theta_j)\left(1-\frac{\lambda_g}{\rho}\right) z^2(1-\rho\lambda_g)(1-\theta_{\neg j}z)h_j(z) \\
&- \sum_{j=1}^{2} \alpha_{gj}(1-\theta_j)\left(1-\frac{\lambda_g}{\rho}\right) z(1-\rho\lambda_g)(1-\theta_{\neg j}z)h_j(z)
\end{aligned}
$$

and the RHS can be rewritten as

$$
\begin{aligned}
d_g(z) = {} & \varphi_g\left(1-\frac{\lambda_g}{\rho}\right) z(z-\lambda_g)(1-\theta_1 z)(1-\theta_2 z) - h_g(0)\beta_g(z-\lambda_g)(1-\rho z)(1-\theta_1 z)(1-\theta_2 z) \\
& - \left\{\left(1-\frac{\lambda_g}{\rho}\right)\sum_{j=1}^{2}\frac{1-\theta_j}{1-\theta_j\lambda_g}[\mu_{gj}\lambda_g^2+\gamma_{gj}\lambda_g+\alpha_{gj}]h_j(\lambda_g)\right\} z(1-\rho z)(1-\theta_1 z)(1-\theta_2 z) \\
= {} & \varphi_g\left(1-\frac{\lambda_g}{\rho}\right) z(z-\lambda_g)(1-\theta_1 z)(1-\theta_2 z) - h_g(0)\beta_g(z-\lambda_g)(1-\rho z)(1-\theta_1 z)(1-\theta_2 z) \\
& - \widetilde{h}_g z(1-\rho z)(1-\theta_1 z)(1-\theta_2 z)
\end{aligned}
$$

where $\widetilde{h}_g = \left(1-\frac{\lambda_g}{\rho}\right)\sum_{j=1}^{2}\frac{1-\theta_j}{1-\theta_j\lambda_g}[\mu_{gj}\lambda_g^2+\gamma_{gj}\lambda_g+\alpha_{gj}]h_j(\lambda_g)$. I can write the system in matrix form as $\boldsymbol{C}(z)\boldsymbol{h}(z) = \boldsymbol{d}(z)$, where

$$
\boldsymbol{C}(z) = \begin{bmatrix} C_{11}(z) & C_{12}(z) \\ C_{21}(z) & C_{22}(z) \end{bmatrix}, \qquad \boldsymbol{h}(z) = \begin{bmatrix} h_1(z) \\ h_2(z) \end{bmatrix}, \qquad \boldsymbol{d}(z) = \begin{bmatrix} d_1(z) \\ d_2(z) \end{bmatrix}
$$

$$
\begin{aligned}
C_{gg}(z) &= (z-\beta_g)(z-\lambda_g)(1-\lambda_g z)(1-\theta_1 z)(1-\theta_2 z) \\
&\qquad - (1-\theta_g)\left(1-\frac{\lambda_g}{\rho}\right)(1-\rho\lambda_g)(1-\theta_{\neg g}z)z(\mu_{gg}z^2+\gamma_{gg}z+\alpha_{gg}) \\
C_{gn}(z) &= -(1-\theta_n)\left(1-\frac{\lambda_g}{\rho}\right)(1-\rho\lambda_g)(1-\theta_g z)(\mu_{gn}z^3+\gamma_{gn}z^2+\alpha_{gn}z) \\
d_g(z) &= \left[\varphi_g\left(1-\frac{\lambda_g}{\rho}\right) z(z-\lambda_g) - h_g(0)\beta_g(z-\lambda_g)(1-\rho z) - \widetilde{h}_g z(1-\rho z)\right](1-\theta_1 z)(1-\theta_2 z)
\end{aligned}
$$

Cancelling out parameters equal to zero to simplify the expressions, I can write

$$
\begin{aligned}
C_{11}(z) &= \left[(z-\beta_1)(z-\lambda_1)(1-\lambda_1 z) - \left(1-\frac{\lambda_1}{\rho}\right)(1-\rho\lambda_1)\gamma_{11}z^2\right](1-\theta_2 z) \\
C_{12}(z) &= -(1-\theta_2)\left(1-\frac{\lambda_1}{\rho}\right)(1-\rho\lambda_1)z(\mu_{12}z^2+\gamma_{12}z+\alpha_{12}) \\
C_{21}(z) &= -\left(1-\frac{\lambda_2}{\rho}\right)(1-\rho\lambda_2)(1-\theta_2 z)\gamma_{21}z^2
\end{aligned}
$$

$$C_{22}(z) = (z-\beta_2)(z-\lambda_2)(1-\lambda_2 z)(1-\theta_2 z) - (1-\theta_2)\left(1-\frac{\lambda_2}{\rho}\right)(1-\rho\lambda_2)\gamma_{22}z^2$$
$$d_1(z) = \left[\varphi_1\left(1-\frac{\lambda_1}{\rho}\right)z(z-\lambda_1) - h_1(0)\beta_1(z-\lambda_1)(1-\rho z) - \widetilde{h}_1 z(1-\rho z)\right](1-\theta_2 z)$$
$$d_2(z) = \left[-h_g(0)\beta_2(z-\lambda_2)(1-\rho z) - \widetilde{h}_2 z(1-\rho z)\right](1-\theta_2 z)$$

and the solution to the policy functions is given by $\boldsymbol{h}(z) = \boldsymbol{C}(z)^{-1}\boldsymbol{d}(z) = \frac{\operatorname{adj}\boldsymbol{C}(z)}{\det\boldsymbol{C}(z)}\boldsymbol{d}(z)$.

Note that the degree of $\boldsymbol{C}(z)$ is 8, given that $\theta_1 = 0$. Denote the inside roots of $\det\boldsymbol{C}(z)$ as $\{\zeta_1, \zeta_2, ..., \zeta_{n_1}\}$ and the outside roots as $\{\vartheta_1^{-1}, \vartheta_2^{-1}, ..., \vartheta_{n_1}^{-1}\}$. Because agents cannot use future signals, the inside roots have to be removed. Note that the number of free constants in $\boldsymbol{d}$ is 4: $\{h_g(0), \widetilde{h}_g\}_{g=1}^2$. For a unique solution, it must be the case that the number of outside roots is $n_2 = 4$. Also note that by Cramer's rule, $h_g(z)$ is given by

$$h_1(z) = \frac{\det\begin{bmatrix} d_1(z) & C_{12}(z) \\ d_2(z) & C_{22}(z)\end{bmatrix}}{\det\boldsymbol{C}(z)}, \qquad h_2(z) = \frac{\det\begin{bmatrix} C_{11}(z) & d_1(z) \\ C_{21}(z) & d_2(z)\end{bmatrix}}{\det\boldsymbol{C}(z)}$$

The degree of the numerator is 7, as the highest degree of $d_g(z)$ is 1 degree less than $C_{gg}(z)$. By choosing the constants $\{h_g(0), \widetilde{h}_g\}_{g=1}^2$, the 4 inside roots will be removed. Therefore, the 4 constants are solutions to the following system of linear equations[6]

$$\det\begin{bmatrix} d_1(\zeta_n) & C_{12}(\zeta_n) \\ d_2(\zeta_n) & C_{22}(\zeta_n)\end{bmatrix} = 0, \qquad \text{for } \{\zeta_n\}_{n=1}^4$$

where $n_2 = 4$. After removing the inside roots in the denominator, the degree of the numerator is 3 and the degree of the denominator is 4. As a result, the solution to $h_g(z)$ takes the form

$$h_g(z) = \frac{\widetilde{\psi}_{g1} + \widetilde{\psi}_{g2}z + \widetilde{\psi}_{g3}z^2 + \widetilde{\psi}_{g4}z^3}{(1-\vartheta_1 z)(1-\vartheta_2 z)(1-\vartheta_3 z)(1-\vartheta_4 z)}$$

Given the model conditions, I have that $\vartheta_4 = \theta$. I can write

$$h_g(z) = \frac{\widetilde{\psi}_{g1} + \widetilde{\psi}_{g2}z + \widetilde{\psi}_{g3}z^2 + \widetilde{\psi}_{g4}z^3}{(1-\vartheta_1 z)(1-\vartheta_2 z)(1-\vartheta_3 z)(1-\theta z)} = \frac{\widetilde{\psi}_{g4}(z-\eta_{g1})(z-\eta_{g2})(z-\eta_{g3})}{(1-\vartheta_1 z)(1-\vartheta_2 z)(1-\vartheta_3 z)(1-\theta z)}$$

[6]The set of constants that solve the system of equations for $h_1(z)$ also solves it for $h_2(z)$, since $\{\zeta_n\}_{n=1}^4$ are roots of $\det\boldsymbol{C}(z)$, leaving vectors in $\boldsymbol{C}(\zeta_n)$ being linearly dependent.

$$= \frac{-\widetilde{\psi}_{g4}\eta_{g1}\eta_{g2}\eta_{g3}(1-\eta_{g1}^{-1}z)(1-\eta_{g2}^{-1}z)(1-\eta_{g3}^{-1}z)}{(1-\vartheta_1 z)(1-\vartheta_2 z)(1-\vartheta_3 z)(1-\theta z)} = \frac{-\widetilde{\psi}_{g4}\eta_{g1}\eta_{g2}\eta_{g3}(1-\xi_{g1}z)(1-\xi_{g2}z)(1-\xi_{g3}z)}{(1-\vartheta_1 z)(1-\vartheta_2 z)(1-\vartheta_3 z)(1-\theta z)}$$

where $(\eta_{g1}, \eta_{g2}, \eta_{g3})$ are the roots of $\widetilde{\psi}_{g1} + \widetilde{\psi}_{g2}z + \widetilde{\psi}_{g3}z^2 + \widetilde{\psi}_{g4}z^3$. I also have that $\xi_{13} = \xi_{22} = \xi_{23} = \theta$. Hence, I can write

$$\begin{aligned}
\widetilde{y}_t &= h_1(z)\nu_t = \frac{-\widetilde{\psi}_{14}\eta_{11}\eta_{12}\eta_{13}(1-\xi_{11}z)(1-\xi_{12}z)}{(1-\vartheta_1 z)(1-\vartheta_2 z)(1-\vartheta_3 z)}\nu_t = \phi_1\frac{(1-\xi_{11}z)(1-\xi_{12}z)}{(1-\vartheta_1 z)(1-\vartheta_2 z)(1-\vartheta_3 z)}\nu_t \\
&= \psi_{11}\left(1-\frac{\vartheta_1}{\rho}\right)\frac{1}{1-\vartheta_1 z}\nu_t + \psi_{12}\left(1-\frac{\vartheta_2}{\rho}\right)\frac{1}{1-\vartheta_2 z}\nu_t + \psi_{13}\left(1-\frac{\vartheta_1}{\rho}\right)\frac{1}{1-\vartheta_3 z}\nu_t \\
&= \psi_{11}\widetilde{\vartheta}_{1t} + \psi_{12}\widetilde{\vartheta}_{2t} + \psi_{13}\widetilde{\vartheta}_{3t} \\
p_t &= (1-\theta)h_2(z)\frac{1}{1-\theta z}\nu_t = \frac{-\widetilde{\psi}_{24}\eta_{21}\eta_{22}\eta_{23}(1-\theta)(1-\xi_{21}z)}{(1-\vartheta_1 z)(1-\vartheta_2 z)(1-\vartheta_3 z)}\nu_t = \phi_2\frac{1-\xi_{21}z}{(1-\vartheta_1 z)(1-\vartheta_2 z)(1-\vartheta_3 z)}\nu_t \\
&= \psi_{21}\left(1-\frac{\vartheta_1}{\rho}\right)\frac{1}{1-\vartheta_1 z}\nu_t + \psi_{22}\left(1-\frac{\vartheta_2}{\rho}\right)\frac{1}{1-\vartheta_2 z}\nu_t + \psi_{23}\left(1-\frac{\vartheta_1}{\rho}\right)\frac{1}{1-\vartheta_3 z}\nu_t \\
&= \psi_{21}\widetilde{\vartheta}_{1t} + \psi_{22}\widetilde{\vartheta}_{2t} + \psi_{23}\widetilde{\vartheta}_{3t}
\end{aligned}$$

Using $\pi_t = (1-L)\,p_t$, I can write

$$\begin{aligned}
\pi_t &= (1-\theta)h_2(z)\frac{1-z}{1-\theta z}\nu_t = \frac{-\widetilde{\psi}_{24}\eta_{21}\eta_{22}\eta_{23}(1-\theta)(1-\xi_{21}z)(1-z)}{(1-\vartheta_1 z)(1-\vartheta_2 z)(1-\vartheta_3 z)}\nu_t = \phi_2\frac{(1-\xi_{21}z)(1-z)}{(1-\vartheta_1 z)(1-\vartheta_2 z)(1-\vartheta_3 z)}\nu_t \\
&= \psi_{31}\left(1-\frac{\vartheta_1}{\rho}\right)\frac{1}{1-\vartheta_1 z}\nu_t + \psi_{32}\left(1-\frac{\vartheta_2}{\rho}\right)\frac{1}{1-\vartheta_2 z}\nu_t + \psi_{33}\left(1-\frac{\vartheta_1}{\rho}\right)\frac{1}{1-\vartheta_3 z}\nu_t \\
&= \psi_{31}\widetilde{\vartheta}_{1t} + \psi_{32}\widetilde{\vartheta}_{2t} + \psi_{33}\widetilde{\vartheta}_{3t}
\end{aligned}$$

I can finally write

$$\boldsymbol{a}_t = \begin{bmatrix} \widetilde{y}_t \\ p_t \\ \pi_t \end{bmatrix} = Q\widetilde{\vartheta}_t = \begin{bmatrix} \psi_{11} & \psi_{12} & \psi_{13} \\ \psi_{21} & \psi_{22} & \psi_{23} \\ \psi_{31} & \psi_{32} & \psi_{33} \end{bmatrix} \begin{bmatrix} \widetilde{\vartheta}_{1t} \\ \widetilde{\vartheta}_{2t} \\ \widetilde{\vartheta}_{3t} \end{bmatrix}$$

where $\widetilde{\vartheta}_{kt}(1-\vartheta_k L) = \left(1-\frac{\vartheta_k}{\rho}\right)\nu_t \implies \widetilde{\vartheta}_{kt} = \vartheta_k\widetilde{\vartheta}_{k,t-1} + \left(1-\frac{\vartheta_k}{\rho}\right)\nu_t$, which I can write as a system as $\widetilde{\vartheta}_t = \Lambda\widetilde{\vartheta}_{t-1} + \Gamma\nu_t$, where

$$\Lambda = \begin{bmatrix} \vartheta_1 & 0 & 0 \\ 0 & \vartheta_2 & 0 \\ 0 & 0 & \vartheta_3 \end{bmatrix}, \qquad \Gamma = \begin{bmatrix} 1-\frac{\vartheta_1}{\rho} \\ 1-\frac{\vartheta_2}{\rho} \\ 1-\frac{\vartheta_3}{\rho} \end{bmatrix}$$

Hence, I can write $\boldsymbol{a}_t = Q\widetilde{\theta}_t = Q(\Lambda\widetilde{\theta}_{t-1} + \Gamma\nu_t) = Q\Lambda\widetilde{\theta}_{t-1} + Q\Gamma\nu_t = Q\Lambda Q^{-1}\boldsymbol{a}_{t-1} + Q\Gamma\nu_t = A\boldsymbol{a}_{t-1} + B\xi_t$. $\square$

OA.3. History of Fed's Gradual Transparency

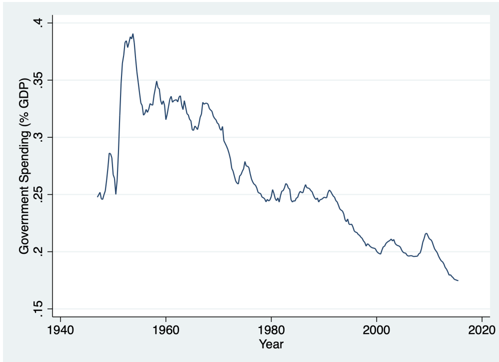
Fed's actions have become more transparent over time. Before 1967 the FOMC only announced policy decisions once a year in the Annual Report. The report also included the Memoranda of Discussion (MOD) containing the minutes of the meeting, released with a 5-year lag since 1935. In 1967, the FOMC decided to release the directive in the PR, 90 days after the decision. The rationale for maintaining a delay was that earlier disclosure would interfere with CB best practices due to political pressure, both from the Administration and from Congress. In a letter from Chairman Burns to Senator Proxmire on August 1972, Burns enumerated six reasons for deferment of availability. Among them, Burns argued that earlier disclosure could interfere with the execution of policies, permit speculators to gain unfair profits by trading in securities, foreign exchange, etc., result in unwarranted disturbances in the asset market, or affect transactions with foreign governments or banks. In the same letter, Burns hypothesized reducing the delay to shorter than 90 days, although stressing that a few hours/days delay would harm the Fed.

In March 1975 David R. Merril, a student at Georgetown University, requested *current* MOD to be disclosed based on the Freedom of Information Act (FOIA). Congressman Patman supported this initiative and officially asked Chairman Burns for the unedited MOD from the period 1971-1974. Burns declined to comply with the request.[7] At the same time, the FOMC formed a subcommittee on the matter, which suggested cutting back substantially on details about the members' forecasts and to allow each member to edit the minutes, but discouraged eliminating the MODs. In May 1976, concerned about the chance of premature disclosure, the FOMC discontinued the MOD arguing that it had not been a useful tool.[8,9] The decision increased the ire of several critics of the Fed. In the coming years, Congress took several actions to protect the premature release of the minutes, in order to convince the Fed to reinstate the MOD, with no success. Contemporaneously to these events, in May 1976 the PR increased its length (expanded to include short-run and long-run members' forecasts) and reduced the delay to 45 days, shortly after the next (monthly) meeting.

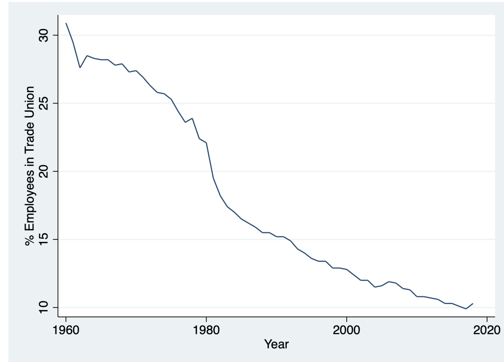
[7]The letter exchange is available at Lindsey (2003), pp. 11-15.

[8]Robert P. Black, former president of the Richmond Fed that served at the FOMC, explained years later that "I did it for the fear that Congress would request access quite promptly" (see Lindsey (2003), p. 22).

[9]Whether meetings were still recorded was unclear to the public, until Chairman Greenspan revealed their existence in October 1993, causing a stir.



A. Real government spending as a share of real GDP.



B. Percentage of workers members of Trade Union.

Figure OA.7. Time series.

Merrill's lawsuit included the request for an immediate release of the directive (the Fed decision). On November 1977 the Court of Appeals for the District of Columbia ruled in Merrill's favor in this regard. In January 1978, Burns asked Senator Proxmire for legislative relief from the requirement. Finally, in June 1979 the Supreme Court ruled in the FOMC's favor.

Between 1976 and 1993 the information contained in the PR was significantly enlarged, without further changes in the announcement delay. In November 1977 the *Federal Reserve Reform Act* officially entitled the Fed with 3 objectives: maximum employment, stable prices, and moderate long-term interest rates. In July 1979, the first individual macroeconomic forecasts on (annual) real GNP growth, GNP inflation, and unemployment from FOMC members were made available. During this period, the Fed was widely criticized for the rise in inflation (see Figure 1). The FOMC stressed in their communication that the increase in inflation was due to excessive fiscal policy stimulus (see Figure OA.7A) and the cost-push shock on real wages coming from the increased worker unionization (see Figure OA.7B).

From October 1979 to November 1989 the policy instrument changed from the fed funds rate to non-borrowed reserves (M1, until Fall 1982) and borrowed reserves (M2 and M3, thereafter), respectively. In the early 1980s, the Fed had not established an inflation target yet. Instead, the focus was on stabilizing monetary aggregates, M1 growth in particular. However, frequent and volatile changes in money demand made it particularly challenging for the Fed to deliver stable monetary aggregates. The aspects of these operational procedures were not explained to the public during 1982.

The "tilt" (predisposition or likelihood regarding possible future action) was introduced in the PR in November 1983. Between March 1985 and December 1991 the Fed introduced the "ranking of policy factors", which after each meeting ranked aggregate macro variables in importance, signaling priorities with regard to possible future adjustments. During this period the FOMC members started discussing internally the possibility of reducing the delay of announcements. An internal report from November 1982 summarizes the benefits, calling for democratic public institutions, reducing the criticism due to excessive secrecy, and the induced misallocation of resources by firms, somehow forced to hire "Fed watchers". Yet, the cons, remained similar to those expressed in 1972. In fact, Chairman Volcker defended the Fed's translucent policy in two letters to Representative Fauntroy in August 1984 and Senator Mattingly in July 1985.

Until then, the FOMC had been successful in convincing politicians and the judicial system that its secrecy was grounded in a purely economic rationale, and was not the result of an arbitrary decision. The first critique from the academic profession came from Goodfriend (1986), which argued that opaqueness reduces the power of monetary policy by distorting agents' reactions. Cukierman and Meltzer (1986) formalize a theoretical framework in which credibility and reputation induce rich dynamics around a low-inflation steady state. Blinder (2000); Bernanke et al. (1999) stressed the benefits of a more transparent policy, such as inflation targeting. Faust and Svensson (2001) build a framework in which the CB cares about its reputation, and identifies a potential conflict between society and the CB: the general public wants full transparency, while the CB prefers minimal transparency. Faust and Svensson (2002) extend their results by endogenizing the choice of transparency and the degree of control that the CB has.

After the successful disinflation episode in the mid-1980s, the Fed gained a reputation, not fearing the criticism of further tightening in the policy stance. As a result, the FOMC was subject to little political interference, which together with the criticism coming from the academic profession led them to increase transparency. The minutes, a revised transcript of the discussions during the meeting were reintroduced into the PR in March 1993 under Chairman Greenspan. In 1994 the FOMC introduced the immediate release of the PR after a meeting if there had been a decision, coupled with an immediate release of the "tilt" since 1999. Since January 2000 there is an immediate announcement and press conference after each meeting, regardless of the decision.

OA.4. Derivation of the General New Keynesian Model

OA.4.1. Households

There is a continuum of infinitely-lived, ex-ante identical households indexed by $i \in \mathcal{I}_h = [0, 1]$ seeking to maximize $\mathbb{E}_{i0} \sum_{t=0}^{\infty} \beta^t U(C_{it}, N_{it})$, where the utility function takes a standard CRRA shape $U(C,N) = \frac{C^{1-\sigma}}{1-\sigma} - \frac{N^{1+\varphi}}{1+\varphi}$. Notice that I relax the benchmark framework and assume that households might differ in their beliefs and their expectation formation. Furthermore, the consumption index C_{it} is given by $C_{it} = \left(\int_{\mathcal{I}_f} C_{ijt}^{\frac{\epsilon-1}{\epsilon}} dj\right)^{\frac{\epsilon}{\epsilon-1}}$, with C_{ijt} denoting the quantity of good j consumed by household i in period t, and ϵ denotes the elasticity between goods. Here I have assumed that each consumption good is indexed by $j \in \mathcal{I}_f = [0,1]$. Given the different good varieties, the household must decide how to optimally allocate its limited expenditure on each good j. A cost-minimization problem yields

$$C_{ijt} = \left(\frac{P_{jt}}{P_t}\right)^{-\epsilon} C_{it} \tag{OA.22}$$

where the aggregate price index is defined as $P_t \equiv \left(\int_{\mathcal{I}_f} P_{jt}^{1-\epsilon} dj\right)^{\frac{1}{1-\epsilon}}$. Using the above conditions, one can show that $\int_{\mathcal{I}_f} P_{jt} C_{ijt}\, dj = P_t C_{it}$.

I can now state the household-level budget constraint. In real terms, households decide how much to consume, work and save subject to the following restriction

$$C_{it} + B_{it} = R_{t-1} B_{i,t-1} + W_t^r N_{it} + D_t \tag{OA.23}$$

where N_{it} denotes employment (or hours worked) by household i, B_{it} denotes savings (or bond purchases) by household i, R_{t-1} denotes the gross real return on savings, W_t^r denotes the real wage at time t, and D_t denotes dividends received from the profits produced by firms. The optimality conditions from the household problem satisfy $C_{it}^{-\sigma} = \beta \mathbb{E}_{it}\left(R_t C_{i,t+1}^{-\sigma}\right)$ and $C_{it}^{\sigma} N_{it}^{\varphi} = \mathbb{E}_{it} W_t^r$.

Let us now focus on the budget constraint. Define $A_{it} = R_{t-1} B_{i,t-1}$ as consumer i's initial asset position in period t. Rewrite (OA.23) at $t+1$

$$C_{it+1} + B_{it+1} = R_t B_{i,t} + W_{t+1}^r N_{it+1} + D_{t+1} \tag{OA.24}$$

Combining (OA.23) and (OA.24) I can write $C_{it} + (C_{it+1} + B_{it+1}) R_t^{-1} = A_{it} + W_t^r N_{it} + D_t + (W_{t+1}^r N_{it+1} + D_{t+1}) R_t^{-1}$. Doing this until $T \to \infty$ I obtain $\sum_{k=0}^{\infty} \prod_{j=1}^{k} \frac{1}{R_{t+j-1}} C_{it+k} = A_{it} + \sum_{k=0}^{\infty} \prod_{j=1}^{k} \frac{1}{R_{t+j-1}} (W_{t+k}^r N_{it+k} + D_{t+k})$. Log-linearizing the above condition around a zero inflation steady-state I obtain

$$\sum_{k=0}^{\infty} \beta^k c_{it+k} = a_{it} + \Omega_i \sum_{k=0}^{\infty} \beta^k (w_{t+k}^r + n_{it+k}) + (1-\Omega_i) \sum_{k=0}^{\infty} \beta^k d_{t+k} \tag{OA.25}$$

where a lower case letter denotes the log deviation from steady state, i.e., $x_t = \log X_t - \log X$, except for the initial asset position, defined as $a_{it} = A_{it}/C_i$; and Ω_i denotes the labor income share for household i.

The optimal intratemporal labor supply condition can be log-linearized to

$$\mathbb{E}_{it} w_t^r = \sigma c_{it} + \varphi n_{it} \tag{OA.26}$$

and the intertemporal Euler condition can be log-linearized to

$$c_{it} = -\frac{1}{\sigma}\mathbb{E}_{it} r_t + \mathbb{E}_{it} c_{it+1} \tag{OA.27}$$

where I define the ex-post real interest rate as $r_t = i_t - \pi_{t+1}$.

I want to obtain the optimal expenditure of household i in period t as a function of the current a future expected wages, dividends and real interest rates. Using (OA.26) and taking expectations, I can rearrange (OA.25) as

$$\begin{aligned}\sum_{k=0}^{\infty} \beta^k \mathbb{E}_{it} c_{it+k} &= a_{it} + \Omega_i \sum_{k=0}^{\infty} \beta^k \mathbb{E}_{it}\left(\frac{1+\varphi}{\varphi} w_{t+k}^r - \frac{\sigma}{\varphi} c_{it+k}\right) + (1-\Omega_i)\sum_{k=0}^{\infty} \beta^k \mathbb{E}_{it} d_{t+k} \\ &= \frac{\varphi}{\varphi+\sigma\Omega_i} a_{it} + \sum_{k=0}^{\infty} \beta^k \mathbb{E}_{it}\left[\frac{\Omega_i(1+\varphi)}{\varphi+\sigma\Omega_i} w_{t+k}^r + \frac{(1-\Omega_i)\varphi}{\varphi+\sigma\Omega_i} d_{t+k}\right] \end{aligned} \tag{OA.28}$$

Let us now focus on the left-hand side. Taking individual expectations, I can rewrite it as $\sum_{k=0}^{\infty} \beta^k \mathbb{E}_{it} c_{it+k}$. Keeping this aside, I can rearrange (OA.27) as $\mathbb{E}_{it} c_{it+1} = c_{it} + \frac{1}{\sigma}\mathbb{E}_{it} r_t$. Iterating (OA.27) one period forward, I can similarly write $\mathbb{E}_{it} c_{it+2} = c_{it} + \frac{1}{\sigma}\mathbb{E}_{it}(r_t + r_{t+1})$ and, for a general k, $\mathbb{E}_{it} c_{it+k} = c_{it} + \frac{1}{\sigma}\sum_{j=0}^{k} \mathbb{E}_{it} r_{t+j}$. That is, I can write

$$\sum_{k=0}^{\infty} \beta^k \mathbb{E}_{it} c_{it+k} = \sum_{k=0}^{\infty} \beta^k c_{it} + \frac{1}{\sigma}\sum_{k=0}^{\infty}\sum_{j=0}^{k} \beta^k \mathbb{E}_{it} r_{t+j} = \frac{1}{1-\beta} c_{it} + \frac{\beta}{\sigma(1-\beta)}\sum_{k=0}^{\infty} \beta^k \mathbb{E}_{it} r_{t+k}$$

Inserting this last condition into (OA.28), I can write

$$c_{it} = -\frac{\beta}{\sigma}\sum_{k=0}^{\infty} \beta^k \mathbb{E}_{it} r_{t+k} + \frac{\varphi(1-\beta)}{\varphi+\sigma\Omega} a_{it} + \sum_{k=0}^{\infty} \beta^k \mathbb{E}_{it}\left[\frac{\Omega_i(1+\varphi)(1-\beta)}{\varphi+\sigma\Omega} w_{t+k}^r + \frac{(1-\Omega_i)\varphi(1-\beta)}{\varphi+\sigma\Omega} d_{t+k}\right]$$

Aggregating, using the fact that assets are in zero net supply, $\int_{\mathcal{I}_h} a_{it}\, di = a_t = 0$,

$$c_t = -\frac{\beta}{\sigma}\sum_{k=0}^{\infty} \beta^k \overline{\mathbb{E}}_t^h r_{t+k} + \sum_{k=0}^{\infty} \beta^k \left[\frac{\Omega(1+\varphi)(1-\beta)}{\varphi+\sigma\Omega}\overline{\mathbb{E}}_t^h w_{t+k}^r + \frac{(1-\Omega)\varphi(1-\beta)}{\varphi+\sigma\Omega}\overline{\mathbb{E}}_t^h d_{t+k}\right] \tag{OA.29}$$

where $\overline{\mathbb{E}}_t^h(\cdot) = \int_{\mathcal{I}_c} \mathbb{E}_{it}(\cdot)\, di$ is the average household expectation operator in period t.

OA.4.2. Firms

As in the household sector, I assume a continuum of firms indexed by $j \in \mathcal{I}_f = [0,1]$. Each firm is a monopolist producing a differentiated intermediate-good variety, producing output Y_{jt} and setting nominal price P_{jt} and making real profit D_{jt}. Technology is represented by the production function

$$Y_{jt} = A_t N_{jt}^{1-\alpha} \tag{OA.30}$$

where A_t is the level of technology, common to all firms, which evolves according to

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a \tag{OA.31}$$

where $\varepsilon_t^a \sim \mathcal{N}(0, \sigma_a^2)$.

Aggregate Price Level Dynamics. As in the benchmark NK model, price rigidities take the form of Calvo-lottery friction. At every period, each firm is able to reset their price with probability $(1-\theta)$, independent of the time of the last price change. That is, only a measure $(1-\theta)$ of firms is able to reset their prices in a given period, and the average duration of a price is given by $1/(1-\theta)$. Such an environment implies that aggregate price level dynamics are given (in log-linear terms) by

$$\pi_t = \int_{\mathcal{I}_f} \pi_{jt}\, dj = (1-\theta)\left[\int_{\mathcal{I}_f} p_{jt}^*\, dj - p_{t-1}\right] = (1-\theta)\left(p_t^* - p_{t-1}\right) \tag{OA.32}$$

Optimal Price Setting. A firm re-optimizing in period t will choose the price P_{jt}^* that maximizes the current market value of the profits generated while the price remains effective. Formally, $P_{jt}^* = \arg\max_{P_{jt}} \sum_{k=0}^{\infty} \theta^k \mathbb{E}_{jt}\left\{\Lambda_{t,t+k}\frac{1}{P_{t+k}}\left[P_{jt}Y_{j,t+k|t} - \mathcal{C}_{t+k}(Y_{j,t+j|t})\right]\right\}$ subject to the sequence of demand schedules $Y_{j,t+k|t} = \left(\frac{P_{jt}}{P_{t+k}}\right)^{-\epsilon} C_{t+k}$, where $\Lambda_{t,t+k} \equiv \beta^k \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma}$ is the stochastic discount factor, $\mathcal{C}_t(\cdot)$ is the (nominal) cost function, and $Y_{j,t+k|t}$ denotes output in period $t+k$ for a firm j that last reset its price in period t. The First-Order Condition is $\sum_{k=0}^{\infty} \theta^k \mathbb{E}_{jt}\left[\Lambda_{t,t+k}Y_{j,t+k|t}\frac{1}{P_{t+k}}\left(P_{jt}^* - \mathcal{M}\Psi_{j,t+k|t}\right)\right] = 0$, where $\Psi_{j,t+k|t} \equiv \mathcal{C}_{t+k}^{\intercal}(Y_{j,t+j|t})$ denotes the (nominal) marginal cost for firm j, and $\mathcal{M} = \frac{\epsilon}{\epsilon-1}$. Log-linearizing around the zero inflation steady-state, I obtain the familiar price-setting rule

$$p^*_{jt} = (1 - \beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k \mathbb{E}_{jt}\left(\psi_{j,t+k|t} + \mu\right) \tag{OA.33}$$

where $\psi_{j,t+k|t} = \log \Psi_{j,t+k|t}$ and $\mu = \log \mathcal{M}$.

OA.4.3. Equilibrium

Market clearing in the goods market implies that $Y_{jt} = C_{jt} = \int_{\mathcal{I}_h} C_{ijt}\, di$ for each j good/firm. Aggregating across firms, I obtain the aggregate market clearing condition: since assets are in zero net supply and there is no capital, investment, government consumption nor net exports, production equals consumption: $\int_{\mathcal{I}_f} Y_{jt}\, dj = \int_{\mathcal{I}_h}\int_{\mathcal{I}_f} C_{ijt}\, dj\, di \implies Y_t = C_t$. Aggregate employment is given by the sum of employment across firms, and must meet aggregate labor supply $N_t = \int_{\mathcal{I}_h} N_{it}\, di = \int_{\mathcal{I}_f} N_{jt}\, dj$. Using the production function (OA.30) and (OA.22) together with goods market clearing, $N_t = \int_{\mathcal{I}_f}\left(\frac{Y_{jt}}{A_t}\right)^{\frac{1}{1-\alpha}} dj = \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}}\int_{\mathcal{I}_f}\left(\frac{P_{jt}}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}} dj$. Log-linearizing the above expression yields to $n_t = \frac{1}{1-\alpha}(y_t - a_t)$.

The (log) marginal cost for firm j at time $t+k|t$ is $\psi_{j,t+k|t} = w_{t+k} - mpn_{j,t+k|t}$ $= w_{t+k} - [a_{t+k} - \alpha n_{j,t+k|t} + \log(1-\alpha)]$, where $mpn_{j,t+k|t}$ and $n_{j,t+k|t}$ denote (log) marginal product of labor and (log) employment in period $t+k$ for a firm that last reset its price at time t, respectively. Let $\psi_t \equiv \int_{\mathcal{I}_f}\psi_{jt}$ denote the (log) average marginal cost. I can then write $\psi_t = w_t - [a_t - \alpha n_t + \log(1-\alpha)]$. Thus, the following relation holds

$$\psi_{j,t+k|t} = \psi_{t+k} + \alpha(n_{jt+k|t} - n_{t+k}) = \psi_{t+k} + \frac{\alpha}{1-\alpha}(y_{jt+k|t} - y_{t+k}) = \psi_{t+k} - \frac{\alpha\epsilon}{1-\alpha}(p^*_{jt} - p_{t+k}) \tag{OA.34}$$

Introducing (OA.34) into (OA.33), I can rewrite the firm price-setting condition as

$$p^*_{jt} = (1 - \beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k \mathbb{E}_{jt}\left(p_{t+k} - \Theta\widehat{\mu}_{t+k}\right), \tag{OA.35}$$

where $\widehat{\mu} = \mu_t - \mu$ is the deviation between the average and desired markups, where $\mu_t = -(\psi_t - p_t)$, and $\Theta = \frac{1-\alpha}{1-\alpha+\alpha\epsilon}$.

Individual and Aggregate Phillips curve. Note that I can write the deviation between average and desired markups as

$$\begin{aligned}\mu_t &= p_t - \psi_t = p_t - w_t + w_t - \psi_t = -(w_t - p_t) + w_t - [w_t - a_t + \alpha n_t - \log(1-\alpha)] \\ &= -(\sigma y_t + \varphi n_t) + [a_t - \alpha n_t + \log(1-\alpha)] = -\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) y_t + \frac{1+\varphi}{1-\alpha} a_t + \log(1-\alpha)\end{aligned}$$

As in the benchmark model, under flexible prices ($\theta = 0$) the average markup is constant and equal to the desired μ. Consider the natural level of output, y_t^n as the equilibrium level under flexible prices and FIRERE. Rewriting the above condition under the natural equilibrium, $\mu = -\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) y_t^n + \frac{1+\varphi}{1-\alpha} a_t + \log(1-\alpha)$, which I can write as $y_t^n = \psi a_t + \psi_y$, where $\psi = \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}$ and $\psi_y = -\frac{(1-\alpha)[\mu-\log(1-\alpha)]}{\sigma(1-\alpha)+\varphi+\alpha}$. Therefore, I can write $\widehat{\mu}_t = -\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) \widetilde{y}_t$, where $\widetilde{y}_t = y_t - y_t^n$ is defined as the output gap. Finally, plugging this expression into (OA.35), I obtain (6).

Individual and Aggregate DIS curve. In order to derive the DIS curve, let us first log-linearize the profit of the monopolist. The profit D_{jt} of monopolist j at time t is $D_{jt} = \frac{1}{P_t}\left(P_{jt}Y_{jt} - W_t N_{jt}\right) = \frac{P_{jt}}{P_t} Y_{jt} - W_t^r N_{jt}$. Log-linearizing around a zero-inflation steady state, $D_j d_{jt} = \frac{P_j}{P} Y_j (p_{jt} + y_{jt} - p_t) - \frac{W^r}{P} N_j (w_t^r + n_{jt})$. Aggregating the above expression across firms

$$y_t = \frac{W^r N}{Y}(w_t^r + n_t) + \frac{D}{Y} d_t = \Omega(w_t^r + n_t) + (1-\Omega) d_t \tag{OA.36}$$

Aggregating the labor supply condition (OA.26) across households, and using the goods market clearing condition $w_t^r = \sigma y_t + \varphi n_t$. Inserting the above condition in (OA.36), I can write $y_t = \frac{\Omega(1+\varphi)}{\varphi+\Omega\sigma} w_t^r + \frac{(1-\Omega)\varphi}{\varphi+\Omega\sigma} d_t$. Introducing this last expression into the aggregate consumption function (OA.29), using again the goods market clearing condition

$$y_t = -\frac{\beta}{\sigma} \sum_{k=0}^{\infty} \beta^k \overline{\mathbb{E}}_t^h r_{t+k} + (1-\beta) \sum_{k=0}^{\infty} \beta^k \overline{\mathbb{E}}_t^h y_{t+k} \tag{OA.37}$$

Let us now derive the DIS curve. Substracting the natural level of output from (OA.37), I obtain

$$\widetilde{y}_t = -\frac{\beta}{\sigma} \sum_{k=0}^{\infty} \beta^k \overline{\mathbb{E}}_t^h (r_{t+k} - r_{t+k}^n) + (1-\beta) \sum_{k=0}^{\infty} \beta^k \overline{\mathbb{E}}_t^h \widetilde{y}_{t+k} \tag{OA.38}$$

I now need to derive an expression for the natural real interest rate. Recall that in a natural equilibrium with no price nor information frictions, the natural real interest rate is given by

$$r_t^n = \sigma \mathbb{E}_t \Delta y_{t+1}^n = \sigma\psi \mathbb{E}_t \Delta a_{t+1} = \sigma\psi(\rho_a - 1)a_t \tag{OA.39}$$

Finally, the aggregate DIS curve is given by

$$\widetilde{y}_t = -\frac{\beta}{\sigma}\sum_{k=0}^{\infty}\beta^k \overline{\mathbb{E}}_t^h(i_{t+k} - \pi_{t+k+1}) + (1-\beta)\sum_{k=0}^{\infty}\beta^k \overline{\mathbb{E}}_t^h \widetilde{y}_{t+k} - \psi(1-\rho_a)\sum_{k=0}^{\infty}\beta^k \overline{\mathbb{E}}_t^h a_{t+k} \tag{OA.40}$$

Notice that in this case there is no direct individual DIS curve. However, one can show that the following consumption function

$$c_{it} = -\frac{\beta}{\sigma}\mathbb{E}_{it} r_t + (1-\beta)\mathbb{E}_{it} c_t + \beta \mathbb{E}_{it} c_{i,t+1} - \psi(1-\rho_a)\mathbb{E}_{it} a_t, \quad \text{with } c_t = \int c_{it}\, di \tag{OA.41}$$

is equivalent to (OA.40) provided that $\lim_{T\to\infty}\beta^T \mathbb{E}_{it} c_{i,t+T}$, which is broadly assumed in the literature given $\beta < 1$.

Discussion on Model Derivation and FIRE. Notice that throughout the model derivation I have not discussed how are beliefs and expectations formed. Therefore, the model derived above, consisting of equations (OA.40), (6), (9), (10) and (OA.31), should be interpreted as a general framework.

Under the assumption that expectations satisfy the Law of Iterated expectations, $\mathbb{E}_t[\mathbb{E}_{t+k}(\cdot)] = \mathbb{E}_t(\cdot)$ for $k > 0$, and that they are common across agents, $\overline{\mathbb{E}}_t^h(\cdot) = \overline{\mathbb{E}}_t^f(\cdot) = \mathbb{E}_t(\cdot)$, I can write the model in its usual form: $\widetilde{y}_t = -\frac{1}{\sigma}(i_t - \mathbb{E}_t\pi_{t+1}) + \mathbb{E}_t\widetilde{y}_{t+1} + \psi(\rho_a - 1)a_t$, (4), together with (9), (10) and (OA.31).

OA.5. Useful Mathematical Concepts

OA.5.1. Wiener-Hopf Filter

Consider the non-causal prediction of $f_t = A(L)\widehat{\boldsymbol{s}}_{it}$ given the whole stream of signals $\mathbb{E}(f_t|x_i^\infty) = \rho_{yx}(L)\rho_{xx}^{-1}(L)x_{it} = \rho_{yx}(L)\boldsymbol{B}(L^{-1})^{-1}\boldsymbol{V}^{-1}\boldsymbol{B}(L)^{-1}x_{it} = \rho_{yx}(L)\boldsymbol{B}(L^{-1})^{-1}\boldsymbol{V}^{-1}\boldsymbol{w}_{it} = \sum_{k=-\infty}^{\infty} h_k \boldsymbol{w}_{it-k}$, where $\rho_{yx}(z) = A(z)\boldsymbol{M}^\top(z^{-1})$ and $\rho_{xx}(z) = \boldsymbol{B}(z)\boldsymbol{V}\boldsymbol{B}^\top(z^{-1})$. Notice that I am using future values of $\boldsymbol{w}_{it}$. If the agent only observes events or signals up to time t, the best prediction is

$$\mathbb{E}(f_t|x_i^t) = \left[\sum_{k=-\infty}^{\infty} h_k \boldsymbol{w}_{it-k}\right]_+ = \sum_{k=0}^{\infty} h_k \boldsymbol{w}_{it-k} = \left[\rho_{yx}(L)\boldsymbol{B}(L^{-1})^{-1}\right]_+ \boldsymbol{V}^{-1}\boldsymbol{w}_{it} = \left[\rho_{yx}(L)\boldsymbol{B}(L^{-1})^{-1}\right]_+ \boldsymbol{V}^{-1}\boldsymbol{B}(L)^{-1}x_{it}$$

OA.5.2. Annihilator Operator

The annihilator operator $[\cdot]_+$ eliminates the negative powers of the lag polynomial: $[A(z)]_+ = \left[\sum_{k=-\infty}^{\infty} a_k z^k\right]_+ = \sum_{k=0}^{\infty} a_k z^k$. Suppose that I am interested in obtaining $[A(z)]_+$, where $A(z)$ takes this particular form, $A(z) = \frac{\phi(z)}{z-\lambda}$ with $|\lambda| < 1$, and $\phi(z)$ only contains positive powers of z. I can rewrite $A(z)$ as $A(z) = \frac{\phi(z)-\phi(\lambda)}{z-\lambda} + \frac{\phi(\lambda)}{z-\lambda}$. Let us first have a look at the second term, I can write

$$\frac{\phi(\lambda)}{z-\lambda} = -\frac{\phi(\lambda)}{\lambda}\frac{1}{1-\lambda^{-1}z} = -\frac{\phi(\lambda)}{\lambda}(1+\lambda^{-1}z+\lambda^{-2}z^2+\ldots)$$

which is not converging. Alternatively, I can write it as a converging series as

$$\frac{\phi(\lambda)}{z-\lambda} = \phi(\lambda)z^{-1}\frac{1}{1-\lambda z^{-1}} = \phi(\lambda)z^{-1}(1+\lambda z^{-1}+\lambda^2 z^{-2}+\ldots)$$

Notice that all the power terms are on the negative side of z. As a result, $\left[\frac{\phi(\lambda)}{z-\lambda}\right]_+ = 0$. Let us now move to the first term. I can write $\phi(z) - \phi(\lambda) = \sum_{k=0}^{\infty} \phi_k(z^k - \lambda^k) = \phi_0 \prod_{k=1}^{\infty}(z-\xi_k)$, where $\{\xi_k\}$ are the roots of this difference polynomial. Since I know that λ is a root of the LHS, I can set $\xi^k = \lambda$ and write

$$\phi(z) - \phi(\lambda) = \phi_0(z-\lambda)\prod_{k=2}^{\infty}(z-\xi_k) \implies \frac{\phi(z)-\phi(\lambda)}{z-\lambda} = \prod_{k=2}^{\infty}(z-\xi_k)$$

which only contains positive powers of z. Hence, I have that $\left[\frac{\phi(z)}{z-\lambda}\right]_+ = \frac{\phi(z)-\phi(\lambda)}{z-\lambda}$.

Consider now instead the case $A(z) = \frac{\phi(z)}{(z-\lambda)(z-\beta)}$. Making use of partial fractions, I can write

$$\frac{\phi(z)}{(z-\lambda)(z-\beta)} = \frac{1}{\lambda-\beta}\left[\frac{\phi(z)}{z-\lambda} - \frac{\phi(z)}{z-\beta}\right] = \frac{1}{\lambda-\beta}\left[\frac{\phi(z)-\phi(\lambda)}{z-\lambda} - \frac{\phi(z)-\phi(\beta)}{z-\beta} + \frac{\phi(\lambda)}{z-\lambda} - \frac{\phi(\beta)}{z-\beta}\right]$$

Following the same steps as in the previous case, I can solve

$$\left[\frac{\phi(z)}{(z-\lambda)(z-\beta)}\right]_+ = \frac{\phi(z)-\phi(\lambda)}{(\lambda-\beta)(z-\lambda)} - \frac{\phi(z)-\phi(\beta)}{(\lambda-\beta)(z-\beta)}$$

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