Reconciling Empirics and Theory: The Behavioral Hybrid New Keynesian Model*

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March 30, 2022

Abstract

Structural estimates of the standard New Keynesian model are at odds with the microeconomic estimates. To reconcile these findings, we develop and estimate a behavioral New Keynesian model augmented with backward-looking households and firms. We find (i) strong evidence for bounded rationality, with a cognitive discount factor estimate of 0.4 at a quarterly frequency; and (ii) that the behavioral setting with backward-looking agents helps us harmonize the New Keynesian theory with empirical studies. We suggest that both cognitive discounting and anchoring are essential, first, to match the empirical estimates for certain parameters of interest and, second, to obtain the hump-shaped and initially muted impulse-response functions that we observe in empirical studies.

Keywords: New Keynesian, Bounded Rationality, Bayesian Estimation.

JEL Classifications: E27, E52, E71.

*We would like to thank Xavier Gabaix, Mark Gertler, Per Krusell, Jesper Lindé, Lars E.O. Svensson, Jörgen Weibull and seminar participants at the Macro Workshop in Bogotá, the IIES, and the Stockholm School of Economics for useful feedback and comments. Declarations of interest: none. The views expressed in this paper are those of the authors and do not necessarily reflect the position of the Central Bank of Ireland or the European System of Central Banks.

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1 Introduction

> *"Despite the advances in theoretical modeling, accompanying econometric analysis of the 'new Phillips curve' has been rather limiting [...]. The work to date has generated some useful findings, but these findings have raised some troubling questions about the existing theory."*
>
> J. Galí and M. Gertler, *Inflation dynamics: A structural econometric analysis (1999).*

The most important characteristic of the standard New Keynesian (NK) model is that it can be synthesized in a system of two first-order stochastic difference equations that are easy to interpret: the Dynamic IS curve or the demand side, and the Phillips curve or the supply side. Every slope in these curves is a combination of different parameters in the model, namely the discount factor, the degree of risk aversion, the Frisch elasticity and the Calvo-fairy probability. As a result, by estimating the slopes of the final system of equations, one can retrieve the structural parameters of the model. However, when the monetary economics literature performed such analyses, the estimated parameters were at odds with microeconometric studies.

Reconciling the NK theory with the data has proved to be a difficult exercise. One of the main criticisms of the benchmark NK model is that it is purely forward-looking, and therefore lacks the ability to capture any sort of endogenous persistence in output and inflation (see Christiano et al. 2005; Fuhrer and Moore 1995; Fuhrer 2010; Galí and Gertler 1999). As a result, the model does not produce the usual anchoring that we observe in the data. The main approach to enforce anchoring in the model is to include backward-looking households and firms, either assuming a backward-looking utility function for households or a sticky price indexation for firms. Unfortunately, the parameter values that characterize the frictions required to produce the degree of anchoring that the data suggests are at odds with the micro evidence. To reconcile these differences between empirics and theory, we put forward a behavioral NK model, similar in spirit to the one described in Gabaix (2020), that is additionally extended with *external habit persistent* households and backward-looking firms.[1] We show that the combination of backward-looking agents *and* bounded rationality helps reduce the discrepancy between macro and micro estimates.

Our contribution to the literature is threefold. First, we extend the behavioral NK setting in Gabaix (2020) to allow for household habit persistence and firm price indexa-

[1] These ingredients are necessary to obtain hump-shaped IRFs in the theoretical model, which is what we observe in empirical research (see e.g., Altig et al. 2011; Christiano et al. 2005).

tion, inducing anchoring in the model dynamics. Second, we estimate all the structural parameters behind the coefficients in the behavioral DIS and hybrid NK Phillips curves using Bayesian techniques. Thus, we reconcile three key parameters in the theory that were at odds with the empirical evidence: the subjective discount factor, the degree of external habits, and the degree of price stickiness. Third, we also find empirical evidence for considerable bounded rationality behavior, supporting the deviation from the standard fully rational behavioral framework. A salient feature of our model is that it can be easily reduced to the ones described in Galí and Gertler (1999), Galí (2008) or Gabaix (2020) by turning off certain key parameters such as the degree of habit persistence, the degree of price indexation, or the bounded rationality parameter. As a result, our model nests those frameworks and allows us to easily compare estimates.

The first approach to induce anchoring in the NK framework dates back to Galí and Gertler (1999). The authors only focus on the supply side of the model and estimate two different (micro-founded) NK Phillips curves: the standard curve (NKPC) and the Hybrid curve (H-NKPC), which has a backward-looking component. In an empirical exercise, they show that the H-NKPC produces dynamics closer to what the data suggests. However, even in the hybrid version, the structural parameter estimates are at odds with the micro evidence. For example, the discount factor estimate at a quarterly frequency is generally below 0.9. In subsequent research, Christiano et al. (2005) suggest to induce inflation persistence by assuming that firms that cannot re-optimize their prices update them according to past inflation. Other approaches able to generate intrinsic inflation persistence can be found in Roberts (1997) and Milani (2007) through the modeling of adaptive expectations and learning, respectively; in the form of sticky information as in Mankiw and Reis (2002); incomplete information as in Woodford (2003), or by relaxing the Calvo assumption of a random selection of firms that are able to change their prices as in Sheedy (2010).[2] Likewise, Christiano et al. (2005) also extend the backward-looking behavior to households by including internal habits.[3] They find that the degree of habits

[2]Other studies have stressed alternative mechanisms. For example, Grauwe (2011) generates non-fundamental (animal spirit) business cycles by introducing optimistic and pessimistic agents. In the same vein, Hommes and Lustenhouwer (2019) describe how Central Bank credibility can be affected by the share of rational and naïve agents, each agent type optimally decided at the individual level. They provide the conditions under which a self-fulfilling liquidity trap can occur, and how Central Bank credibility affects the equilibrium. Finally, in a model that features both rational and naïve agents, Cornea-Madeira et al. (2019) generate anchoring and myopia in aggregate inflation dynamics. Importantly, their mean estimate of myopia, 0.353, is similar to the estimates presented in this paper.

[3]Christiano et al. (2005) model the backward-looking behavior by means of internal habits (each agent cares about its own consumption growth). In this paper, we instead focus on external habits (each agent cares about the difference between its consumption today and aggregate consumption yesterday). We take this route motivated by the meta-analysis in Havranek et al. (2017).

necessary to match the impulse response after a monetary shock is three or four times larger than the one estimated in the micro literature (for an extensive meta-analysis, see Havranek et al. 2017). From these extensions we take the lesson that adding a backward-looking behavior is no panacea, at least on its own, for a reconciliation between micro and macro estimates.

We include backward-looking firms along the lines of Galí and Gertler (1999) and Christiano et al. (2005). This is done in order to obtain a hybrid NK Phillips curve that is closer to empirical evidence, in the sense that it also includes lags of inflation and helps explain its persistence. The motivation for this is mostly empirical, since previous studies have found the inflation equation to be largely inertial.[4] We include household habit persistence in the light of Christiano et al. (2005) and Blanchard et al. (2015). Christiano et al. (2005) find a quantitatively important degree of household habit persistence for the US. Most importantly, they show that including habit persistence is critical to obtain hump-shaped impulse responses in the model, as the empirical VAR literature has observed. Given that our intention is to build a model that is closer to the data, we follow their approach in order to consistently estimate the behavioral DIS curve.

Our departure from the standard Full-Information Rational Expectations (FIRE) assumption is motivated by empirical evidence. Using survey data from households' and firms' expectations, Coibion and Gorodnichenko (2015) test for the null of FIRE, which is rejected by the data. However, their empirical findings are inconclusive on the direction of the FIRE departure, whether it is Full-Information or Rational Expectations that is rejected. This leaves room for different extensions beyond FIRE, some being more empirically robust than others.

The consideration of departures from full information undoubtedly has a long history in the literature. On the one hand, Phelps (1969) and Lucas (1972) put forth the idea that imperfect information can have real effects in the economy. Mankiw and Reis (2002) and Ball et al. (2005) assume that information is sticky, with a share of the population's expectations not being up to date. They show that accounting for information stickiness is sufficient to generate a non-neutral monetary policy. This framework has recently been adapted to a HANK economy in Auclert et al. (2020). Woodford (2003), Lorenzoni (2009), Nimark (2008) and Angeletos and Huo (2018) show that an imperfect information economy inherits anchoring from the sluggish expectations, and would thus not require backward-looking households and firms to generate hump-shaped dynamics.

On the other hand, Sims (1998, 2003, 2006) endogenizes the attention choice and finds

[4] Since the seminal paper by Fuhrer and Moore (1995), a sizable literature has tried to estimate the NKPC. See, Mavroeidis et al. (2014), for an extensive review.

that the limited capacity of processing information prevents agents from fully incorporating available information into their decision making process.[5] Maćkowiak and Wiederholt (2009) assume limited information acquisition in order to explain the real effect of monetary policy shocks. Their model is able to produce impulse responses of prices that are sticky, without the need for Calvo frictions, and the Phillips curve, consistent with Lucas (1973), becomes steeper as the variance of nominal aggregate demand increases as a result of increased attention. Other work that tries to explain macroeconomic dynamics under the lens of rational inattention includes Maćkowiak and Wiederholt (2009, 2015), Matějka (2016), Steiner et al. (2017) and Zorn (2020).[6]

There have been other approaches within the rubric of bounded rationality, where the model of level-k thinking is one notable example. In this framework, agents have access to all information about the state of nature, but agents only reason up to a level k, in the sense that their best responses are only iterated k times. That is, agents satisfy the FIRE assumption up to a level k, after which they just play the default action. Recent applications to macroeconomics include Farhi and Werning (2019), García-Schmidt and Woodford (2019) and Iovino and Sergeyev (2017), showing that level-k thinking produces anchoring.

One notable approach that lies between the models of less than full information and the models of less than full rationality has been developed in a series of papers by Gabaix (2014, 2016, 2020), which provides an operational, very tractable framework by incorporating the behavioral assumption that the decision makers allocate their attention optimally according to a simplified version of the full model, where their utility is replaced by a linear-quadratic approximation, and then solve the full model with this partial attention vector. The framework captures some of the essential features of the rational inattention framework, namely the under-reaction of beliefs and actions, while it allows tractability for dynamic models beyond linear-quadratic forms.

[5]These papers find that imperfect information can explain the sluggishness observed in macro aggregates either through incomplete information or optimally chosen inattention.

[6]This rational inattention paradigm has gained attraction ever since, especially after Matêjka and McKay (2015) and Caplin et al. (2019a) helped the application of the static framework beyond the linear-quadratic settings. In its canonical form, the rational inattention framework models the cost of information acquisition as a function of reduction in uncertainty, which is measured as the expected mutual information between the prior and the posterior belief. The decision-maker then chooses the conditional distribution of the noisy information in each possible state of the world in order to maximize the expected utility subject to the information cost. When specified by using the Shannon entropy and linear cost, this model exhibits a stochastic choice in the multinomial logit form, endogenously biased towards ex-ante favorable alternatives. The resulting stochastic choice comes from the fact that the elimination of a priori possible states is prohibitively costly. One major qualitative implication is the under-reaction of beliefs and actions. See Caplin et al. (2019b) and Maćkowiak et al. (2018) for comprehensive reviews of the framework and its behavioral implications.

In this paper, we follow this last strand of the literature by assuming an attention coefficient that the decision-makers on both sides of the economy assign to a piece of newly arriving information, so that the posterior expectation is a convex combination of the prior mean and the realization (i.e. full-information) value. We follow this reduced-form approach for two main reasons. First, our core interest is to reconcile the theory with empirical evidence, and this behavioral approximation of a limited attention model affords us to arrive at the simple closed-form solutions that are typical of the standard New Keynesian model while incorporating the first-order effects of limited inattention. Second, since we estimate this coefficient in our two-sided economy, explicitly modeling the cost structure and estimating the cost coefficient would add an extra layer without providing any further insight.

Cognitive discounting, as presented in Gabaix (2020), is successful in producing myopia but does not produce anchoring *on its own*. In fact, when we estimate the forward-looking model in Gabaix (2020), we find an excessively low cognitive discount factor, biased towards zero due to the anchoring that we observe in the data and thus, the model is unable to produce. We find that the cognitive discount factor, *together* with habit persistence and price indexation, is key to obtain macro estimates that align with their micro counterpart, and its estimated coefficient is nearly twice as large as in the benchmark case with no backward-looking agents. The cognitive discount factor increases the relative weight of the past (anchoring) and reduces the weight of the future (myopia).

For the estimation of the structural parameters, and being able to compare our New Keynesian models, we follow a Bayesian approach as in Fernández-Villaverde and Rubio-Ramírez (2004), Rabanal and Rubio-Ramírez (2005) and Milani (2007). This approach has some advantages over limited-information methods such as GMM. For example, Bayesian estimation mitigates the misspecification problem and allows a transparent comparison across models. In particular, we estimate four different models using US data: (i) the standard NK model; (ii) the hybrid NK model; (iii) the behavioral NK model, and (iv) the behavioral hybrid NK model. We find that the latter model reports estimates that are micro-consistent with previous empirical evidence. Likewise, in order to test the ability of our set of models to replicate empirical impulse-response functions, we also estimate a monetary policy shock by means of a Bayesian VAR using narrative sign restrictions as in Antolín-Díaz and Rubio-Ramírez (2018). We find that only our Behavioral NK model with both habit formation and backward-looking firms is able to generate, at the same time, hump-shaped responses and enough inflation persistence as we observe in the data.

The paper proceeds as follows. In section 2 we introduce the behavioral model. In section 3 we estimate the structural parameters of the model. In section 4 we discuss our findings. Section 5 concludes the paper.

2 The Behavioural Agents and Firms Setting

2.1 Bounded Rationality Assumptions

Before introducing a behavioral version of the New Keynesian model, we here briefly explain the cognitive discounting approach à la Gabaix (2016, 2020) that we operationalize in this paper. Let $X_t \in \Omega$ be the state vector at period t, that might include TFP shocks and announced monetary policy actions for that period, and $\epsilon_t \in E$ is an additive stochastic noise with 0 mean.

Now let $G\colon \Omega \times E \to \Omega$ be the function that represents the equilibrium law of motion for the state variable,

$$X_{t+1} = G(X_t, \epsilon_{t+1})$$

Let us assume that the deterministic economy has a unique non-exploratory steady-state, and is denoted by $\bar{X}$. Likewise, let $\hat{X}_t := X_t - \bar{X}$ denote the period t deviation from the steady state.

Here, the cognitive discounting assumption states that the agents do not fully internalize the expected equilibrium deviations from the steady state by partially anchoring their belief to the steady state. Let $\bar{m} \in [0,1]$ denote the degree of cognitive discounting, and let $G^B\colon \Omega \times E \to \Omega$

$$G^B(X_t, \epsilon_t) = \bar{m}G(X_t, \epsilon_t) + (1 - \bar{m})\bar{X}$$

denote the law of motion perceived by the behavioral agent.

For notational simplicity, in the rest of this section, we will assume that the state vector is de-meaned, i.e. the steady state is given by the zero vector; however, the analysis holds true for the generic case as well. Under this assumption the above expression simplifies to

$$G^B(X_t, \epsilon_t) = \bar{m}G(X_t, \epsilon_t)$$

Observe that the linearization of the actual law of motion and renormalization gives

$$X_{t+1} = \Gamma X_t + \epsilon_{t+1}$$

for some matrix Γ. Likewise, the perceived law of motion by the behavioral agents linearizes to

$$X_{t+1} = \bar{m}(\Gamma X_t + \epsilon_{t+1})$$

However, since the noise parameter has 0 mean, we have the following relation between the expectation of a behavioral agent, denoted by the expectation operator with a superscript B, and the rational expectation

$$\mathbb{E}_t^B[X_{t+1}] = \bar{m}\Gamma X_t = \bar{m}\mathbb{E}_t[X_{t+1}]$$

Likewise, iterating for k periods we obtain

$$\mathbb{E}_t^B[X_{t+k}] = \bar{m}^k\mathbb{E}_t[X_{t+k}]$$

Throughout the paper, we will assume that all forecasts, made by households or firms and across different macroeconomic variables, are cognitively discounted by the same factor $\overline{m}$.

2.2 Households

2.2.1 Rational Preferences

Let us first describe the full rational case. There is a population of households that is treated as a continuum of unit mass. Each household chooses its consumption and labor supply level for each period. We assume identical preferences over expected lifetime utility and hence omit indexing for notational ease. The preference of a representative household can be given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(C_t - h\overline{C}_{t-1})^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right] \tag{1}$$

where C_t is a consumption index given by

$$C_t \equiv \left(\int_0^1 C_{it}^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$$

with C_{it} denoting the quantity of good $i \in [0,1]$ consumed by the household in period t, N_t denotes employment or labor supply, $\overline{C}_{t-1}$ denotes the average consumption level in the economy (which is taken as given by the individual household), σ is the intertemporal elasticity of substitution and $1/\varphi$ is the Frisch elasticity. The period-t consumption utility of each household is affected by a reference level, which we assume to be given by a linear function of the average consumption level in the previous period. Thus, the household preferences exhibit a *keeping up with the Joneses* element.[7] $h \in [0,1]$ represents the sensitivity towards this reference point. The household's budget constraint is given by

$$P_t C_t + Q_t B_t = B_{t-1} + W_t N_t + T_t \tag{2}$$

where P_t is the price of the consumption good, B_t stands for bond holdings at the household, Q_t is the price of each bond, W_t is the wage rate for each unit of labor supply and T_t are transfers to households. We show in Appendix A that the demand for good i is given by

$$Y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\epsilon} Y_t \tag{3}$$

where $Y_t = C_t$ (since we are in a representative household economy), and the aggregate price index P_t is given by

$$P_t = \left(\int_0^1 P_{it}^{1-\epsilon} di\right)^{\frac{1}{1-\epsilon}}$$

which is also derived in Appendix A.

The optimization problem of the household is represented as maximizing lifetime utility (1) subject to its budget constraint (2) and the usual transversality condition $\lim_{t\to\infty} \beta^t u'(C_t)B_t = 0$. The rational household optimality conditions, derived in Appendix A, are

$$\frac{W_t}{P_t} = \frac{N_t^{\varphi}}{(C_t - h\overline{C}_{t-1})^{-\sigma}} \tag{4}$$

$$Q_t = \beta \mathbb{E}_t \left[\left(\frac{C_{t+1} - h\overline{C}_t}{C_t - h\overline{C}_{t-1}}\right)^{-\sigma} \frac{P_t}{P_{t+1}} \right] \tag{5}$$

Notice that, since households are identical and of unit mass, we can take the average consumption of the past period as the consumption of the representative agent in that period, $C_t = \overline{C}_t$ for all periods t.

2.2.2 Behavioral Preferences

Let us now focus on the behavioral household. The behavioral households exhibits the cognitive discounting as described in Section 2.1, hence its mean expectation of the stochastic variables in the economy is dampened towards its steady-state values compared to the expectation of a rational agent. This effect is even more nuanced, especially for events

[7]The consequences of such an assumption are similar to assuming habit persistence, albeit simplifying the computation.

that are far into the future. We can rewrite conditions (4)-(5) as

$$\frac{W_t}{P_t} = \frac{N_t^{\varphi}}{(C_t - hC_{t-1})^{-\sigma}} \tag{6}$$

$$Q_t = \beta \mathbb{E}_t^B \left[\left(\frac{C(X_{t+1}) - hC(X_t)}{C(X_t) - hC(X_{t-1})} \right)^{-\sigma} \frac{P(X_t)}{P(X_{t+1})} \right] \tag{7}$$

As one can see, the labor supply condition is kept unchanged: since it is an intratemporal condition, cognitive discounting plays no role here. Importantly, fully rational and behavioral households do not differ in intratemporal considerations, but in their perception of the future. On the other hand, the Euler condition now has a different expectation operator. The log-linearized version of both optimality conditions, derived in Appendix B, is

$$\hat{w}_t - \hat{p}_t = \varphi \hat{n}_t + \frac{\sigma}{1-h}\hat{c}_t - \frac{\sigma h}{1-h}\hat{c}_{t-1} \tag{8}$$

$$\hat{c}_t = \frac{h}{1+h}\hat{c}_{t-1} + \frac{1}{1+h}\overline{m}\mathbb{E}_t\hat{c}_{t+1} - \frac{1-h}{\sigma(1+h)}\left(\hat{i}_t - \overline{m}\mathbb{E}_t\pi_{t+1}\right) \tag{9}$$

where a hat on top of a variable denotes the log deviation from steady-state $x_t = \frac{X_t - X}{X}$, and $\hat{i}_t = -\log Q_t$ is the short-term nominal interest rate. Here we have made use of the BR assumptions described in the previous section, setting $\mathbb{E}_t^B \widehat{c}_{t+1} = \overline{m}\mathbb{E}_t\widehat{c}_{t+1}$ and $\mathbb{E}_t^B \pi_{t+1} = \overline{m}\mathbb{E}_t\pi_{t+1}$. The Euler condition (9) can be rewritten in terms of the output gap as the Behavioral DIS (BDIS) curve

$$\tilde{y}_t = \lambda_b \tilde{y}_{t-1} + \lambda_f \mathbb{E}_t \tilde{y}_{t+1} + \lambda_r \left(\widehat{i}_t - \overline{m}\mathbb{E}_t\pi_{t+1} - r_t^n\right) \tag{BDIS}$$

where $\lambda_b = \frac{h}{1+h}$, $\lambda_f = \frac{1}{1+h}\overline{m}$, $\lambda_r = -\frac{1-h}{\sigma(1+h)}$, r_t^n is the natural interest rate; and a tilde denotes the log deviation with respect to the natural level $\tilde{z}_t = z_t - z_t^n$.

2.3 Firms

There is a continuum of firms with unit mass, each producing a different type of good. Good i is produced by a monopolistic firm i with technology

$$Y_{it} = A_t N_{it} \tag{10}$$

where A_t represents the level of technology, assumed to be common across firms. Given $Y_t = C_t$ and $Y_{it} = C_{it}$,[8] we know that the final good is produced competitively in quantity Y_t.

Each firm chooses the price level of the good that it produces. Prices are set subject to Calvo-style friction, i.e., in each period, a firm is only allowed to reset its price with probability $1-\theta$, independent of the time elapsed since it last adjusted its price. Thus, in each period a measure $1-\theta$ of producers reset their prices freely. However -and departing from the standard NK setting- it is assumed that when a firm is unable to reoptimize, its price is partially indexed to past inflation as in Christiano et al. (2005), i.e.,

$$P_{it} = P_{it-1}\Pi_{t-1}^{\omega} \tag{11}$$

where $\Pi_t = \frac{P_t}{P_{t-1}}$ is the gross rate of inflation between $t-1$ and t, and ω is the elasticity of prices with respect to past inflation.[9] As a result, a firm that last reset its price in period t will in period $t+k$ have a nominal price of $P_t^*\chi_{t,t+k}$, where

$$\chi_{t,t+k} = \begin{cases} \Pi_t^{\omega}\Pi_{t+1}^{\omega}\Pi_{t+2}^{\omega}\cdots\Pi_{t+k-1}^{\omega} & \text{if } k \geq 1 \\ 1 & \text{if } k = 0 \end{cases}$$

2.3.1 Rational Preferences

The rational firm's problem is to maximize its discounted profit stream

$$\mathbb{E}_0 \sum_{t=0}^{\infty} Q_t[P_{it}Y_{it} - W_tN_{it}] \tag{12}$$

subject to the sequence of demand constraints (3) and technology constraints (10). We can rewrite the objective function (profits) as

$$\begin{aligned} P_{it}Y_{it} - W_tN_{it} &= P_{it}Y_{it} - W_t\frac{Y_{it}}{A_t} \\ &= \left[P_{it} - \frac{W_t}{A_t}\right]Y_{it} \\ &= [P_{it} - P_t\text{MC}_t]\left[\frac{P_{it}}{P_t}\right]^{-\varepsilon} Y_t \end{aligned}$$

[8] No firm will choose to produce more that what is demanded.

[9] This assumption is equivalent to the more ad-hoc derivation of backward-looking firms in Galí and Gertler (1999). However, since firms are identical, its consequences are equivalent: ω could be interpreted as the share of backward-looking firms.

where the marginal costs are defined as $\mathrm{MC}_t = (W_t/P_t)(\partial Y_t/\partial N_t) = W_t/(P_t A_t)$.

Consider a firm reoptimizing its price at time t. Let the firm's optimal price be denoted $P_t^*(i)$, such that in this setting at time $t+k$ its price will be $P_{it}^*\chi_{t,t+k}$. Ignoring states in which reoptimization is allowed, its maximization program is

$$\max_{P_{it}^*} \mathbb{E}_t \sum_{k=0}^{\infty} \theta^k Q_{t+k}[P_{it}^*\chi_{t,t+k} - P_{t+k}\mathrm{MC}_{t+k}] \left[\frac{P_{it}^*\chi_{t,t+k}}{P_t}\right]^{-\epsilon} Y_{t+k}$$

which yields the following first-order condition[10]

$$P_{it}^* = \mathcal{M}\frac{\mathbb{E}_t \sum_{k=0}^{\infty}(\theta\beta)^k(C_{t+k} - hC_{t+k-1})^{-\sigma}C_{t+k}P_{t+k}^{\epsilon}P_{t+k-1}^{-\omega\epsilon}\mathrm{MC}_{t+k}}{\mathbb{E}_t \sum_{k=0}^{\infty}(\theta\beta)^k(C_{t+k} - hC_{t+k-1})^{-\sigma}C_{t+k}P_{t+k}^{\epsilon-1}P_{t+k-1}^{\omega(1-\epsilon)}}P_{t-1}^{\omega} \tag{13}$$

where we have used the Euler condition (7), $\chi_{t,t+k} = \left(\frac{P_{t+k-1}}{P_{t-1}}\right)^{\omega}$ and $\mathcal{M} = \frac{\epsilon}{\epsilon-1}$, which stands for the mark-up. Notice that with flexible prices (i.e., $\theta = 0$), the optimal pricing condition (13) collapses to the familiar monopolistic competition price-setting rule

$$P_{it}^* = \mathcal{M}P_t\mathrm{MC}_t \tag{14}$$

where (14) is the frictionless mark-up. Since all firms who get to reset are facing an identical environment (i.e., we can treat them as if they were a representative firm), they choose to set the same price: $P_{it}^* = P_t^*$ $\forall i$. The log-linearized version of the optimal pricing condition (13) is

$$p_t^* = p_t + (1-\theta\beta)\mathbb{E}_t \sum_{k=0}^{\infty}(\theta\beta)^k \left[(\pi_{t+1} + \dots + \pi_{t+k}) - \omega(\pi_t + \dots + \pi_{t+k-1}) + \widehat{\mathrm{mc}}_{t+k}\right] \tag{15}$$

where $\widehat{\mathrm{mc}}_t = \mathrm{mc}_t - \mathrm{mc} = \mathrm{mc}_t + \mu$ and $\mu = -\mathrm{mc} = -\log \mathrm{MC} = -\log\frac{1}{\mathcal{M}} = \log\mathcal{M}$. That is, a resetting firm will choose a price that corresponds to the desired mark-up over a convex combination of current and expected future prices and nominal marginal costs, in addition to the prices in the previous period.

2.3.2 Behavioral Preferences

The behavioral firm faces the same problem, with a less accurate view of reality. Most importantly, the behavioral firm perceives the future via the cognitive discounting mechanism discussed in Section 2.1. To be precise, we model that at time t, the firm perceives

[10] A detailed derivation can be found in Appendix C.

the future inflation and marginal costs at date $t+k$ as

$$\mathbb{E}_t^B[\pi_{t+k}] = \overline{m}^k \mathbb{E}_t[\pi_{t+k}]$$
$$\mathbb{E}_t^B[\widehat{\text{mc}}_{t+k}] = \overline{m}^k \mathbb{E}_t[\widehat{\text{mc}}_{t+k}]$$

Note that the cognitive discount factor is not required to be the same across households and firms. The equivalent condition of equation (15) for a behavioral firm is

$$\begin{aligned} p_t^* &= p_t + (1-\theta\beta)\sum_{k=0}^{\infty}(\theta\beta)^k \mathbb{E}_t^B\left[(\pi_{t+1}+\ldots+\pi_{t+k}) - \omega(\pi_t+\ldots+\pi_{t+k-1}) + \widehat{\text{mc}}_{t+k}\right] \\ &= p_t + (1-\theta\beta)\sum_{k=0}^{\infty}(\theta\beta\overline{m})^k \mathbb{E}_t\left[(\pi_{t+1}+\ldots+\pi_{t+k}) - \omega(\pi_t+\ldots+\pi_{t+k-1}) + \widehat{\text{mc}}_{t+k}\right] \end{aligned} \quad (16)$$

where the future is additionally discounted by a cognitive discount factor $\overline{m}$.

2.4 Aggregate Price Dynamics

In this economy, in every period, there are two types of firms: those allowed to reset their price and those who are not, whose price is updated with previous aggregate inflation. We can describe the price dynamics as

$$P_t = \left[\Pi_{t-1}^{\omega(1-\epsilon)}\theta P_{t-1}^{1-\epsilon} + (1-\theta)(P_t^*)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}$$

Notice that all firms resetting their price in any given period will choose the same price because they face an identical problem. A log-linear approximation to the aggregate price index around a zero inflation steady-state yields[11]

$$\pi_t = \theta\omega\pi_{t-1} + (1-\theta)(p_t^* - p_{t-1}) \quad (17)$$

2.5 The Behavioural Hybrid New Keynesian Curve

After some algebra relegated to Appendix E, a rearrangement of expressions (16) and (17) yields the Behavioral Hybrid NK Phillips curve in terms of marginal costs

$$\pi_t = \gamma_b\pi_{t-1} + \gamma_\mu\widehat{\text{mc}}_t + \gamma_f\mathbb{E}_t\pi_{t+1} \quad (18)$$

[11] Derived in Appendix D.

where

$$\gamma_b = \frac{\omega}{1+\omega\beta\overline{m}\left[\theta+(1-\theta)\frac{1-\theta\beta}{1-\theta\beta\overline{m}}\right]}$$
$$\gamma_\mu = \frac{(1-\theta)(1-\theta\beta)}{\theta\left\{1+\omega\beta\overline{m}\left[\theta+(1-\theta)\frac{1-\theta\beta}{1-\theta\beta\overline{m}}\right]\right\}}$$
$$\gamma_f = \frac{\beta\overline{m}\left[\theta+(1-\theta)\frac{1-\theta\beta}{1-\theta\beta\overline{m}}\right]}{1+\omega\beta\overline{m}\left[\theta+(1-\theta)\frac{1-\theta\beta}{1-\theta\beta\overline{m}}\right]}$$

In order to obtain the Behavioural Hybrid NK Phillips curve in terms of the output gap, recall that in this economy with technological progress, $\mathrm{MC}_t = W_t/(A_tP_t)$. Taking logs, we can write $\mathrm{mc}_t = w_t - p_t - a_t$. Additionally, firm technology implies $y_t = a_t + n_t$ and the aggregate resource constraint implies $y_t = c_t$. Finally, thanks to the labor supply condition (6) we know $w_t - p_t = \varphi n_t + \frac{\sigma}{1-h}c_t - \frac{\sigma h}{1-h}c_{t-1}$. Using these four expressions together yields

$$\widehat{\mathrm{mc}}_t = \left(\varphi + \frac{\sigma}{1-h}\right)\widetilde{y}_t - \frac{\sigma h}{1-h}\widetilde{y}_{t-1} \tag{19}$$

Introducing (19) into (18) leads to the Behavioral Hybrid NK Phillips curve

$$\pi_t = \gamma_b\pi_{t-1} + \alpha_b\widetilde{y}_{t-1} + \alpha_c\widetilde{y}_t + \gamma_f\mathbb{E}_t\pi_{t+1} \tag{BHNKPC}$$

where $\alpha_b = -\gamma_\mu\frac{\sigma h}{1-h}$ and $\alpha_c = \gamma_\mu\left(\varphi + \frac{\sigma}{1-h}\right)$. The Behavioral Hybrid NK Phillips curve (BHNKPC), together with the Behavioral Dynamic IS curve (BDIS) and a reaction function for the Central Bank (an ad-hoc inertial Taylor rule)

$$\widehat{i}_t = (1-\rho_r)(\phi_\pi\pi_t + \phi_y\widetilde{y}_t) + \rho_r\widehat{i}_{t-1} + e_t \tag{TR}$$

constitute the Behavioral New Keynesian framework with *keeping up with the Joneses* households and hybrid firms. Finally, we can write the model in system form as

$$\mathbf{A_c}\mathbf{x}_t = \mathbf{A_b}\mathbf{x}_{t-1} + \mathbf{A_f}\mathbb{E}_t\mathbf{x}_{t+1} + \mathbf{A_s}\mathbf{u}_t \tag{20}$$

where

$$\mathbf{x}_t = \begin{bmatrix}\widetilde{y}_t\\ \pi_t\\ \widehat{i}_t\end{bmatrix}, \qquad \mathbf{u}_t = \begin{bmatrix} r_t^n\\ e_t\end{bmatrix}$$

and

$$
\mathbf{A_c} = \begin{bmatrix} 1 & 0 & -\lambda_r \\ -\alpha_c & 1 & 0 \\ -(1-\rho_r)\phi_y & -(1-\rho_r)\phi_\pi & 1 \end{bmatrix}, \qquad \mathbf{A_b} = \begin{bmatrix} \lambda_b & 0 & 0 \\ \alpha_b & \gamma_b & 0 \\ 0 & 0 & \rho_r \end{bmatrix}
$$

$$
\mathbf{A_f} = \begin{bmatrix} \lambda_f & -\lambda_r \overline{m} & 0 \\ 0 & \gamma_f & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad \mathbf{A_s} = \begin{bmatrix} -\lambda_r & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}
$$

3 Estimation

This section lays out the approach we follow for the estimation of our structural parameters of interest through Bayesian techniques. First, we discuss the time series data we use and their transformation, for stationarity. Second, we describe our estimation procedure, that is, prior selection, calibration of certain parameters, and evaluation of the likelihood function using the Kalman Filter and the Metropolis-Hastings algorithm for finding posterior distributions as well as moments for our structural parameters.

As we have previously introduced, one of the objectives of this paper is to compare our results with those in Galí and Gertler (1999). In their seminal paper, they exclusively estimate different versions of the Phillips curve by GMM methods, whereas we estimate complete versions of the New Keynesian model. Given our strategy, we instead rely on Bayesian inference.

There are some advantages associated with full-information methods such as Bayesian estimation and that is the route we follow below.[12] For example, Bayesian approaches can improve the estimator precision and can lessen the identification problems, at least asymptotically; can reduce the risk of misspecification and can deal with model uncertainty and, finally, the results can be easily compared to the point estimates from standard BVARs.[13]

3.1 The Data

We estimate the model using three US time series at the quarterly frequency: 1) the log of real GDP per capita, 2) the log-difference of the inflation rate, and 3) the nominal interest rate. In particular, to proxy for the output gap, we apply a one-sided HP filter to the log

[12] Mavroeidis et al. (2014) present an excellent survey of studies using limited-information methods for the estimation of the New Keynesian Phillips curve.

[13] See Rabanal and Rubio-Ramírez (2005) and Fernández-Villaverde and Rubio-Ramírez (2004), for instance, for a more detailed discussion.

of real GDP per capita.[14] We demean both the inflation rate and the nominal interest rate which is the effective Federal Funds rate. The underlying data comes from FRED.[15]

We use two different samples that differ regarding their time spans. The first sample starts in 1960:I and ends in 1997:IV. We choose this period to be able to compare our results to those reported by Galí and Gertler (1999). The second sample, on the other hand, starts earlier in 1955:I and ends later in 2007:III. The only purpose of extending the sample is to improve the quality of the estimations.

3.2 A Bayesian Approach

Before estimating the model by using the aforementioned data, we introduce two additional sources of disturbance in our equations (BDIS) and (BHNKPC): an aggregate demand shock and an aggregate supply shock because we should have at least as many structural shocks as observable variables. As usual in Bayesian analysis, we need to specify the prior distributions for the structural parameters. To avoid identification problems we also decide to fix a small set of parameters at particular values. Finally, using prior information and the observable variables, we apply the Kalman Filter to evaluate the likelihood function of each model and the Metropolis-Hastings algorithm to draw from the posterior distributions and estimate their moments.[16]

3.2.1 Calibration and Prior Selection

To reduce the dimensionality of our problem and identification concerns, we fix the value for the coefficient of risk aversion and the inverse of the Frisch elasticity at unity, that is, $\sigma = \varphi = 1$. These parameters are not of any interest for this paper, so we simply set them to their standard values in the literature. The prior distribution for the others parameters is also standard (see e.g., Smets and Wouters 2007) and reported in Table 1. For the subjective discount factor, β, we use a Beta distribution with mean 0.85 and standard

[14] In the spirit of Bouakez et al. (2005), we use the per capita series to control for population growth. Nonetheless, our results do not depend on this particular specification. We also estimate the Behavioral Hybrid NK model by using output growth as an observable instead of the one-sided HP filtered GDP series (see Appendix F).

[15] We obtain real GDP from the U.S. Bureau of Economic Analysis (retrieved from FRED), "Real Gross Domestic Product [GDPC1]"; the price index from the U.S. Bureau of Labor Statistics (retrieved from FRED), "Consumer Price Index for All Urban Consumers: All Items in U.S. City Average [CPIAUCSL]"; and the nominal interest rate from the Board of Governors of the Federal Reserve System (retrieved from FRED), "Effective Federal Funds Rate [FEDFUNDS]". To convert real GDP in per capita terms we use population from the U.S. Bureau of Labor Statistics (retrieved from FRED), "Population Level [CNP16OV]".

[16] As usual, the posterior distribution can be approximated by the product of the prior and the likelihood function.

deviation 0.10.[17] Along the lines of Smets and Wouters (2007), for price stickiness, θ, and price indexation, ω, we also choose a Beta distribution with mean 0.5 and standard deviation 0.10.[18] For habit persistence, h, and inattention, $\overline{m}$, we set a Beta distribution with mean 0.75 and standard deviation 0.15.[19]

For the parameters entering the Taylor rule, we use a Normal distribution with mean 1.5 and standard deviation 0.15 for the response to changes in inflation, ϕ_π. Likewise, for the response to deviations from potential output, ϕ_y, we set a normally distributed prior with mean 0.15 and standard deviation 0.10. As in Smets and Wouters (2007), we use a Beta distribution for the persistence parameters using a mean of 0.5 and a standard deviation of 0.2. Finally, for the standard deviation of the shocks we use an Inverse Gamma Distribution with a mean of 0.01 and infinite standard deviation.

3.2.2 Bayesian Inference

We solve the model and estimate the remaining parameters for each specification using Dynare.[20] We use the CMA-ES algorithm for computing the mode which is robust to multiple local maxima (Hansen et al. 2003). To sample and estimate the moments of the posterior distributions, we use a Markov Chain Monte Carlo with 500.000 draws from the Metropolis-Hastings algorithm and burn-in the first 125.000 (25%). The acceptance rate was of 24%. Since we only employ one chain for the Metropolis-Hastings algorithm to reduce estimation time, convergence is checked using the test proposed by Geweke (1991).

4 Findings

This section discusses the estimation results for our parameters of interest and their implications for the business cycle. First, we pay attention to how those estimates change when we include new features into the standard New Keynesian model. Second, we examine the ability of our model specifications to reconcile previous empirical estimates. Finally, we consider whether our analytical models can replicate the responses of output gap and

[17] We also provide evidence that including σ in the estimation and fixing β to the standard value of 0.99 does not alter our main results (see Appendix F).

[18] This prior would imply that the average length of price contracts is 6 months.

[19] Notice that for the habit persistence parameter, Smets and Wouters (2007) use slightly different numbers: a mean of 0.7 and a standard deviation of 0.10. For the parameter measuring inattention, we have less prior information available. For example, Ilabaca et al. (2020) set a Beta distribution with a mean of 0.8 and a standard deviation of 0.15. Since these two key parameters interact in distinct ways as discussed below, we use the same prior for both parameters over a mean range. Overall, the estimations are not sensitive to small changes in either the mean or the standard deviations of the priors.

[20] See Adjemian et al. (2011) for more details.

inflation to a monetary policy shock estimated by means of a BVAR using narrative sign restrictions.

4.1 Posterior Distributions and Moments

Standard New Keynesian Model

Table 1 displays the main results. We report the posterior median and the 90% error bands. We begin by estimating an otherwise standard New Keynesian model using the first sample that goes from 1960:I to 1997:IV. The first column reports the estimation of the standard NK model. That is, we estimate the system (20) restricting $h = \omega = 0$ and $\overline{m} = 1$. In this standard framework there is no aggregate anchoring in the system since $\lambda_b = \gamma_b = \alpha_b = 0$, and the model exhibits an extreme forward-looking behavior. We find that this extreme forward-looking behavior in the theory can only be reproduced by the data if the agents discount the future at a very high rate, resulting in a discount factor of 0.805 at a quarterly frequency.

Even though there has not been any consensus on the level of the discount rate, or even the shape of time preferences in general (see Frederick et al. 2002 and Cohen et al. 2020 for excellent reviews), there is a sizeable literature, both experimental and empirical, which suggests that: (i) individuals do not exhibit any significant present-bias in monetary choices (see Augenblick et al. 2015) and (ii) individuals do not exhibit any high future discounting in such long-term monetary choices, with the estimated discount factors above 0.9 (see Angeletos et al. 2001, Laibson et al. 2015, Andreoni and Sprenger 2012, Andersen et al. 2014 and Attema et al. 2016). Such a high discount rate is also at odds with the common practice in macroeconomics, where a very high discount factor is usually assumed. For instance, the textbook NK model in Galí (2008) assumes a discount factor β of 0.99 at a quarterly frequency.

Hybrid New Keynesian Model

In order to provide a microfoundation for the value of the subjective discount factor β that is closer to its standard measure (close to unity), we add a backward-looking dimension into the model that produces aggregate anchoring. In particular, we add external habits on the household side ("*keeping up with the Joneses*") and inflation indexation on the firm side. That is, we estimate the system (20) relaxing h and ω. We also make this choice guided by a well-known failure of the benchmark NK model with respect to the data: it is unable to reproduce the hump-shaped IRFs observed in empirical macro studies, due to its extreme forward-looking behavior.

Table 1: Estimated Structural Parameters

	Prior Distribution		Posterior Distribution				
		Mean	*1960:I-1997:IV*				*1955:I-2007:III*
		(S.d)	**NK**	**HNK**	**BNK**	**BHNK**	**BHNK**
β	*Beta*	0.85	0.805	0.964	0.935	0.960	0.967
		(0.10)	(0.621, 0.978)	(0.905, 0.999)	(0.842, 0.998)	(0.898, 0.998)	(0.916, 0.999)
ϕ_π	*Normal*	1.50	1.802	1.344	1.341	1.351	1.351
		(0.15)	(1.610, 1.995)	(1.148, 1.546)	(1.141, 1.550)	(1.137, 1.562)	(1.145, 1.556)
ϕ_y	*Normal*	0.15	0.084	0.293	0.337	0.275	0.333
		(0.10)	(0.001, 0.175)	(0.188, 0.400)	(0.230, 0.449)	(0.156, 0.392)	(0.223, 0.449)
θ	*Beta*	0.50	0.622	0.916	0.795	0.882	0.901
		(0.10)	(0.545, 0.697)	(0.885, 0.946)	(0.743 0.847)	(0.841, 0.922)	(0.867, 0.935)
h	*Beta*	0.75	–	0.870	–	0.651	0.631
		(0.15)	(–)	(0.809, 0.927)	(–)	(0.483, 0.814)	(0.455, 0.802)
ω	*Beta*	0.50	–	0.835	–	0.786	0.778
		(0.15)	(–)	(0.722, 0.936)	(–)	(0.152, 0.896)	(0.699, 0.860)
$\overline{m}$	*Beta*	0.75	–	–	0.203	0.394	0.360
		(0.15)	(–)	(–)	(0.087, 0.326)	(0.189, 0.614)	(0.153, 0.582)
ρ_i	*Beta*	0.50	0.407	0.787	0.797	0.838	0.857
		(0.20)	(0.235, 0.765)	(0.745, 0.844)	(0.737, 0.854)	(0.788, 0.885)	(0.821, 0.893)
ρ_d	*Beta*	0.50	0.832	0.309	0.859	0.708	0.647
		(0.20)	(0.765, 0.894)	(0.170, 0.456)	(0.798, 0.918)	(0.595, 0.820)	(0.522, 0.760)
ρ_s	*Beta*	0.50	0.970	0.133	0.858	0.129	0.061
		(0.20)	(0.939, 0.990)	(0.023, 0.264)	(0.796, 0.921)	(0.017, 0.776)	(0.010, 0.127)
ρ_{e_i}	*Beta*	0.50	0.458	0.208	0.226	0.190	0.174
		(0.20)	(0.368, 0.539)	(0.085, 0.337)	(0.087, 0.373)	(0.071, 0.304)	(0.071, 0.274)
σ_d	*Inv. gamma*	0.01	0.0024	0.0030	0.0069	0.0051	0.0057
		(∞)	(0.0017, 0.0032)	(0.0025, 0.0036)	(0.0059, 0.0078)	(0.0040, 0.0062)	(0.0047, 0.0069)
σ_s	*Inv. gamma*	0.01	0.0048	0.0024	0.0037	0.0033	0.0035
		(∞)	(0.0029, 0.0068)	(0.0021, 0.0028)	(0.0032, 0.0042)	(0.0027, 0.0038)	(0.0029, 0.0040)
σ_i	*Inv. gamma*	0.01	0.0046	0.0025	0.0025	0.0025	0.0021
		(∞)	(0.0034, 0.0058)	(0.0023, 0.0028)	(0.0023, 0.0028)	(0.0022, 0.0027)	(0.0020, 0.0023)

Note: Results are reported at the posterior median. 90% confidence intervals in parenthesis.

To understand how these two new parameters, h and ω, affect the model dynamics, let us see how the hyper-parameters are affected by them. We begin with the Dynamic IS curve. Recall that in the standard model there is no anchoring in the Dynamic IS curve when $h = 0$ since $\lambda_b = 0$. However, once we relax h and allow it to be in the closed unit interval, we find that $\lambda_b = \frac{h}{1-h}$. One can then show that an increase in h from its initial 0 value leads to a rise in the backward-looking hyper-parameter in the Dynamic IS curve: $\partial\lambda_b/\partial h > 0$. In our model, this is a consequence of the "keeping up with the Joneses" utility function that we assume, in which agents maximize a quasi-difference in consumption. Because large increases in current consumption can be harming tomorrow's felicity, agents take into account past consumption levels and, after aggregating across agents, this results in anchoring in the Dynamic IS curve. We can similarly interpret the forward-looking hyper-parameter λ_f. Once we relax h we find that $\lambda_f = \frac{1}{1-h}$, which is decreasing in h: $\partial\lambda_f/\partial h < 0$. This is an expected result after our prior discussion. An increase in the backward-looking behavior, parameterized by h, implies that the agent assigns less importance to the future. Likewise, regarding the hyper-parameters of the contemporaneous variables, we find that $\lambda_r = -\frac{1-h}{\sigma(1+h)}$. One can then show that the introduction of h reduces the importance of the present: $\partial(-\lambda_r)/\partial h < 0$. That is, overall, the past becomes more important at the cost of the present and the future. As a result, the forward-looking dimension of the Dynamic IS curve is dampened, and anchoring gains momentum.

Turning now to the NK Philips curve, a similar argument follows. Recall that in the standard model there is no price indexation, so that $\omega = 0$. Extending the model to price indexation generates anchoring in the Philips curve, because current pricing decisions by firms are indexed to prior inflation. Notice also that there are two backward-looking terms in the Phillips curve, the output gap and the inflation rate. The additional backward-looking output gap term appears after inserting the output gap into the marginal costs, which establish a direct relation between the marginal rate of substitution, driven by households' preferences, and the output gap.

Let us start with the backward-looking inflation term. Relaxing h and ω, thus allowing each of them to be in the closed unit interval, we find that $\gamma_b = \frac{\omega}{1+\omega\beta}$. The backward-looking nature of the inflation term is affected by ω, the degree of indexation. As in the Dynamic IS curve, increasing the degree of price indexation from its benchmark value $\omega = 0$ enlarges the backward-looking hyper-parameter attached to inflation: $\partial\gamma_b/\partial\omega > 0$. The backward-looking output gap term α_b is affected both by h, since marginal costs are endogenous to the marginal rate of substitution, and ω. It is increasing in h, decreasing in ω and decreasing in both: $\partial(-\alpha_b)/\partial\omega < 0, \partial(-\alpha_b)/\partial h > 0$ and $\partial(-\alpha_b)/(\partial\omega\partial h) < 0$, thus generating anchoring when external habits and price-indexation are introduced into the model. As in the Dynamic IS curve, the forward-looking term γ_f is decreasing in ω: the relative importance of hyper-parameters is transferred from forward-looking to backward-looking ones. Moreover, the hyper-parameter interacting with the contemporaneous output gap, α_c, is increasing in h, decreasing in ω and in both: $\partial\alpha_c/\partial\omega < 0, \partial\alpha_c/\partial h > 0$ and $\partial\alpha_c/(\partial\omega\partial h) < 0$. As a result of anchoring, aggregate dynamics are more determined by the past and less by the present, as is the case in the standard NK model.

Our estimates of $h = 0.870$ and $\omega = 0.835$ are on the upper bound of the estimated values in the literature. In a meta-analysis, Havranek et al. (2017) find that the standard value of external habits in the macro literature is around 0.7, while the micro-consistent estimate is 0.4. There is less micro-empirical evidence on the true value of ω, since this form of indexation is a model artifact. This value is nevertheless not far from the standard assumed value in the literature (see e.g., Christiano et al. 2005 and Auclert et al. 2020). On top of these, we estimate an excessively high pricing friction $\theta = 0.916$: on average firms change prices every 12 quarters. For instance, Bils and Klenow (2004) find that, on average, prices change every six-to-nine months, and Nakamura and Steinsson (2008) find that, on average, prices change every seven-to-nine months. This lack of micro-consistency of aggregate parameters leads us to our last extension: a Behavioral Hybrid NK model à la Gabaix (2020).

Behavioral Hybrid New Keynesian Model

Despite being successful in obtaining hump-shaped impulse responses (see the next subsection), these are obtained assuming an excessive degree of household external habits and a too large a pricing friction. The above results then motivate our departure from full rationality. Here, we assume that agents (households and firms) in the model are bounded rational and discount the future with a cognitive discount factor $\overline{m}$. Because the cognitive discount factor interacts with the degree of external habits h and the Calvo price rigidity parameter θ backward-, contemporaneous and forward-looking terms, relaxing the cognitive discount factor, to be different from one, helps match the other parameters to their micro empirical estimates.

To understand how this new parameter $\overline{m}$ is affecting the model dynamics, let us see how the hyper-parameters are influenced by its introduction. Let us start with the Dynamic IS curve. Recall that in the HNK model there is no role for bounded rationality. However, once we relax $\overline{m}$ and allow it to be in the closed unit interval, we find that $\lambda_f = \frac{\overline{m}}{1-h}$ and $\lambda_r = -\frac{(1-h)\overline{m}}{\sigma(1+h)}$, while the other two hyper-parameters are unaffected by $\overline{m}$.

One can see that both affected hyper-parameters are increasing in the degree of attention $\overline{m}$.

Because attention in our last model takes the form of cognitive discounting, introducing inattention generates household myopia and reduces the importance of the parameters interacting with forward-looking variables, doing so without affecting the parameters linked to past and contemporaneous variables. Turning now to the NK Philips curve, a similar argument follows. The main difference relies on the fact that hyper-parameters interacting with backward-looking and contemporaneous variables are now also affected. Recall that in the standard model ($\overline{m} = 1$) there is no role for inattention. Extending the model to bounded rationality generates anchoring in the Philips curve, since cognitive discounting also interacts with price-indexation. To see the implications of this feature, let us start with the backward-looking inflation term. Relaxing $\overline{m}$ by allowing it to be in the closed unit interval, we find that γ_b is decreasing in attention: $\partial\gamma_b/\partial\overline{m} < 0$ as long as

$$(1 - \beta\theta)(1 - \beta\theta^2\overline{m}^2) + \beta\theta^2(1 - \overline{m})^2 > 0 \tag{21}$$

which is always satisfied.

Differently from the Dynamic IS curve, the backward-looking output gap term α_b is now affected by inattention, and is decreasing in $\overline{m}$ as long as condition (21) is satisfied. Again, as in the previous discussion, inattention and anchoring coming from price indexation lead to *more* anchoring. As in the Dynamic IS curve, the forward looking term γ_f is increasing in $\overline{m}$: with inattentive firms the relative importance of hyper-parameters is transferred from forward-looking to backward-looking ones. Besides, the hyper-parameter interacting with the contemporaneous output gap, α_c, is decreasing in $\overline{m}$, as in the Dynamic IS curve. We now find values of external habits and price frictions that are closer to their microfounded and standard values in the literature. Regarding the degree of inattention in the economy, we estimate a posterior median of 0.394.[21] Although we find this estimated value to be in the lower range, there is no consensus among previous studies regarding the value of this parameter.[22]

As a robustness check, we estimate the full model using the second sample that goes from 1955:I to 2007:III, right before the Great Recession. Our estimates are quite stable and we do not observe any considerable differences.

[21] Notice that when we consider the behavioral model without backward-looking agents, we estimate an even lower value of 0.203 for the bounded rationality parameter.

[22] For instance, Ilabaca et al. (2020) estimate a cognitive discount factor that is around 0.5 using Bayesian techniques, and Andrade et al. (2019) report a value of 0.67 using maximum likelihood inference.

4.2 Monetary Policy Shocks: NK Models vs. a Narrative BVAR

A failure of the standard model is that it does not produce hump-shaped impulse responses after an exogenous monetary policy shock, which is at odds with empirical macro evidence (see e.g., Christiano et al. 2005 and Altig et al. 2011). Applied macro studies generally find that the peak effect of a monetary policy shock appears after 4-8 quarters, whereas in the standard model without anchoring the peak effect occurs instantaneously, and the impulse responses are monotonically decreasing over time.

To see this more clearly, in Figure 1 we plot the impulse response functions (IRFs) of output, inflation and nominal interest rates after an exogenous monetary policy shock of $-25bp$ for the standard model which we label "NK". The excessively low discount factor and the counterfactual shape of the impulse responses motivate our departure from the benchmark model and the introduction of new features. For instance, Figure 1 also displays the IRFs for this model with anchoring labelled "HNK". The inclusion of those new features leads to strong hump-shaped responses for output gap and inflation following an exogenous monetary policy shock.

In order to compare our set of models to the data, we also estimate a BVAR and reproduce the impulse responses after a negative $25bp$ monetary policy shock, using the narrative sign restrictions approach followed by Antolín-Díaz and Rubio-Ramírez (2018) (see Appendix G for details on the identification strategy). As we show in Figure 1, including the backward-looking elements in the HNK model generates aggregate anchoring and improves the match between theoretical and empirical impulse responses' hump shapes. In addition, since the model is not as extremely forward-looking as the benchmark NK model, we are able to reconcile the discount factor to its microfounded value thanks to the introduction of h and ω. We are not successful, however, in matching other parameters with their established values in the literature. Concretely, the two key parameters that we use to generate the factual anchoring in the impulse responses are excessively high for the HNK model.

When we introduce behavioral features into the HNK model, now labelled "BHNK", we observe that cognitive discounting dampens the aggregate response to a monetary policy shock. Adding the backward-looking behavior together with cognitive discounting helps obtain impulse responses that align much better with the data — the reaction is smaller, more persistent and closer to the empirical counterpart while maintaining the hump-shaped dynamics. Overall, as we see in the output gap and inflation responses in Figure 1 for the BHNK model, there will be smaller responses than in the NK and HNK models. Intuitively, because there is anchoring due to price indexation, less attentive firms' actions will be determined by past aggregates to a larger extent.

For completeness, we also report the results for the behavioral New Keynesian model without anchoring which we label "BNK". We observe that the exclusion of external habits and inflation indexation implies that the IRFs are not hump-shaped. We then argue that we need both cognitive discounting and anchoring, first to match the empirical estimates for certain parameters of interest, and second to obtain hump-shaped IRFs and initially muted responses for both output gap and inflation. In particular, the strong inflation persistence obtained in VAR frameworks is exclusively present in the BHNK model.

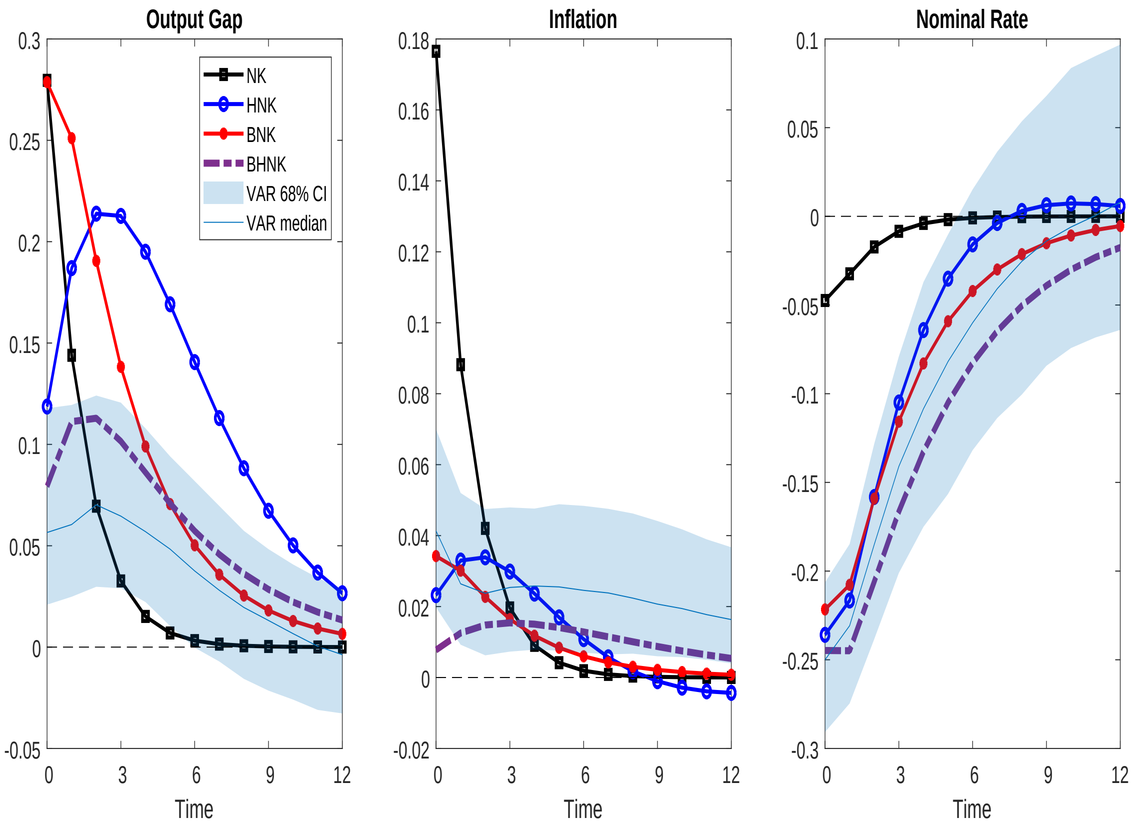

Figure 1: Dynamic Responses to a Monetary Policy Shock

Note: The dynamic paths for the variables are reported under different model specifications after an expansionary 25*bp* monetary policy shock: (i) a standard NK model in black lines (squares), (ii) a hybrid NK model in blue lines (circles), (iii) a behavioral NK model in red lines (asterisks), and (iv) a behavioral hybrid NK model in purple lines (dashed). The VAR-based monetary policy shock is identified by means of narrative sign restrictions as in Antolín-Díaz and Rubio-Ramírez (2018). Appendix G details the identification strategy. The horizontal axis displays the time which is measured in quarters. Vertical axis values refer to deviations from steady state in percentage.

5 Conclusion

The benchmark NK model is purely forward looking, and therefore, it lacks the ability to capture any sort of endogenous persistence in output and inflation that we observe in the data. In order to avoid this, the literature has included backward-looking agents, either assuming a backward-looking utility function for households or sticky price indexation for firms. Unfortunately, the parameter values that characterize the frictions required to produce the degree of anchoring that the data suggests are at odds with empirical evidence. In this paper, we harmonize these discrepancies between empirics and theory by building and estimating a New Keynesian model augmented with backward-looking agents *and* cognitive discounting. We find strong evidence for aggregate myopia, with a cognitive discount factor estimate of 0.4 at a quarterly frequency, and we reconcile three key parameters in the theory that were at odds with the empirical evidence: the subjective discount factor, the degree of external habits, and the degree of price stickiness.

For the estimation of the structural parameters, we follow a Bayesian approach that allows a transparent comparison across models. We estimate four different models: the standard NK model, the hybrid NK model, the behavioral NK model, and the behavioral hybrid NK model. We show that cognitive discounting is successful in producing myopia but does not produce anchoring on its own. In fact, when we estimate the behavioral NK model, we find an excessively low bounded rationality parameter, biased towards zero due to the anchoring that we observe in the data and the model is unable to produce. We find that the cognitive discount factor, *together* with habit persistence and price indexation, is key to obtain macro estimates that align better with their micro counterpart, and its estimated coefficient is nearly twice as large as in the benchmark case with no backward-looking agents. Finally, in order to test the ability of our set of models to replicate empirical impulse-response functions, we compare them with an estimated monetary policy shock. We find that only our Behavioral NK model with both habit formation and backward-looking firms is able to generate, at the same time, hump-shaped responses and enough output and inflation persistence as we observe in the data.

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Appendices

A Demand for good i, Aggregate Price Index and Optimality Conditions

The representative household derives utility from consumption of different goods, indexed $i \in I = [0,1]$, according to the consumption index. Let $\mathcal{C} = \{C_t \in \mathcal{L}^1 \colon C_t \colon I \to \mathbb{R}$ is quasi-concave and Borel measurable, $t \in \mathbb{Z}_+\}$ be the set of consumption choice functions over the set of goods I in the economy at a given period t.

Given the price function $P_t \colon I \to \mathbb{R}_+$ with $\|P_t\|_\infty < \infty$, and for a fixed endowment $Z_t \in \mathbb{R}_+$, the representative household's maximization problem at period t is:

$$\tilde{C}_t = \max_{C_t \in \mathcal{C}} \left[\int_0^1 C_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}} \tag{22}$$

subject to the budget constraint:

$$\int_0^1 P_t(i) C_t(i) \ di \le Z_t \tag{23}$$

which will be satisfied with equality in the optimum. The derivative of the Lagrangian with respect to $C_t(i)$, the consumption level of good i, yields:

$$\left[\int_0^1 C_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{1}{\epsilon-1}} C_t(i)^{-\frac{1}{\epsilon}} - \lambda_t P_t(i) = 0 \implies \tilde{C}_t^{\frac{1}{\epsilon}} C_t(i)^{-\frac{1}{\epsilon}} = \lambda_t P_t(i)$$

where λ_t is the sequence of Lagrange multipliers attached to the sequence of restrictions (23). By dividing the last expression for two different goods $i, j \in I$, we find relation between the optimal consumption levels of two different goods:

$$C_t(i) = \left[\frac{P_t(j)}{P_t(i)} \right]^{\epsilon} C_t(j) \tag{24}$$

and inserting (24) into (23),

$$Z_t = \int_0^1 P_t(i) \left[\frac{P_t(j)}{P_t(i)} \right]^{\epsilon} C_t(j) \ di \implies C_t(j) = \frac{Z_t P_t(j)^{-\epsilon}}{\int_0^1 P_t(i)^{1-\epsilon} di} \tag{25}$$

we obtain an expression for the optimal consumption levels of almost all goods in terms of prices and the initial endowment.

Integrating the last equation over all goods gives the optimal aggregate consumption level:

$$\tilde{C}_t = \left[\int_0^1 \left(\frac{Z_t P_t(i)^{-\epsilon}}{\int_0^1 P_t(i)^{1-\epsilon} di} \right)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}} = Z_t \left[\int_0^1 P_t(i)^{1-\epsilon} di \right]^{\frac{1}{\epsilon-1}}$$

Now, let's define $\tilde{P}_t$ as the unit cost of the aggregate consumption level $\tilde{C}_t$ at endowment level Z, $\tilde{P}_t \tilde{C}_t = Z_t$. Hence,

$$\tilde{P}_t Z_t \left[\int_0^1 P_t(i)^{1-\epsilon} di \right]^{\frac{1}{\epsilon-1}} = Z_t \implies \tilde{P}_t = \left[\int_0^1 P_t(i)^{1-\epsilon} di \right]^{\frac{1}{1-\epsilon}} \tag{26}$$

where (26) is the price index. Inserting (26) into (25)

$$C_t(j) = \frac{Z_t P_t(j)^{-\epsilon}}{\tilde{P_t}^{1-\epsilon}} = \frac{Z_t}{\tilde{P}_t} \left[\frac{\tilde{P}_t}{P_t(i)} \right]^{\epsilon} \tag{27}$$

And finally replacing Z_t, we find the desired optimal consumption for good i in terms of the aggregate good and the aggregate price:

$$C_t(i) = \left[\frac{P_t(i)}{\tilde{P}_t} \right]^{-\epsilon} \tilde{C}_t \tag{28}$$

With market clearing and a representative household setting, $C_t(i) = Y_t(i)$ and $\tilde{C}_t = \tilde{Y}_t$, and we obtain expression (3). Since we deal with the aggregate quantities in the rest of the paper, with a slight abuse of notation we drop the tilde from the aggregate terms.

Finally, in order to obtain the optimality conditions we form the lagrangian

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(C_t - h\overline{C}_{t-1})^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} + \lambda_t \left[B_{t-1} + W_t N_t + T_t - P_t C_t - Q_t B_t \right] \right]$$

The FOCs with respect to C_t, B_t and N_t yield

$$\begin{aligned} C_t : \quad & \lambda_t P_t = (C_t - h\overline{C}_{t-1})^{-\sigma} \\ N_t : \quad & \lambda_t W_t = N_t^{\varphi} \\ B_t : \quad & \lambda_t Q_t = \lambda_{t+1} \end{aligned}$$

Combining them and cancelling the lagrange multiplier λ_t we obtain the optimality conditions

$$\frac{W_t}{P_t} = \frac{N_t^{\varphi}}{(C_t - h\overline{C}_{t-1})^{-\sigma}}$$
$$Q_t = \beta \mathbb{E}_t \left[\left(\frac{C_{t+1} - h\overline{C}_t}{C_t - h\overline{C}_{t-1}} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right]$$

B Log-linearization of Behavioural Household's Optimality Conditions

We now proceed to log-linearize (6)-(7). Starting with (6), taking a first order Taylor approximation,

$$\frac{W}{P} + \frac{1}{P}(W_t - W) - \frac{W}{P^2}(P_t - P) = \frac{N^{\varphi}}{[C(1-h)]^{-\sigma}} + \varphi \frac{N^{\varphi-1}}{[C(1-h)]^{-\sigma}}(N_t - N) +$$
$$+ \sigma \frac{N^{\varphi}}{[C(1-h)]^{1-\sigma}}(C_t - C) - \sigma h \frac{N^{\varphi}}{[C(1-h)]^{1-\sigma}}(C_{t-1} - C)$$
$$\frac{W}{P}\left\{1 + \frac{W_t - W}{W} - \frac{P_t - P}{P}\right\} = \frac{N^{\varphi}}{[C(1-h)]^{-\sigma}} \times \left\{1 + \varphi \frac{N_t - N}{N} + \frac{\sigma}{1-h}\frac{C_t - C}{C} - \frac{\sigma h}{1-h}\frac{C_{t-1} - C}{C}\right\}$$
$$\hat{w}_t - \hat{p}_t = \varphi \hat{n}_t + \frac{\sigma}{1-h}\hat{c}_t - \frac{\sigma h}{1-h}\hat{c}_{t-1}$$

where any variable x satisfies $\hat{x}_t = x_t - x = \log X_t - \log X = \frac{X_t - X}{X}$. Turning to (7). Taking logs

$$\log Q_t = \log \beta + \mathbb{E}_t^B \big\{ -\sigma \log[C(X_{t+1}) - hC(X_t)] + \sigma \log[C(X_t) - hC(X_{t-1})] +$$
$$+ \log P(X_t) - \log P(X_{t+1}) \big\}$$

Since $Q_t = 1/(1+i_t)$, one can show that $i_t \approx -\log Q_t$. We then set $\rho = -\log \beta$ and $\pi(X_{t+1}) = \log P(X_{t+1})/P(X_t)$. Let us now log-linearize the terms that include consumption

$$\log[C(X_{t+1}) - hC(X_t)] \approx \log[(1-h)C] + \frac{1}{(1-h)C}[C(X_{t+1}) - C] - \frac{h}{(1-h)C}[C(X_t) - C]$$
$$= \log[(1-h)C] + \frac{1}{1-h}[c(X_{t+1}) - c] - \frac{h}{1-h}[c(X_t) - c]$$
$$= \log[(1-h)C] + \frac{1}{1-h}\widehat{c}(X_{t+1}) - \frac{h}{1-h}\widehat{c}(X_t)$$

proceeding in a similar manner with the other consumption term, and plugging into the above expression leads to

$$0 = \mathbb{E}_t^B \left\{ i(X_t) - \rho - \frac{\sigma}{1-h} \left[\widehat{c}(X_{t+1}) - h\widehat{c}(X_t) - \widehat{c}(X_t) + h\widehat{c}(X_{t-1}) \right] - \pi(X_{t+1}) \right\}$$

By Lemma 1, $\mathbb{E}_t^B[z(X_{t+k})] = m_z \overline{m}^k \mathbb{E}_t[z(X_{t+k})]$. Hence,

$$0 = m_i \widehat{i}(X_t) - \frac{\sigma}{1-h} m_y \overline{m} \mathbb{E}_t \left[\widehat{c}(X_{t+1}) \right] - \frac{(1+h)\sigma}{1-h} m_y c(X_t) - \frac{h\sigma}{1-h} c(X_{t-1}) - m_\pi \overline{m} \mathbb{E}_t [\pi(X_{t+1})]$$

which, denoting $z_t \equiv z(X_t)$ and $\widehat{i}_t = i_t - i = i_t - \rho$, leads to (9). Written in natural terms and denoting $r_t = m_i \widehat{i}_t - m_\pi \overline{m} \mathbb{E}_t \pi_{t+1}$ yields

$$\hat{c}_t^n = \frac{h}{1+h} \frac{1}{m_y} \hat{c}_{t-1}^n + \frac{1}{1+h} \overline{m} \mathbb{E}_t \hat{c}_{t+1}^n - \frac{1-h}{\sigma(1+h)} \frac{1}{m_y} r_t^n$$

Since $\widehat{c}_t = \widehat{y}_t$, we can rewrite it in terms of the output gap $\tilde{y}_t = y_t - y_t^n$ and it yields (BDIS).

C Solving the Firm Problem

Taking the first-order condition of (2.3.1) with respect to $P_t^*(i)$ yields

$$P_t^*(i): \quad \mathbb{E}_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} \left[\frac{P_t^*(i) \chi_{t,t+k}}{P_{t+k}} \right]^{-\epsilon} C_{t+k} \left[\chi_{t,t+k}(1-\epsilon) + \epsilon \frac{W_{t+k}}{P_t^*(i) A_{t+k}} \right] = 0 \implies$$

$$\underset{\times \frac{P_t^*(i)}{1-\epsilon}}{\implies} \mathbb{E}_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} \left[\frac{P_t^*(i) \chi_{t,t+k}}{P_{t+k}} \right]^{-\epsilon} C_{t+k} \left[P_t^*(i) \chi_{t,t+k} - \mathcal{M} \frac{W_{t+k}}{A_{t+k}} \right] = 0$$

where $\mathcal{M} = \frac{\epsilon}{\epsilon - 1}$. Separating both sides,

$$\mathbb{E}_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} \left[P_t^*(i) \chi_{t,t+k} \right]^{1-\epsilon} P_{t+k}^{\epsilon} C_{t+k} = \mathbb{E}_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} \left[\frac{P_t^*(i) \chi_{t,t+k}}{P_{t+k}} \right]^{-\epsilon} C_{t+k} \mathcal{M} \frac{W_{t+k}}{A_{t+k}}$$

Inserting $Q_{t,t+k} = \beta^k \left(\frac{C_{t+k}-hC_{t+k-1}}{C_t-hC_{t-1}}\right)^{-\sigma} \frac{P_t}{P_{t+k}}$ and $\chi_{t,t+k} = \left(\frac{P_{t+k-1}}{P_{t-1}}\right)^{\omega}$, and solving for P_t^*, we can write the left-hand side as

$$\begin{aligned}
&\mathbb{E}_t \sum_{k=0}^{\infty} \theta^k \beta^k \left(\frac{C_{t+k}-hC_{t+k-1}}{C_t-hC_{t-1}}\right)^{-\sigma} \frac{P_t}{P_{t+k}} \left[P_t^*(i)\left(\frac{P_{t+k-1}}{P_{t-1}}\right)^{\omega}\right]^{1-\epsilon} P_{t+k}^{\epsilon} C_{t+k} \\
&\quad = \mathbb{E}_t \sum_{k=0}^{\infty} (\theta\beta)^k \left(\frac{C_{t+k}-hC_{t+k-1}}{C_t-hC_{t-1}}\right)^{-\sigma} C_{t+k} P_t P_{t+k}^{\epsilon-1} P_t^*(i)^{1-\epsilon} \left(\frac{P_{t+k-1}}{P_{t-1}}\right)^{\omega(1-\epsilon)} \\
&\quad = (C_t - hC_{t-1})^{\sigma} P_t P_t^*(i)^{1-\epsilon} P_{t-1}^{-\omega(1-\epsilon)} \mathbb{E}_t \sum_{k=0}^{\infty} (\theta\beta)^k (C_{t+k}-hC_{t+k-1})^{-\sigma} C_{t+k} P_{t+k}^{\epsilon-1} P_{t+k-1}^{\omega(1-\epsilon)}
\end{aligned}$$

Similarly with the right-hand side

$$\begin{aligned}
&\mathbb{E}_t \sum_{k=0}^{\infty} \theta^k \beta^k \left(\frac{C_{t+k}-hC_{t+k-1}}{C_t-hC_{t-1}}\right)^{-\sigma} \frac{P_t}{P_{t+k}} \left[\frac{P_t^*(i)\left(\frac{P_{t+k-1}}{P_{t-1}}\right)^{\omega}}{P_{t+k}}\right]^{-\epsilon} C_{t+k} \mathcal{M} \frac{W_{t+k}}{A_{t+k}} \\
&\quad = \mathcal{M} \mathbb{E}_t \sum_{k=0}^{\infty} (\theta\beta)^k \left(\frac{C_{t+k}-hC_{t+k-1}}{C_t-hC_{t-1}}\right)^{-\sigma} C_{t+k} P_t P_{t+k}^{\epsilon-1} P_t^*(i)^{-\epsilon} P_{t-1}^{\omega\epsilon} P_{t+k-1}^{-\omega\epsilon} \frac{W_{t+k}}{A_{t+k}} \\
&\quad = \mathcal{M} (C_t - hC_{t-1})^{\sigma} P_t P_t^*(i)^{-\epsilon} P_{t-1}^{\omega\epsilon} \mathbb{E}_t \sum_{k=0}^{\infty} (\theta\beta)^k (C_{t+k}-hC_{t+k-1})^{-\sigma} C_{t+k} P_{t+k}^{\epsilon-1} P_{t+k-1}^{-\omega\epsilon} \frac{W_{t+k}}{A_{t+k}}
\end{aligned}$$

Finally, equating both sides of the equality gives

$$\begin{aligned}
&(C_t - hC_{t-1})^{\sigma} P_t P_t^*(i)^{1-\epsilon} P_{t-1}^{-\omega(1-\epsilon)} \mathbb{E}_t \sum_{k=0}^{\infty} (\theta\beta)^k (C_{t+k}-hC_{t+k-1})^{-\sigma} C_{t+k} P_{t+k}^{\epsilon-1} P_{t+k-1}^{\omega(1-\epsilon)} \\
&\quad = \mathcal{M} (C_t - hC_{t-1})^{\sigma} P_t P_t^*(i)^{-\epsilon} P_{t-1}^{\omega\epsilon} \mathbb{E}_t \sum_{k=0}^{\infty} (\theta\beta)^k (C_{t+k}-hC_{t+k-1})^{-\sigma} C_{t+k} P_{t+k}^{\epsilon-1} P_{t+k-1}^{-\omega\epsilon} \frac{W_{t+k}}{A_{t+k}}
\end{aligned}$$

And solving for the optimal reset prices gives

$$\begin{aligned}
P_t^*(i) &= \mathcal{M} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} (\theta\beta)^k (C_{t+k}-hC_{t+k-1})^{-\sigma} C_{t+k} P_{t+k}^{\epsilon-1} P_{t+k-1}^{-\omega\epsilon} \frac{W_{t+k}}{A_{t+k}}}{\mathbb{E}_t \sum_{k=0}^{\infty} (\theta\beta)^k (C_{t+k}-hC_{t+k-1})^{-\sigma} C_{t+k} P_{t+k}^{\epsilon-1} P_{t+k-1}^{\omega(1-\epsilon)}} P_{t-1}^{\omega} = \\
&= \mathcal{M} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} (\theta\beta)^k (C_{t+k}-hC_{t+k-1})^{-\sigma} C_{t+k} P_{t+k}^{\epsilon} P_{t+k-1}^{-\omega\epsilon} MC_{t+k}}{\mathbb{E}_t \sum_{k=0}^{\infty} (\theta\beta)^k (C_{t+k}-hC_{t+k-1})^{-\sigma} C_{t+k} P_{t+k}^{\epsilon-1} P_{t+k-1}^{\omega(1-\epsilon)}} P_{t-1}^{\omega} \qquad (29)
\end{aligned}$$

where we have used $MC_{t+k} = \frac{W_{t+k}}{A_{t+k}P_{t+k}}$. With flexible prices, (29) collapses to

$$P_t^*(i) = \mathcal{M}\frac{(C_t - hC_{t-1})^{-\sigma}C_t P_t^{\epsilon} P_{t-1}^{-\omega\epsilon} MC_t}{(C_t - hC_{t-1})^{-\sigma}C_t P_t^{\epsilon-1} P_{t-1}^{\omega(1-\epsilon)}} P_{t-1}^{\omega} = \mathcal{M}P_t MC_t \tag{30}$$

where (30) is the frictionless mark-up. To simplify computation, I now log-linearize (29): separating (back) both sides,

$$P_t^* \mathbb{E}_t \sum_{k=0}^{\infty} (\theta\beta)^k (C_{t+k} - hC_{t+k-1})^{-\sigma} C_{t+k} P_{t+k}^{\epsilon-1} P_{t+k-1}^{\omega(1-\epsilon)} =$$
$$= \mathcal{M}\mathbb{E}_t \sum_{k=0}^{\infty} (\theta\beta)^k (C_{t+k} - hC_{t+k-1})^{-\sigma} C_{t+k} P_{t+k}^{\epsilon} P_{t+k-1}^{-\omega\epsilon} MC_{t+k} P_{t-1}^{\omega} \tag{31}$$

We know that, in steady-state, $P_t^* = P_t = P_{t-1} = P$, $\Pi_t = \Pi = 1$, $C_t = C$, $Q_{t,t+k} = \beta^k$ and $MC_t = MC$. It lasts to find MC. To obtain it, write (29) in steady-state and solve for MC,

$$\begin{aligned} P &= \mathcal{M}\frac{\sum_{k=0}^{\infty}(\theta\beta)^k C^{1-\sigma}(1-h)^{-\sigma} P^{\epsilon(1-\omega)} MC}{\sum_{k=0}^{\infty}(\theta\beta)^k C^{1-\sigma}(1-h)^{-\sigma} P^{-(1-\epsilon)(1-\omega)}} P^{\omega} = \\ &= \mathcal{M}\frac{C^{1-\sigma}(1-h)^{-\sigma} P^{\epsilon(1-\omega)} MC \frac{1}{1-\theta\beta}}{C^{1-\sigma}(1-h)^{-\sigma} P^{-(1-\epsilon)(1-\omega)} \frac{1}{1-\theta\beta}} P^{\omega} = \\ &= \mathcal{M}P^{1-\omega} MC P^{\omega} \end{aligned}$$

Hence, $MC = \frac{1}{\mathcal{M}}$. Before log-linearizing, divide (32) by P_{t-1},

$$\frac{P_t^*}{P_{t-1}} \mathbb{E}_t \sum_{k=0}^{\infty} (\theta\beta)^k (C_{t+k} - hC_{t+k-1})^{-\sigma} C_{t+k} P_{t+k}^{\epsilon-1} P_{t+k-1}^{\omega(1-\epsilon)} =$$
$$= \mathcal{M}\mathbb{E}_t \sum_{k=0}^{\infty} (\theta\beta)^k (C_{t+k} - hC_{t+k-1})^{-\sigma} C_{t+k} P_{t+k}^{\epsilon} P_{t+k-1}^{-\omega\epsilon} MC_{t+k} P_{t-1}^{\omega-1} \tag{32}$$

Log-linearizing the LHS,

$$\frac{P_t^*}{P_{t-1}}\mathbb{E}_t\sum_{k=0}^{\infty}(\theta\beta)^k(C_{t+k}-hC_{t+k-1})^{-\sigma}C_{t+k}P_{t+k}^{\epsilon-1}P_{t+k-1}^{\omega(1-\epsilon)}\simeq$$

$$\begin{aligned}
&\simeq\sum_{k=0}^{\infty}(\theta\beta)^kC^{1-\sigma}(1-h)^{-\sigma}P^{-(1-\epsilon)(1-\omega)}+\\
&+\frac{1}{P}\sum_{k=0}^{\infty}(\theta\beta)^kC^{1-\sigma}(1-h)^{-\sigma}P^{-(1-\epsilon)(1-\omega)}(P_t^*-P)-\\
&-\frac{P}{P^2}\sum_{k=0}^{\infty}(\theta\beta)^kC^{1-\sigma}(1-h)^{-\sigma}P^{-(1-\epsilon)(1-\omega)}(P_{t-1}-P)+\\
&+\sum_{k=0}^{\infty}(\theta\beta)^kC^{1-\sigma}(1-h)^{-\sigma}(\epsilon-1)P^{\epsilon-2}P^{\omega(1-\epsilon)}(P_{t+k}-P)+\\
&+\sum_{k=0}^{\infty}(\theta\beta)^kC^{1-\sigma}(1-h)^{-\sigma}P^{\epsilon-1}\omega(1-\epsilon)P^{\omega(1-\epsilon)-1}(P_{t+k-1}-P)+\\
&+\sum_{k=0}^{\infty}(\theta\beta)^k\{\underbrace{(-\sigma)[C(1-h)]^{-\sigma-1}C+[C(1-h)]^{-\sigma}}_{C^{-\sigma}(1-h)^{-\sigma-1}(1-h-\sigma)}\}P^{-(1-\epsilon)(1-\omega)}(C_{t+k}-C)+\\
&+\sum_{k=0}^{\infty}(\theta\beta)^k\underbrace{(-\sigma)[C(1-h)]^{-\sigma-1}(-h)C}_{\sigma hC^{-\sigma}(1-h)^{-\sigma-1}}P^{-(1-\epsilon)(1-\omega)}(C_{t+k-1}-C)=\\
&=\sum_{k=0}^{\infty}(\theta\beta)^kC^{1-\sigma}(1-h)^{-\sigma}P^{-(1-\epsilon)(1-\omega)}\Bigg\{1+p_t^*-p-p_{t-1}+p-(1-\epsilon)(p_{t+k}-p)+\\
&\qquad+\omega(1-\epsilon)(p_{t+k-1}-p)+\left(1-\frac{\sigma}{1-h}\right)(c_{t+k}-c)+\frac{\sigma h}{1-h}(c_{t+k-1}-c)\Bigg\}
\end{aligned}$$

Log-linearizing the RHS,

$$\mathcal{M}\mathbb{E}_t\sum_{k=0}^{\infty}(\theta\beta)^k(C_{t+k}-hC_{t+k-1})^{-\sigma}C_{t+k}P_{t+k}^{\epsilon}P_{t+k-1}^{-\omega\epsilon}MC_{t+k}P_{t-1}^{\omega-1}\simeq$$

$$\begin{aligned}
\simeq&\mathcal{M}\sum_{k=0}^{\infty}(\theta\beta)^kC^{1-\sigma}(1-h)^{-\sigma}P^{-(1-\epsilon)(1-\omega)}MC+\\
&+\mathcal{M}\sum_{k=0}^{\infty}(\theta\beta)^kC^{1-\sigma}(1-h)^{-\sigma}P^{\epsilon}P^{-\omega\epsilon}MC(\omega-1)P^{\omega-2}(P_{t-1}-P)+\\
&+\mathcal{M}\sum_{k=0}^{\infty}(\theta\beta)^kC^{1-\sigma}(1-h)^{-\sigma}\epsilon P^{\epsilon-1}P^{-\omega\epsilon}MCP^{\omega-1}(P_{t+k}-P)+\\
&+\mathcal{M}\sum_{k=0}^{\infty}(\theta\beta)^kC^{1-\sigma}(1-h)^{-\sigma}P^{\epsilon}(-\omega\epsilon)P^{-\omega\epsilon-1}MCP^{\omega-1}(P_{t+k-1}-P)+\\
&+\mathcal{M}\sum_{k=0}^{\infty}(\theta\beta)^kC^{-\sigma}(1-h)^{-\sigma-1}(1-h-\sigma)P^{-(1-\epsilon)(1-\omega)}MC(C_{t+k}-C)+\\
&+\mathcal{M}\sum_{k=0}^{\infty}(\theta\beta)^kC^{-\sigma}(1-h)^{-\sigma-1}\sigma hP^{-(1-\epsilon)(1-\omega)}MC(C_{t+k-1}-C)+\\
&+\mathcal{M}\sum_{k=0}^{\infty}(\theta\beta)^kC^{1-\sigma}(1-h)^{-\sigma}P^{-(1-\epsilon)(1-\omega)}(MC_{t+k}-MC)=\\
=&\sum_{k=0}^{\infty}(\theta\beta)^kC^{1-\sigma}(1-h)^{-\sigma}P^{-(1-\epsilon)(1-\omega)}\Big\{1-(1-\omega)(p_{t-1}-p)+\epsilon(p_{t+k}-p)-\\
&-\omega\epsilon(p_{t+k-1}-p)+\left(1-\frac{\sigma}{1-h}\right)(c_{t+k}-c)+\frac{\sigma h}{1-h}(c_{t+k-1}-c)+mc_{t+k}-mc\Big\}
\end{aligned}$$

Set LHS=RHS, eliminating $C^{1-\sigma}(1-h)^{-\sigma}P^{-(1-\epsilon)(1-\omega)}$ on both sides,

$$\begin{aligned}
&\sum_{k=0}^{\infty}(\theta\beta)^k\Big\{1+p_t^*-p-p_{t-1}+p-(1-\epsilon)(p_{t+k}-p)+\omega(1-\epsilon)(p_{t+k-1}-p)+\\
&\qquad+\left(1-\frac{\sigma}{1-h}\right)(c_{t+k}-c)+\frac{\sigma h}{1-h}(c_{t+k-1}-c)\Big\}=\\
&=\sum_{k=0}^{\infty}(\theta\beta)^k\Big\{1-(1-\omega)(p_{t-1}-p)+\epsilon(p_{t+k}-p)-\omega\epsilon(p_{t+k-1}-p)+\\
&\qquad+\left(1-\frac{\sigma}{1-h}\right)(c_{t+k}-c)+\frac{\sigma h}{1-h}(c_{t+k-1}-c)+mc_{t+k}-mc\Big\}
\end{aligned}$$

Rearranging and cancelling terms, we end up with

$$\mathbb{E}_t \sum_{k=0}^{\infty} (\theta\beta)^k \left[p_t^* - \omega p_{t-1} - p_{t+k} + \omega p_{t+k-1}\right] = \mathbb{E}_t \sum_{k=0}^{\infty} (\theta\beta)^k \left[mc_{t+k} - mc\right] \implies$$

$$\implies p_t^* = (1-\theta\beta) \sum_{k=0}^{\infty} (\theta\beta)^k \mathbb{E}_t \left[p_{t+k} - \omega(\omega p_{t+k-1} - p_{t-1}) + mc_{t+k} - mc\right]$$
$$= (1-\theta\beta) \sum_{k=0}^{\infty} (\theta\beta)^k \mathbb{E}_t \left[p_{t+k} - \omega(\omega p_{t+k-1} - p_{t-1}) + \widehat{mc}_{t+k}\right]$$
$$= p_t + (1-\theta\beta) \sum_{k=0}^{\infty} (\theta\beta)^k \mathbb{E}_t \left[(p_{t+k} - p_t) - \omega(\omega p_{t+k-1} - p_{t-1}) + \widehat{mc}_{t+k}\right]$$

which can be rewritten as (15).

D Aggregate Price Dynamics

Let S_t denote the subset of firms not reoptimizing at time t,

$$P_t = \left[\int_0^1 P_t(i)^{1-\epsilon} di\right]^{\frac{1}{1-\epsilon}} =$$
$$= \Bigg\{ \underbrace{\int_{S_t} [P_{t-1}(i)\Pi_{t-1}^{\omega}]^{1-\epsilon} di}_{\Pi_{t-1}^{\omega(1-\epsilon)} \int_{S_t} P_{t-1}(i)^{1-\epsilon} di} + \int_{S_t^C} (P_t^*)^{1-\epsilon} di \Bigg\}^{\frac{1}{1-\epsilon}} =$$
$$= \left[\Pi_{t-1}^{\omega(1-\epsilon)} \theta \int_0^1 P_{t-1}(i)^{1-\epsilon} di + (1-\theta) \int_0^1 (P_t^*)^{1-\epsilon} di\right]^{\frac{1}{1-\epsilon}} =$$
$$= \left[\Pi_{t-1}^{\omega(1-\epsilon)} \theta P_{t-1}^{1-\epsilon} + (1-\theta)(P_t^*)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}$$

Moving the exponent from the RHS to the LHS, and dividing in both sides by $P_{t-1}^{1-\epsilon}$,

$$\left(\frac{P_t}{P_{t-1}}\right)^{1-\epsilon} = \Pi_{t-1}^{\omega(1-\epsilon)} \theta + (1-\theta) \left(\frac{P_t^*}{P_{t-1}}\right)^{1-\epsilon} \implies$$
$$\implies \Pi_t^{1-\epsilon} = \left(\frac{P_{t-1}}{P_{t-2}}\right)^{\omega(1-\epsilon)} \theta + (1-\theta) \left(\frac{P_t^*}{P_{t-1}}\right)^{1-\epsilon} \tag{33}$$

To simplify computation, I now log-linearize the left-hand side of (33),

$$\Pi_t^{1-\epsilon} \simeq \Pi^{1-\epsilon} + (1-\epsilon)\Pi^{-\epsilon} \underbrace{(\Pi_t - \Pi)}_{\pi_t} = \\ = 1 + (1-\epsilon)\pi_t$$

since $\Pi = \frac{P}{P} = 1$. A log-linearization of the right-hand side around a zero-inflation steady-state yields

$$\begin{aligned}
\left(\frac{P_{t-1}}{P_{t-2}}\right)^{\omega(1-\epsilon)} \theta + (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\epsilon} \simeq & \left(\frac{P}{P}\right)^{\omega(1-\epsilon)} \theta + (1-\theta)\left(\frac{P^*}{P}\right)^{1-\epsilon} + \\
& + (1-\theta)(1-\epsilon)P^{-\epsilon}P^{\epsilon-1}(P_t^* - P) + \\
& + \left[\theta\omega(1-\epsilon)P^{\omega(1-\epsilon)-1}P^{-\omega(1-\epsilon)} - (1-\theta)(1-\epsilon)P^{1-\epsilon}P^{2-\epsilon}\right] \times \\
& \times (P_{t-1} - P) - \theta\omega(1-\epsilon)P^{\omega(1-\epsilon)}P^{-\omega(1-\epsilon)-1}(P_{t-2} - P) = \\
= & \theta + 1 - \theta + (1-\theta)(1-\epsilon)\hat{p}_t^* + \\
& + [\theta\omega(1-\epsilon) - (1-\theta)(1-\epsilon)]\hat{p}_{t-1} - \theta\omega(1-\epsilon)\hat{p}_{t-2} = \\
= & 1 + (1-\theta)(1-\epsilon)\hat{p}_t^* - (1-\epsilon)[1-\theta(1+\omega)]\hat{p}_{t-1} - \\
& - \theta\omega(1-\epsilon)\hat{p}_{t-2} = \\
= & 1 + (1-\theta)(1-\epsilon)p_t^* - (1-\epsilon)[1-\theta(1+\omega)]p_{t-1} - \\
& - \theta\omega(1-\epsilon)p_{t-2}
\end{aligned}$$

Writing $\hat{x}_t = x_t - x$, luckily happens that all p's are cancelled.

$$\begin{aligned}
\text{LHS=RHS:} \quad 1 + (1-\epsilon)\pi_t = 1 + (1-\theta)(1-\epsilon)p_t^* - (1-\epsilon)[1-\theta(1+\omega)]p_{t-1} - \theta\omega(1-\epsilon)p_{t-2} \implies \\
\implies \pi_t = (1-\theta)p_t^* - [1-\theta(1+\omega)]p_{t-1} - \theta\omega p_{t-2} \\
= \theta\omega\pi_{t-1} + (1-\theta)(p_t^* - p_{t-1})
\end{aligned}$$

E Deriving the Behavioural Hybrid New Keynesian Phillips Curve

Rewriting $\theta\beta\overline{m} = \delta$, the firm's problem optimality condition (16) reads

$$
\begin{aligned}
p_t^* &= p_t + (1-\theta\beta)\sum_{k=0}^{\infty}\delta^k \mathbb{E}_t\left[\overline{m}(\pi_{t+1}+...+\pi_{t+k}) - \omega\overline{m}(\pi_t+...+\pi_{t+k-1}) + \overline{m}\widehat{mc}_{t+k}\right] \\
&= p_t + (1-\theta\beta)\mathbb{E}_t\left[\overline{m}\sum_{k=0}^{\infty}\delta^k(\pi_{t+1}+...+\pi_{t+k}) - \omega\overline{m}\sum_{k=0}^{\infty}\delta^k(\pi_t+...+\pi_{t+k-1}) + \overline{m}\sum_{k=0}^{\infty}\delta^k\widehat{mc}_{t+k}\right]
\end{aligned}
\tag{34}
$$

We can calculate the following

$$
\begin{aligned}
H_t &= \sum_{k=1}^{\infty}\delta^k(\pi_{t+1}+...+\pi_{t+k}) = \sum_{j=1}^{\infty}\pi_{t+j}\sum_{k=j}^{\infty}\delta^k = \sum_{j=1}^{\infty}\pi_{t+j}\frac{\delta^j}{1-\delta} = \frac{1}{1-\delta}\sum_{j=1}^{\infty}\pi_{t+j}\delta^j = \\
&= \frac{1}{1-\delta}\sum_{k=0}^{\infty}\pi_{t+k}\delta^k 1_{\{k>0\}} \\
\widetilde{H}_t &= \sum_{k=1}^{\infty}\delta^k(\pi_t+...+\pi_{t+k-1}) = \sum_{j=1}^{\infty}\pi_{t+j-1}\sum_{k=j}^{\infty}\delta^k = \sum_{j=1}^{\infty}\pi_{t+j-1}\frac{\delta^j}{1-\delta} = \frac{1}{1-\delta}\sum_{j=1}^{\infty}\pi_{t+j-1}\delta^j = \\
&= \frac{1}{1-\delta}\sum_{k=0}^{\infty}\pi_{t+k-1}\delta^k 1_{\{k>0\}}
\end{aligned}
$$

Rewriting (34),

$$
\begin{aligned}
p_t^* - p_t &= (1-\theta\beta)\mathbb{E}_t\left[\overline{m}H_t - \omega\overline{m}\widetilde{H}_t + \overline{m}\sum_{k=0}^{\infty}\delta^k\widehat{mc}_{t+k}\right] \\
&= (1-\theta\beta)\mathbb{E}_t\left[\overline{m}\frac{1}{1-\delta}\sum_{k=0}^{\infty}\pi_{t+k}\delta^k 1_{\{k>0\}} - \omega\overline{m}\frac{1}{1-\delta}\sum_{k=0}^{\infty}\pi_{t+k-1}\delta^k 1_{\{k>0\}} + \overline{m}\sum_{k=0}^{\infty}\delta^k\widehat{mc}_{t+k}\right] \\
&= (1-\theta\beta)\mathbb{E}_t\sum_{k=0}^{\infty}\delta^k\left[\overline{m}\frac{1}{1-\delta}\pi_{t+k}1_{\{k>0\}} - \omega\overline{m}\frac{1}{1-\delta}\pi_{t+k-1}1_{\{k>0\}} + \overline{m}\widehat{mc}_{t+k}\right] \\
&= \mathbb{E}_t\sum_{k=0}^{\infty}\delta^k\left[\overline{m}\frac{1-\theta\beta}{1-\delta}\pi_{t+k}1_{\{k>0\}} - \omega\overline{m}\frac{1-\theta\beta}{1-\delta}\pi_{t+k-1}1_{\{k>0\}} + \overline{m}(1-\theta\beta)\widehat{mc}_{t+k}\right] \\
&= \mathbb{E}_t\sum_{k=0}^{\infty}\delta^k\left[\widetilde{m}_\pi\pi_{t+k}1_{\{k>0\}} - \omega\widetilde{m}_\pi\pi_{t+k-1}1_{\{k>0\}} + \widetilde{m}_\mu\widehat{mc}_{t+k}\right]
\end{aligned}
\tag{35}
$$

where $\widetilde{m}_\pi = \overline{m}\frac{1-\theta\beta}{1-\delta}$ and $\widetilde{m}_\mu = \overline{m}(1-\theta\beta)$. Rewriting the price evolution expression

(17),

$$p_t^* - p_{t-1} + p_t - p_t = \frac{\pi_t - \theta\omega\pi_{t-1}}{1-\theta} \implies p_t^* - p_t = \frac{\theta}{1-\theta}(\pi_t - \omega\pi_{t-1})$$

Hence, we can rewrite (35) as

$$\frac{\theta}{1-\theta}(\pi_t - \omega\pi_{t-1}) = \mathbb{E}_t \sum_{k=0}^{\infty} \delta^k \left[\widetilde{m}_\pi \pi_{t+k} 1_{\{k>0\}} - \omega \widetilde{m}_\pi \pi_{t+k-1} 1_{\{k>0\}} + \widetilde{m}_\mu \widehat{mc}_{t+k}\right] \tag{36}$$

Let us now introduce the forward operator F such that $F^k x_t = x_{t+k}$. Using the forward operator, we can write

$$\sum_{k=0}^{\infty} \delta^k x_{t+k} = \sum_{k=0}^{\infty} \delta^k F^k x_t = \sum_{k=0}^{\infty} (\delta F)^k x_t = \frac{x_t}{1-\delta F} \tag{37}$$

Rewriting (36) using (37)

$$\begin{aligned}
\frac{\theta}{1-\theta}(\pi_t - \omega\pi_{t-1}) &= \widetilde{m}_\pi \mathbb{E}_t \left[\sum_{k=0}^{\infty} \delta^k \pi_{t+k} 1_{\{k>0\}}\right] - \omega\widetilde{m}_\pi \mathbb{E}_t \left[\sum_{k=0}^{\infty} \delta^k \pi_{t+k-1} 1_{\{k>0\}}\right] + \widetilde{m}_\mu \mathbb{E}_t \left[\sum_{k=0}^{\infty} \delta^k \widehat{mc}_{t+k}\right] \\
&= \widetilde{m}_\pi \mathbb{E}_t \left[\sum_{k=0}^{\infty} (\delta F)^k \pi_t 1_{\{k>0\}}\right] - \omega\widetilde{m}_\pi \mathbb{E}_t \left[\sum_{k=0}^{\infty} (\delta F)^k \pi_{t-1} 1_{\{k>0\}}\right] + \widetilde{m}_\mu \mathbb{E}_t \left[\sum_{k=0}^{\infty} (\delta F)^k \widehat{mc}_t\right] \\
&= \widetilde{m}_\pi \mathbb{E}_t \left[\sum_{k=0}^{\infty} (\delta F)^k \pi_t - \pi_t\right] - \omega\widetilde{m}_\pi \mathbb{E}_t \left[\sum_{k=0}^{\infty} (\delta F)^k \pi_{t-1} - \pi_{t-1}\right] + \widetilde{m}_\mu \mathbb{E}_t \left[\sum_{k=0}^{\infty} (\delta F)^k \widehat{mc}_t\right] \\
&= \widetilde{m}_\pi \mathbb{E}_t \left[\frac{\pi_t}{1-\delta F} - \pi_t\right] - \omega\widetilde{m}_\pi \mathbb{E}_t \left[\frac{\pi_{t-1}}{1-\delta F} - \pi_{t-1}\right] + \widetilde{m}_\mu \mathbb{E}_t \left[\frac{\widehat{mc}_t}{1-\delta F}\right] \\
&= \widetilde{m}_\pi \mathbb{E}_t \left[\frac{\delta F \pi_t}{1-\delta F}\right] - \omega\widetilde{m}_\pi \mathbb{E}_t \left[\frac{\delta F \pi_{t-1}}{1-\delta F}\right] + \widetilde{m}_\mu \mathbb{E}_t \left[\frac{\widehat{mc}_t}{1-\delta F}\right]
\end{aligned}$$

Premultiplying by $(1-\delta F)$,

$$\frac{\theta}{1-\theta}(1-\delta F)(\pi_t - \omega\pi_{t-1}) = \widetilde{m}_\pi \mathbb{E}_t [\delta F \pi_t] - \omega\widetilde{m}_\pi \mathbb{E}_t [\delta F \pi_{t-1}] + \widetilde{m}_\mu \mathbb{E}_t [\widehat{mc}_t]$$

which can be rearranged to (18). Let us now derive the Behavioural Hybrid New Keynesian Phillips curve. We have the following expressions

$$mc_t = w_t - p_t - a_t \tag{38}$$

$$y_t = a_t + n_t \tag{39}$$

$$w_t - p_t = \varphi n_t + \frac{\sigma}{1-h} c_t - \frac{\sigma h}{1-h} c_{t-1} \tag{40}$$

$$c_t = y_t \tag{41}$$

Hence, we can write

$$\begin{aligned} mc_t &= w_t - p_t - a_t \\ &= \varphi n_t + \frac{\sigma}{1-h} c_t - \frac{\sigma h}{1-h} c_{t-1} - a_t \\ &= \varphi(y_t - a_t) + \frac{\sigma}{1-h} c_t - \frac{\sigma h}{1-h} c_{t-1} - a_t \\ &= \varphi(y_t - a_t) + \frac{\sigma}{1-h} y_t - \frac{\sigma h}{1-h} y_{t-1} - a_t \\ &= \left(\varphi + \frac{\sigma}{1-h}\right) y_t - \frac{\sigma h}{1-h} y_{t-1} - (1+\varphi) a_t \end{aligned}$$

In the natural equilibrium (with no price frictions), the marginal cost is constant at its steady-state level

$$mc_t^r = mc = -\mu = \left(\varphi + \frac{\sigma}{1-h}\right) y_t^n - \frac{\sigma h}{1-h} y_{t-1}^n - (1+\varphi) a_t$$

hence, we can write

$$\widehat{mc}_t = mc_t - mc = \left(\varphi + \frac{\sigma}{1-h}\right) \widetilde{y}_t - \frac{\sigma h}{1-h} \widetilde{y}_{t-1}$$

which, inserted into the (18), yields the Behavioural Hybrid New Keynesian Phillips curve (BHNKPC).

F Robustness Checks

Table 2: Estimated Structural Parameters: Robustness Checks

	Prior Distribution		Posterior Distribution	
		Mean (S.d)	**Output Growth**	**Estimating σ**
β	*Beta*	0.85	0.956	–
		(0.10)	(0.890, 0.998)	(–)
σ	*Normal*	1.50	–	1.109
		(0.375)	(–)	(0.487, 1.712)
ϕ_π	*Normal*	1.50	1.320	1.349
		(0.15)	(1.107, 1.531)	(1.137, 1.567)
ϕ_y	*Normal*	0.15	0.303	0.273
		(0.10)	(0.171, 0.437)	(0.154, 0.391)
θ	*Beta*	0.50	0.865	0.878
		(0.10)	(0.823, 0.907)	(0.833, 0.919)
h	*Beta*	0.75	0.513	0.664
		(0.15)	(0.354, 0.666)	(0.476, 0.844)
ω	*Beta*	0.50	0.795	0.784
		(0.15)	(0.704, 0.892)	(0.165, 0.899)
$\overline{m}$	*Beta*	0.75	0.323	0.404
		(0.15)	(0.147, 0.508)	(0.188, 0.634)
ρ_i	*Beta*	0.50	0.821	0.838
		(0.20)	(0.766, 0.872)	(0.787, 0.885)
ρ_d	*Beta*	0.50	0.102	0.703
		(0.20)	(0.017, 0.204)	(0.583, 0.817)
ρ_s	*Beta*	0.50	0.102	0.129
		(0.20)	(0.013, 0.209)	(0.014, 0.769)
ρ_{e_i}	*Beta*	0.50	0.208	0.185
		(0.20)	(0.090, 0.335)	(0.068, 0.303)
σ_d	*Inv. gamma*	0.01	0.0082	0.0051
		(∞)	(0.0072, 0.0091)	(0.0039, 0.0062)
σ_s	*Inv. gamma*	0.01	0.0035	0.0032
		(∞)	(0.0029, 0.0040)	(0.0026, 0.0038)
σ_i	*Inv. gamma*	0.01	0.0025	0.0025
		(∞)	(0.0023, 0.0027)	(0.0022, 0.0027)

Note: Results are reported at the posterior median. 90% confidence intervals in parenthesis. The column labelled *Output Growth* estimates the model by using output growth as an observable instead of the one-sided HP filtered GDP series. The second column *Estimating σ* includes the estimation of the risk parameter σ, while fixing β to the standard value of 0.99. Both columns refer to the benchmark HBNK model.

G Narrative VAR Identification

We identified the VAR monetary policy shock by means of sign restrictions. Table 3 displays the signs imposed for the standard sign restriction approach. Besides the monetary policy shock, we control for an aggregate demand shock and an aggregate supply shock. The table imposes well-known sign restrictions required to identify these three different shocks. We assume that an expansionary monetary policy shock is the one that reduces the nominal rate and rises output gap, inflation, non-borrowed reserves and total reserves for the first two quarters. We also include non-borrowed and total reserves as Uhlig (2005) for the sake of completeness. The timing restriction is similar to the one in the aforementioned studies.

In addition to pure sign restrictions, we impose narrative sign restrictions as in Antolín-Díaz and Rubio-Ramírez (2018). Therefore, it is required that the identified monetary policy shock series and the historical decomposition are constrained on particular dates. In particular, we consider the Volcker reform in 1979:IV as a period of an exogenous monetary policy change. For this event we impose the following restrictions:

- ***Narrative Restriction 1:*** The monetary policy shock must be positive for the observation in 1979:IV.
- ***Narrative Restriction 2:*** The monetary policy shock is the most important contributor to the observed changes in the federal funds rate in 1979:IV.

The VAR includes the same observables as in the theoretical model over the period 1960:I through 1997:IV. It features three lags (given the Akaike information criterion) and is estimated by Bayesian methods under a conjugate normal inverse-wishart prior following Antolín-Díaz and Rubio-Ramírez (2018).

Table 3: Sign Restrictions

	MP shock	Demand shock	Supply shock
Output gap	+	+	+
Inflation	+	+	−
Nominal interest rate	−	+	−
Non-borrowed reserves	+	?	?
Total reserves	+	?	?

Note: Sign restrictions are imposed for the first two quarters. Symbols + and − refer to the direction of the response for the considered period of time. When agnostic about the sign, the symbol ? is employed.