Reconciling Empirics and Theory: The Behavioral Hybrid New Keynesian Model

Atahan Afsar
Stockholm School of Economics

José-Elías Gallegos
Banco de España

Richard Jaimes
Pontificia Universidad Javeriana

Edgar Silgado-Gómez
Central Bank of Ireland

January 30, 2023

Structural estimates of the standard New Keynesian model are at odds with the microeconomic estimates. To reconcile these findings, we develop and estimate a behavioral New Keynesian model augmented with backward-looking households and firms. We find (i) evidence for bounded rationality, with a cognitive discount factor estimate around 0.34 at a quarterly frequency, consistent with the underrevision coefficient in survey expectations; and (ii) that the behavioral setting with backward-looking agents helps us harmonize the New Keynesian theory with empirical studies. We suggest that both cognitive discounting and intrinsic persistence are essential, first, to match the empirical estimates for certain parameters of interest and, second, to obtain the hump-shaped and initially muted impulse-response functions that we observe in empirical studies.

Keywords: New Keynesian, Bounded Rationality, Bayesian Estimation.
JEL Classifications: E27, E52, E71.

E-mails: atahan.afsar@phdstudent.hhs.se, jose.elias.gallegos@bde.es, jaimes_rv@javeriana.edu.co, edgar.silgado-gomez@centralbank.ie. We are grateful for the useful comments provided by the Associate Editor and two anonymous referees. We also thank Xavier Gabaix, Gaetano Gaballo, Mark Gertler, Per Krusell, Jesper Lindé, Anna Seim, Ulf Söderström, Lars E.O. Svensson, Karl Walentin, Jörgen Weibull and seminar participants at the LACEA Annual Meeting, the Macro Workshop in Bogotá, the IIES, and the Stockholm School of Economics for useful feedback and comments. This paper was included as Chapter 2 in José-Elías Gallegos's dissertation at the IIES, Stockholm University. The views expressed in this paper are those of the authors and do not necessarily reflect the position of Banco de España, the Central Bank of Ireland or the European System of Central Banks.

1. Introduction

"Despite the advances in theoretical modeling, accompanying econometric analysis of the 'new Phillips curve' has been rather limiting [...]. The work to date has generated some useful findings, but these findings have raised some troubling questions about the existing theory."

J. Galí and M. Gertler, *Inflation dynamics: A structural econometric analysis (1999)*.

An important characteristic of the standard New Keynesian (NK) model is that it can be synthesized in a system of two first-order stochastic difference equations that are easy to interpret: the Dynamic IS curve or the demand side, and the Phillips curve or the supply side. Every slope in these curves is a combination of different parameters in the model, namely the discount factor, the degree of risk aversion, the Frisch elasticity and the Calvo-inaction probability. As a result, by estimating the slopes of the final system of equations, one can retrieve the structural parameters of the model. However, when the monetary economics literature performed such analyses, some estimated parameters were at odds with microeconometric studies.

Reconciling the NK theory with the data has proven to be a difficult exercise. One of the main criticisms of the benchmark NK model is that it is purely forward-looking, and therefore lacks the ability to capture any sort of endogenous persistence in output and inflation (see Galí and Gertler 1999; Fuhrer and Moore 1995; Fuhrer 2010; Christiano et al. 2005; Altig et al. 2011). As a result, the model does not produce the intrinsic persistence (and hump-shaped responses) that we observe in the data. The main approach to enforce intrinsic persistence in the model is to include backward-looking households and firms, either assuming a backward-looking utility function for households or sticky price indexation for firms. Unfortunately, the parameter values that characterize the frictions required to produce the degree of intrinsic persistence that the data suggests are at odds with the micro evidence (see Galí and Gertler 1999; Nakamura and Steinsson 2008; Bils and Klenow 2004; Havranek et al. 2017). To reconcile these differences between empirics and theory, we put forward a behavioral NK model, similar in spirit to the one described in Gabaix (2020), extended with *external habit persistent* households and *price-indexing* firms. We show that the combination of backward-looking agents *and* bounded rationality (BR) helps reduce the discrepancy between macro and micro estimates.

Our contribution to the literature is threefold. First, we extend the behavioral NK setting in Gabaix (2020) to allow for household habit persistence and firm price indexation, inducing intrinsic persistence in the model dynamics. Second, we estimate the structural parameters behind the coefficients in the behavioral Dynamic IS (DIS) and hybrid NK Phillips curves using Bayesian techniques. Third, we also find empirical evidence for considerable BR behavior, supporting the deviation from the standard fully rational behavioral framework. A salient feature of our model is that it can be easily reduced to the ones described in Galí and Gertler (1999), Galí (2008) or Gabaix (2020) by turning off certain key parameters such as the degree of habit persistence, the degree of price indexation, or the BR parameter. As a result, our model nests those frameworks and allows us to easily compare estimates.

The first approach to induce intrinsic persistence in the NK framework dates back to Fuhrer and Moore (1995); Galí and Gertler (1999). The authors focus on the supply side of the model and estimate two different (micro-founded) NK Phillips curves: the standard curve (NKPC) and the Hybrid curve (H-NKPC), which has a backward-looking component. In an empirical exercise, they show that the H-NKPC produces dynamics closer to what the data suggests. However, even in the hybrid version, the structural parameter estimates are at odds with the micro evidence. For example, the discount factor estimate at a quarterly frequency is generally below 0.95 (see Galí and Gertler 1999). In subsequent research, Christiano et al. (2005) suggest to induce inflation persistence by assuming that firms that cannot re-optimize their prices update them according to past inflation. Other approaches able to generate intrinsic inflation persistence can be found in Roberts (1997) and Milani (2007), through the modeling of adaptive expectations and learning, respectively; in the form of sticky information as in Mankiw and Reis (2002); incomplete information as in Woodford (2003); Angeletos et al. (2021); by relaxing the Calvo assumption of a random selection of firms that are able to change their prices as in Sheedy (2010), or in a model of heterogeneous firms that features both rational and naïve agents as in Cornea-Madeira et al. (2019), which generate intrinsic persistence and myopia in aggregate inflation dynamics. Importantly, their mean estimate of myopia, 0.353, is similar to the estimates presented in this paper.[1] Likewise, Christiano et al. (2005) also extend the backward-looking behavior to households by including internal

[1]Other studies have stressed alternative mechanisms. For example, Grauwe (2011) generates non-fundamental (animal spirit) business cycles by introducing optimistic and pessimistic agents. In the same vein, Hommes and Lustenhouwer (2019) describe how Central Bank credibility can be affected by the share of rational and naïve agents, each agent type optimally decided at the individual level. They provide the conditions under which a self-fulfilling liquidity trap can occur, and how Central Bank credibility affects the equilibrium. Finally, Madeira (2014) introduces employment frictions and finds that such a characteristic helps to get a better estimate for the parameter of price stickiness.

habits.[2] They find that the degree of habits necessary to match the impulse response after a monetary shock is three or four times larger than the one estimated in the micro literature.[3] From these extensions we take the lesson that adding a backward-looking behavior is no panacea, at least on its own, for a reconciliation between micro and macro estimates.

We include household habit persistence in the light of Christiano et al. (2005) and Blanchard et al. (2015). Christiano et al. (2005) find a quantitatively important degree of household habit persistence for the US. Most importantly, they show that including habit persistence is critical to obtain hump-shaped impulse responses in the model, as the empirical VAR literature has observed. Given that our intention is to build a model that is closer to the data, we follow their approach in order to consistently estimate the behavioral DIS curve. We include backward-looking firms along the lines of Galí and Gertler (1999) and Christiano et al. (2005).[4] This is done in order to obtain a hybrid NK Phillips curve that is closer to empirical evidence, in the sense that it also includes lags of inflation and helps explain its persistence. The motivation for this is mostly empirical, since previous studies have found the inflation equation to be largely inertial.[5]

Our departure from the standard Full-Information Rational Expectations (FIRE) assumption is motivated by empirical evidence. Using survey data from households' and firms' expectations, Coibion and Gorodnichenko (2015) test for the null of FIRE, which is rejected in the data. However, their empirical findings are inconclusive on the direction of the FIRE departure, whether it is Full-Information or Rational Expectations that is rejected. This leaves room for different extensions beyond FIRE, some being more empirically robust than others.

One notable approach that lies between the models of less than full information and the models of less than full rationality has been developed in a series of papers by Gabaix (2014, 2016, 2020), which provides an operational, tractable framework by incorporating the behavioral assumption that the decision makers allocate their attention optimally according to a simplified version of the full model, where their utility is replaced by a

[2]Christiano et al. (2005) model the backward-looking behavior by means of internal habits (each agent cares about its own consumption growth). In this paper, we instead focus on external habits (each agent cares about the difference between its consumption today and aggregate consumption yesterday). We take this route motivated by the meta-analysis in Havranek et al. (2017).

[3]For an extensive meta-analysis, see Havranek et al. (2017).

[4]Hajdini (2022) introduces myopia as in Gabaix (2020) and finds a smaller role for habits as the endogenous persistence arises due to the expectations themselves. The expectations implied by the model can account for many recently documented facts on expectations (see Angeletos et al. 2021).

[5]Since the seminal paper by Fuhrer and Moore (1995), a sizable literature has tried to estimate the NKPC. See, Mavroeidis et al. (2014), for an extensive review.

linear-quadratic approximation, and then solve the full model with this partial attention vector. The framework captures some of the essential features of the rational inattention framework, namely the under-reaction of beliefs and actions, while it allows tractability for dynamic models beyond linear-quadratic forms.

In this paper, we follow this strand of the literature by assuming an attention coefficient that the decision-makers assign to a piece of newly arriving information, so that the posterior expectation is a convex combination of the prior mean and the realization (i.e. full-information) value. We follow this reduced-form approach since our core interest is to reconcile the theory with empirical evidence, and this behavioral approximation of a limited attention model affords us to arrive at the simple closed-form solutions that are typical of the standard New Keynesian model while incorporating the first-order effects of limited inattention. We calibrate the cognitive discounting parameter to match the empirical evidence on forecast underrevision (Coibion and Gorodnichenko 2015).

Cognitive discounting, as presented in Gabaix (2020), is successful in producing myopia but does not produce intrinsic persistence *on its own*. We find that the cognitive discount factor, *together* with habit persistence and price indexation, is key to obtain macro estimates that align with their micro counterpart, since the cognitive discount factor increases the relative weight of the past (intrinsic persistence) and reduces the weight of the future (myopia).

For the estimation of the structural parameters, and being able to compare our New Keynesian models, we follow a Bayesian approach as in Fernández-Villaverde and Rubio-Ramírez (2004), Rabanal and Rubio-Ramírez (2005) and Milani (2007). This approach has some advantages over limited-information methods such as the generalized method of moments (GMM). For example, Bayesian estimation mitigates the misspecification problem and allows a transparent comparison across models. In particular, we estimate four different models using US data: (i) the standard NK model; (ii) the hybrid NK model; (iii) the behavioral NK model, and (iv) the behavioral hybrid NK model. We find that the latter model reports estimates that are closer to the those of the micro empirical evidence, with a larger log data density. Likewise, in order to test the ability of our set of models to replicate empirical impulse-response functions, we also estimate a monetary policy shock by means of a Bayesian vector autoregression (VAR) using narrative sign restrictions as in Antolín-Díaz and Rubio-Ramírez (2018). We find that only our Behavioral NK model with both habit formation and backward-looking firms is able to generate, at the same time, hump-shaped responses and enough inflation persistence as we observe in the data.

The paper proceeds as follows. In section 2 we introduce the behavioral model. In section 3 we estimate the structural parameters of the model. In section 4 we discuss our findings. Section 5 concludes the paper.

2. The Behavioural Agents and Firms Setting

2.1. Bounded Rationality Assumptions

Before introducing a behavioral version of the New Keynesian model, we here briefly explain the cognitive discounting approach à la Gabaix (2016, 2020) that we operationalize in this paper. Let $X_t \in \Omega$ be the state vector at period t, that might include exogenous shocks, and $\varepsilon_t \in E$ is an additive stochastic noise with zero mean.

Now let $G\colon \Omega \times E \to \Omega$ be the function that represents the equilibrium law of motion for the state variable, $X_{t+1} = G(X_t, \varepsilon_{t+1})$. Let us assume that the deterministic economy has a unique non-exploratory steady-state, and is denoted by X. Here, the cognitive discounting assumption states that the agents do not fully internalize the expected equilibrium deviations from the steady state by partially intrinsic persistence their belief to the steady state. Let $\bar{m} \in [0, 1]$ denote the degree of cognitive discounting, and let $G^B\colon \Omega \times E \to \Omega$, $G^B(X_t, \varepsilon_t) = \bar{m}G(X_t, \varepsilon_t) + (1 - \bar{m})X$ denote the law of motion perceived by the behavioral agent. For notational simplicity, in the rest of this section, we will assume that the state vector is de-meaned, i.e. the steady state is given by the zero vector; however, the analysis holds true for the generic case as well. Under this assumption the above expression simplifies to $G^B(X_t, \varepsilon_t) = \bar{m}G(X_t, \varepsilon_t)$. The linearization of the actual law of motion and renormalization gives $X_{t+1} = \Gamma X_t + \varepsilon_{t+1}$ for some matrix Γ. Likewise, the perceived law of motion by the behavioral agents linearizes to $X_{t+1} = \bar{m}(\Gamma X_t + \varepsilon_{t+1})$.

However, since the noise parameter has zero mean, we have the following relation between the expectation of a behavioral agent, denoted by the expectation operator with a superscript B, and the rational expectation, $\mathbb{E}_t^B[X_{t+1}] = \bar{m}\Gamma X_t = \bar{m}\mathbb{E}_t[X_{t+1}]$. Likewise, iterating for k periods we obtain $\mathbb{E}_t^B[X_{t+k}] = \bar{m}^k \mathbb{E}_t[X_{t+k}]$. Throughout the paper, we will assume that all forecasts, made by households or firms and across different macroeconomic variables, are cognitively discounted by the same factor $\overline{m}$.[6]

[6] In Appendix A.6 we show that intrinsic persistence can be microfounded via beliefs, by assuming $\mathbb{E}_t^B X_{t+h} = \overline{m}^h \mathbb{E}_t X_{t+h} + (1 - \overline{m}^h)X_{t-1}$.

2.2. Households

Rational Agents. We consider a population of households that is treated as a continuum of unit mass. Each household chooses its consumption and labor supply level for each period. We assume identical preferences over expected lifetime utility and hence omit indexing for notational ease. The preference of a representative household can be given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(C_t - h\overline{C}_{t-1})^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right] \tag{1}$$

where C_t is a consumption index given by $C_t \equiv \left(\int_0^1 C_{it}^{\frac{\epsilon_t - 1}{\epsilon_t}} di \right)^{\frac{\epsilon_t}{\epsilon_t - 1}}$, with C_{it} denoting the quantity of good $i \in [0, 1]$ consumed by the household in period t, N_t denotes employment or labor supply, $\overline{C}_{t-1}$ denotes the average consumption level in the economy (which is taken as given by the individual household), σ is the intertemporal elasticity of substitution, $1/\varphi$ is the Frisch elasticity and ϵ_t denotes the elasticity of substitution among goods varies over time.[7] The period-t consumption utility of each household is affected by a reference level, which we assume to be given by a linear function of the average consumption level in the previous period. Thus, the household preferences exhibit a *keeping up with the Joneses* element.[8] The parameter $h \in [0, 1]$ represents the sensitivity towards this reference point. The household's budget constraint is given by

$$P_t C_t + Q_t B_t = B_{t-1} + W_t N_t + T_t \tag{2}$$

where P_t is the price of the consumption good, B_t stands for bond holdings at the household, Q_t is the price of each bond, W_t is the wage rate for each unit of labor supply and T_t are transfers to households. We show in Appendix A.1 that the demand for good i is given by

$$Y_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\epsilon_t} Y_t \tag{3}$$

[7] We microfound cost-push shocks by allowing for a time-varying elasticity of substitution between varieties of goods.

[8] The consequences of such an assumption are similar to assuming habit persistence, albeit simplifying the computation.

where $Y_t = C_t$ (since we are in a representative household economy), and the aggregate price index P_t is given by $P_t = \left(\int_0^1 P_{it}^{1-\epsilon_t} di\right)^{\frac{1}{1-\epsilon_t}}$. The optimization problem of the household is represented as maximizing lifetime utility (1) subject to its budget constraint (2) and the usual transversality condition $\lim_{t\to\infty} \beta^t u'(C_t)B_t = 0$. The rational household optimality conditions, derived in Appendix A.1, are

$$\frac{W_t}{P_t} = \frac{N_t^{\varphi}}{(C_t - h\overline{C}_{t-1})^{-\sigma}} \tag{4}$$

$$Q_t = \beta\mathbb{E}_t\left[\left(\frac{C_{t+1} - h\overline{C}_t}{C_t - h\overline{C}_{t-1}}\right)^{-\sigma} \frac{P_t}{P_{t+1}}\right] \tag{5}$$

Notice that, since households are identical and of unit mass, we can take the average consumption of the past period as the consumption of the representative agent in that period, $C_t = \overline{C}_t$ for all periods t.

Behavioral Agents. The behavioral households exhibit cognitive discounting as described in Section 2.1, hence their mean expectation of the stochastic variables in the economy is dampened towards its steady-state values compared to the expectation of a rational agent. This effect is even more nuanced for events that are far into the future. We can rewrite condition (4)-(5) as

$$\frac{W_t}{P_t} = \frac{N_t^{\varphi}}{(C_t - hC_{t-1})^{-\sigma}} \tag{6}$$

$$Q_t = \beta\mathbb{E}_t^B\left[\left(\frac{C_{t+1} - hC_t}{C_t - hC_{t-1}}\right)^{-\sigma} \frac{P_t}{P_{t+1}}\right] \tag{7}$$

The labor supply condition is unaffected: since it is an intratemporal condition, cognitive discounting plays no role here. Fully rational and behavioral households do not differ in intratemporal considerations, but in their perception of the future. On the other hand, the Euler condition now has a different expectation operator. The log-linearized version of both optimality conditions, derived in Appendix A.2, is

$$\widehat{w}_t - \widehat{p}_t = \varphi\widehat{n}_t + \frac{\sigma}{1-h}\widehat{c}_t - \frac{\sigma h}{1-h}\widehat{c}_{t-1} \tag{8}$$

$$\widehat{c}_t = \frac{h}{1+h}\widehat{c}_{t-1} + \frac{1}{1+h}\overline{m}\mathbb{E}_t\widehat{c}_{t+1} - \frac{1-h}{\sigma(1+h)}\left(\widehat{i}_t - \overline{m}\mathbb{E}_t\pi_{t+1}\right) \tag{9}$$

where a hat on top of a variable denotes the log deviation from the steady state, $\widehat{x}_t = (X_t - X)/X$, and $\widehat{i}_t = -\log Q_t$ is the short-term nominal interest rate. Here we have made use of the BR assumptions described in the previous section, setting $\mathbb{E}_t^B \widehat{c}_{t+1} = \overline{m}\mathbb{E}_t \widehat{c}_{t+1}$ and $\mathbb{E}_t^B \pi_{t+1} = \overline{m}\mathbb{E}_t \pi_{t+1}$. The Euler condition (9) can be rewritten in terms of the output gap as the Behavioral DIS (BDIS) curve

$$\widetilde{y}_t = \lambda_b \widetilde{y}_{t-1} + \lambda_f \mathbb{E}_t \widetilde{y}_{t+1} + \lambda_r \left(\widehat{i}_t - \overline{m}\mathbb{E}_t \pi_{t+1} - r_t^n \right) \tag{10}$$

where $\lambda_b = \frac{h}{1+h}$, $\lambda_f = \frac{1}{1+h}\overline{m}$, $\lambda_r = -\frac{1-h}{\sigma(1+h)}$, r_t^n is the natural interest rate and follows an AR(1) process; and a tilde denotes the log deviation with respect to the natural level $\widetilde{x}_t = \widehat{x}_t - x_t^n$.[9]

2.3. Firms

There is a continuum of firms with unit mass, each producing a different type of good. Good *i* is produced by a monopolistic firm *i* with technology

$$Y_{it} = A_t N_{it} \tag{11}$$

where A_t represents the level of technology, assumed to be common across firms. Given $Y_t = C_t$ and $Y_{it} = C_{it}$,[10] we know that the final good is produced competitively in quantity Y_t.

Each firm chooses the price level of the good that it produces. Prices are set subject to a Calvo-style friction (in each period, a firm is only allowed to reset its price with probability $1 - \theta$, independent of the time elapsed since it last adjusted its price.) Thus, in each period a measure $1 - \theta$ of producers reset their prices freely. However – and departing from the standard NK setting – it is assumed that when a firm is unable to reoptimize, its price is partially indexed to past inflation as in Christiano et al. (2005), i.e.,

$$P_{it} = P_{it-1}\Pi_{t-1}^{\omega} \tag{12}$$

where $\Pi_t = \frac{P_t}{P_{t-1}}$ is the gross rate of inflation between $t - 1$ and t, and ω is the elasticity of prices with respect to past inflation.[11] As a result, a firm that last reset its price in

[9] We define the natural level as the equilibrium under no pricing frictions.

[10] No firm will choose to produce more that what is demanded.

[11] This assumption is equivalent to the more ad-hoc derivation of backward-looking firms in Galí and

period t will in period $t+k$ have a nominal price of $P_t^* \chi_{t,t+k}$, where

$$\chi_{t,t+k} = \begin{cases} \Pi_t^{\omega} \Pi_{t+1}^{\omega} \Pi_{t+2}^{\omega} \cdots \Pi_{t+k-1}^{\omega} & \text{if } k \geq 1 \\ 1 & \text{if } k = 0 \end{cases}$$

Rational Agents. The rational firm's problem is to maximize its discounted profit stream

$$\mathbb{E}_0 \sum_{t=0}^{\infty} Q_t [P_{it} Y_{it} - W_t N_{it}] \tag{13}$$

subject to the sequence of demand constraints (3) and technology constraints (11). We can rewrite the objective function (profits) as

$$P_{it} Y_{it} - W_t N_{it} = P_{it} Y_{it} - W_t \frac{Y_{it}}{A_t} = \left[P_{it} - P_t \mathrm{MC}_t \right] \left[\frac{P_{it}}{P_t} \right]^{-\epsilon_t} Y_t$$

where the marginal costs are defined as $\mathrm{MC}_t = (W_t/P_t)(\partial Y_t/\partial N_t) = W_t/(P_t A_t)$.

Consider a firm reoptimizing its price at time t. Let the firm's optimal price be denoted $P_t^*(i)$, such that in this setting at time $t+k$ its price will be $P_{it}^* \chi_{t,t+k}$. Ignoring states in which reoptimization is allowed, its maximization program is

$$\max_{P_{it}^*} \ \mathbb{E}_t \sum_{k=0}^{\infty} \theta^k Q_{t+k} [P_{it}^* \chi_{t,t+k} - P_{t+k} \mathrm{MC}_{t+k}] \left[\frac{P_{it}^* \chi_{t,t+k}}{P_t} \right]^{-\epsilon_t} Y_{t+k}$$

which yields the following first-order condition,[12]

$$P_{it}^* = \frac{\mathbb{E}_t \sum_{k=0}^{\infty} (\theta\beta)^k (C_{t+k} - h C_{t+k-1})^{-\sigma} C_{t+k} P_{t+k}^{\epsilon_t} P_{t+k-1}^{-\omega\epsilon_t} \mathrm{MC}_{t+k} \mathcal{M}_{t+k}}{\mathbb{E}_t \sum_{k=0}^{\infty} (\theta\beta)^k (C_{t+k} - h C_{t+k-1})^{-\sigma} C_{t+k} P_{t+k}^{\epsilon_t - 1} P_{t+k-1}^{\omega(1-\epsilon)}} P_{t-1}^{\omega} \tag{14}$$

where we have used the Euler condition (7), $\chi_{t,t+k} = \left(\frac{P_{t+k-1}}{P_{t-1}} \right)^{\omega}$ and $\mathcal{M}_t = \frac{\epsilon_t}{\epsilon_t - 1}$, which stands for the mark-up. Notice that with flexible prices (i.e., $\theta = 0$), the optimal pricing condition (14) collapses to the familiar monopolistic competition price-setting rule

$$P_{it}^* = \mathcal{M}_t P_t \mathrm{MC}_t \tag{15}$$

Gertler (1999). However, since firms are identical, its consequences are equivalent: ω could be interpreted as the share of backward-looking firms.

[12] A detailed derivation can be found in Appendix A.3.

where (15) is the frictionless mark-up. Since all firms who get to reset are facing an identical environment (i.e., we can treat them as if they were a representative firm), they choose to set the same price: $P^*_{it} = P^*_t \ \forall i$, and $\mathcal{M}_t = \frac{1}{\mathrm{MC}_t}$. The log-linearized version of the optimal pricing condition (14) is

$$(16) \quad p^*_t = p_t + (1-\theta\beta)\mathbb{E}_t \sum_{k=0}^{\infty} (\theta\beta)^k \left[(\pi_{t+1} + \ldots + \pi_{t+k}) - \omega(\pi_t + \ldots + \pi_{t+k-1}) + \widehat{\mathrm{mc}}_{t+k} + \widehat{\mu}_{t+k} \right]$$

where $\widehat{\mathrm{mc}}_t = \mathrm{mc}_t - \mathrm{mc} = \mathrm{mc}_t + \mu$, $\mu = \log \mathcal{M}$, and $\widehat{\mu}_t = \mu_t - \mu$, which is assumed to follow an AR(1) process. That is, a resetting firm will choose a price that corresponds to the desired mark-up over a convex combination of current and expected future prices and nominal marginal costs, in addition to the prices in the previous period.

Behavioral Agents. The behavioral firm faces the same problem, with a less accurate view of reality. Most importantly, the behavioral firm perceives the future via the cognitive discounting mechanism discussed in Section 2.1. To be precise, we model that at time t, the firm perceives the future inflation and marginal costs at date $t + k$ as $\mathbb{E}^B_t[\pi_{t+k}] = \overline{m}^k \mathbb{E}_t[\pi_{t+k}]$ and $\mathbb{E}^B_t[\widehat{\mathrm{mc}}_{t+k}] = \overline{m}^k \mathbb{E}_t[\widehat{\mathrm{mc}}_{t+k}]$. Note that we assume a common cognitive discount factor across households and firms, since households own the firms, which inherit their belief formation frictions. The equivalent condition of equation (16) for a behavioral firm is

(17)

$$p^*_t = p_t + (1-\theta\beta) \sum_{k=0}^{\infty} (\theta\beta\overline{m})^k \mathbb{E}_t \left[(\pi_{t+1} + \ldots + \pi_{t+k}) - \omega(\pi_t + \ldots + \pi_{t+k-1}) + \widehat{\mathrm{mc}}_{t+k} + \widehat{\mu}_{t+k} \right]$$

where the future is additionally discounted by a cognitive discount factor $\overline{m}$.

Aggregate Price Dynamics and the Behavioral Hybrid New Keynesian Phillips Curve. In this economy, in every period, there are two types of firms: those allowed to reset their price and those who are not, whose price is updated with previous aggregate inflation. We can describe the price dynamics as $P_t = \left[\Pi_{t-1}^{\omega(1-\epsilon_t)} \theta P_{t-1}^{1-\epsilon_t} + (1-\theta)(P^*_t)^{1-\epsilon_t} \right]^{\frac{1}{1-\epsilon_t}}$. All firms resetting their price in any given period will choose the same price because they face an identical problem. A log-linear approximation to the aggregate price index around a zero inflation steady-state, derived in Appendix A.4, yields

$$\pi_t = \theta\omega\pi_{t-1} + (1-\theta)(p_t^* - p_{t-1}). \tag{18}$$

After some algebra relegated to Appendix A.5, a rearrangement of expressions (17) and (18) yields the Behavioral Hybrid NK Phillips curve in terms of marginal costs

$$\pi_t = \gamma_b\pi_{t-1} + \gamma_\mu\widehat{\mathrm{mc}}_t + \gamma_f\mathbb{E}_t\pi_{t+1} + \gamma_\mu\widehat{\mu}_t \tag{19}$$

where

$$\gamma_b = \frac{\omega}{1+\omega\beta\overline{m}\left[\theta + (1-\theta)\frac{1-\theta\beta}{1-\theta\beta\overline{m}}\right]}$$

$$\gamma_\mu = \frac{(1-\theta)(1-\theta\beta)}{\theta\left\{1+\omega\beta\overline{m}\left[\theta + (1-\theta)\frac{1-\theta\beta}{1-\theta\beta\overline{m}}\right]\right\}}$$

$$\gamma_f = \frac{\beta\overline{m}\left[\theta + (1-\theta)\frac{1-\theta\beta}{1-\theta\beta\overline{m}}\right]}{1+\omega\beta\overline{m}\left[\theta + (1-\theta)\frac{1-\theta\beta}{1-\theta\beta\overline{m}}\right]}$$

In order to obtain the Behavioural Hybrid NK Phillips curve in terms of the output gap, recall that in this economy with technological progress, $\mathrm{MC}_t = W_t/(A_tP_t)$. Taking logs, we can write $\mathrm{mc}_t = w_t - p_t - a_t$. Additionally, firm technology implies $y_t = a_t + n_t$ and the aggregate resource constraint implies $y_t = c_t$. Finally, thanks to the labor supply condition (6) we know $w_t - p_t = \varphi n_t + \frac{\sigma}{1-h}c_t - \frac{\sigma h}{1-h}c_{t-1}$. Using these four expressions together yields

$$\widehat{\mathrm{mc}}_t = \left(\varphi + \frac{\sigma}{1-h}\right)\widetilde{y}_t - \frac{\sigma h}{1-h}\widetilde{y}_{t-1} \tag{20}$$

Introducing (20) into (19) leads to the Behavioral Hybrid NK Phillips curve

$$\pi_t = \gamma_b\pi_{t-1} + \alpha_b\widetilde{y}_{t-1} + \alpha_c\widetilde{y}_t + \gamma_f\mathbb{E}_t\pi_{t+1} + \gamma_\mu\widehat{\mu}_t \tag{21}$$

where $\alpha_b = -\gamma_\mu\frac{\sigma h}{1-h}$ and $\alpha_c = \gamma_\mu\left(\varphi + \frac{\sigma}{1-h}\right)$.

2.4. Closing the Model

The Behavioral Hybrid NK Phillips curve (21), together with the Behavioral Dynamic IS curve (10) and a reaction function for the monetary authority (an ad-hoc inertial Taylor rule with an AR(1) monetary policy shock)

$$\widehat{i}_t = (1-\rho_r)(\phi_\pi \pi_t + \phi_y \widetilde{y}_t) + \rho_r \widehat{i}_{t-1} + e_t \tag{22}$$

constitute the Behavioral New Keynesian framework with *keeping up with the Joneses* households and hybrid firms. Finally, we can write the model in system form as

$$\mathbf{A_c x}_t = \mathbf{A_b x}_{t-1} + \mathbf{A_f} \mathbb{E}_t \mathbf{x}_{t+1} + \mathbf{A_s u}_t \tag{23}$$

where $\mathbf{x}_t = \begin{bmatrix} \widetilde{y}_t & \pi_t & \widehat{i}_t \end{bmatrix}^\intercal$ and $\mathbf{u}_t = \begin{bmatrix} r_t^n & \widehat{\mu}_t & e_t \end{bmatrix}^\intercal$, and

$$\mathbf{A_c} = \begin{bmatrix} 1 & 0 & -\lambda_r \\ -\alpha_c & 1 & 0 \\ -(1-\rho_r)\phi_y & -(1-\rho_r)\phi_\pi & 1 \end{bmatrix}, \qquad \mathbf{A_b} = \begin{bmatrix} \lambda_b & 0 & 0 \\ \alpha_b & \gamma_b & 0 \\ 0 & 0 & \rho_r \end{bmatrix}$$

$$\mathbf{A_f} = \begin{bmatrix} \lambda_f & -\lambda_r \overline{m} & 0 \\ 0 & \gamma_f & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad \mathbf{A_s} = \begin{bmatrix} -\lambda_r & 0 & 0 \\ 0 & \gamma_\mu & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2.5. Sensitivity Analysis

Rational Agents. To understand how these two new parameters, h and ω, affect the model dynamics, we study how the composite parameters are affected by them. We begin with the Dynamic IS curve. Recall that in the standard model there is no intrinsic persistence in the Dynamic IS curve when $h = 0$, since $\lambda_b = 0$. Once we relax h and allow it to be in the closed unit interval, we find that $\lambda_b = \frac{h}{1-h}$. An increase in h from its initial 0 value leads to a rise in the backward-looking composite parameter in the Dynamic IS curve: $\partial\lambda_b/\partial h > 0$. In our model, this is a consequence of the "keeping up with the Joneses" utility function that we assume, in which agents maximize a quasi-difference in consumption. Because large increases in current consumption can be harming tomorrow's felicity, agents take into account past consumption levels and, after aggregating across agents, this results in intrinsic persistence in the Dynamic IS curve. We can similarly interpret the forward-looking composite parameter λ_f. Once we relax h, we find that $\lambda_f = \frac{1}{1-h}$, which is decreasing in h: $\partial\lambda_f/\partial h < 0$. An increase in the backward-looking behavior, parameterized by h, implies that the agent assigns less importance to the future. Likewise, regarding the composite parameters of the contemporaneous variables, we find that $\lambda_r = -\frac{1-h}{\sigma(1+h)}$. One can then show that the introduction of h reduces the impact effect of changes in the real interest rate: $\partial(-\lambda_r)/\partial h < 0$. Overall, the forward-looking dimension of the Dynamic IS curve is dampened, and intrinsic persistence gains momentum.

Turning now to the NK Phillips curve, a similar argument follows. In the standard model there is no price indexation, so that $\omega = 0$. Extending the model to price indexation generates intrinsic persistence in the Phillips curve, because current pricing decisions by firms are indexed to prior inflation. Notice also that there are two backward-looking terms in the Phillips curve, the output gap and the inflation rate. The additional backward-looking output gap term appears after inserting the output gap into the marginal costs, which establish a direct relation between the marginal rate of substitution, driven by households' preferences, and the output gap. Let us start with the backward-looking inflation term. Relaxing ω, allowing it to be in the closed unit interval, we find that $\gamma_b = \frac{\omega}{1+\omega\beta}$. The backward-looking nature of the inflation term is affected by ω, the degree of indexation. Increasing the degree of price indexation from its benchmark value $\omega = 0$ enlarges the backward-looking composite parameter attached to inflation: $\partial\gamma_b/\partial\omega > 0$. The backward-looking output gap term α_b is affected both by h, since marginal costs are endogenous to the marginal rate of substitution, and ω. It is increasing in h, decreasing in ω and decreasing in both: $\partial(-\alpha_b)/\partial\omega < 0$, $\partial(-\alpha_b)/\partial h > 0$ and $\partial(-\alpha_b)/(\partial\omega\partial h) < 0$, thus generating intrinsic persistence when external habits and price-indexation are introduced into the model. The forward-looking term γ_f is decreasing in ω: the relative importance of composite parameter is transferred from forward-looking to backward-looking ones. Moreover, the composite parameter interacting with the contemporaneous output gap, α_c, is increasing in h, decreasing in ω and in both: $\partial\alpha_c/\partial\omega < 0$, $\partial\alpha_c/\partial h > 0$ and $\partial\alpha_c/(\partial\omega\partial h) < 0$.

Behavioral Agents. To understand how this new parameter $\overline{m}$ is affecting the model dynamics, let us see how the composite parameters are influenced by its introduction. Let us start with the Dynamic IS curve. Once we relax $\overline{m}$ and allow it to be in the closed unit interval, we find that $\lambda_f = \frac{\overline{m}}{1-h}$ and $-\lambda_r\overline{m} = \frac{(1-h)\overline{m}}{\sigma(1+h)}$, while the other two composite parameters are unaffected by $\overline{m}$. One can see that both affected composite parameters, λ_f and $-\lambda_r\overline{m}$, are increasing in the degree of attention $\overline{m}$. Introducing inattention generates household myopia and reduces the importance of the parameters interacting with forward-looking variables, doing so without affecting the parameters linked to past and contemporaneous variables.

Turning now to the Phillips curve, the main difference relies on the fact that composite parameters interacting with backward-looking and contemporaneous variables are now also affected by $\overline{m}$. Extending the model to BR generates intrinsic persistence in the Phillips curve, since cognitive discounting also interacts with price-indexation. Let us start with the backward-looking inflation term. Relaxing $\overline{m}$ by allowing it to be in the closed unit interval, we find that γ_b is decreasing in attention: $\partial\gamma_b/\partial\overline{m} < 0$ as long as $(1-\beta\theta)(1-\beta\theta^2\overline{m}^2)+\beta\theta^2(1-\overline{m})^2 > 0$, which is always satisfied. The backward-looking output gap term α_b is now affected by inattention, and is decreasing in $\overline{m}$. Again, inattention and intrinsic persistence coming from price indexation lead to *more* intrinsic persistence. The forward looking term γ_f is increasing in $\overline{m}$: with inattentive firms the relative importance of composite parameters is transferred from forward-looking to backward-looking ones. Besides, the composite parameter interacting with the contemporaneous output gap, α_c, is decreasing in $\overline{m}$.

3. Estimation

This section lays out the approach we follow for the estimation of our structural parameters of interest through Bayesian techniques. First, we discuss the time series data we use and their transformation. Second, we describe our estimation procedure, that is, prior selection, and evaluation of the likelihood function using the Kalman Filter and the Metropolis-Hastings algorithm for finding posterior distributions as well as moments for our structural parameters.

As we have previously introduced, one of the objectives of this paper is to compare our results with those in Galí and Gertler (1999). In their seminal paper, they exclusively estimate different versions of the Phillips curve by GMM methods, whereas we estimate complete versions of the New Keynesian model. Given our strategy, we instead rely on Bayesian inference. There are some advantages associated with full-information methods such as Bayesian estimation.[13] For example, Bayesian approaches can improve the estimator precision and can lessen identification problems, at least asymptotically; can reduce the risk of misspecification and can deal with model uncertainty and, finally,

[13] Mavroeidis et al. (2014) present a survey of studies using limited-information methods for the estimation of the New Keynesian Phillips curve.

the results can be easily compared to the point estimates from standard BVARs.[14]

3.1. The Data

We estimate the model using three US time series at the quarterly frequency: 1) the log of real GDP per capita, 2) the log-difference of the CPI inflation rate, and 3) the nominal interest rate.[15] In particular, to proxy for the output gap, we apply a one-sided HP filter to the log of real GDP per capita. We demean both the inflation rate and the nominal interest rate, the effective Federal Funds rate. The underlying data comes from FRED.[16]

We use three different samples that differ regarding their time spans. Our main sample starts in 1960:I and ends in 1997:IV. We choose this period to be able to compare our results to those reported by Galí and Gertler (1999). For robustness, we repeat our estimation for the sample starting in 1985:I until 2007:III, and an extended sample period starting in 1955:I and ending in 2007:III.

3.2. A Bayesian Approach

We need to specify the prior distributions for the structural parameters. Using prior information and the observable variables, we apply the Kalman Filter to evaluate the likelihood function of each model and the Metropolis-Hastings algorithm to draw from the posterior distributions and estimate their moments.[17]

Prior Selection. The prior distribution for the parameters is standard (see e.g., Smets and Wouters 2007) and reported in Table 1. For the subjective discount factor, β, we use a Beta distribution with mean 0.99 and standard deviation 0.001 (similar to Boehl et al. 2022; Kulish et al. 2017). Along the lines of Smets and Wouters (2007), for price

[14] See Rabanal and Rubio-Ramírez (2005) and Fernández-Villaverde and Rubio-Ramírez (2004), for instance, for a more detailed discussion.

[15] We also consider GDP deflator as the price index in the estimation, in table A1. We find similar estimates of bounded rationality. In the spirit of Bouakez et al. (2005), we use the per capita series to control for population growth.

[16] We obtain real GDP from the U.S. Bureau of Economic Analysis (retrieved from FRED), "Real Gross Domestic Product [GDPC1]"; the price index from the U.S. Bureau of Labor Statistics (retrieved from FRED), "Consumer Price Index for All Urban Consumers: All Items in U.S. City Average [CPIAUCSL]"; and the nominal interest rate from the Board of Governors of the Federal Reserve System (retrieved from FRED), "Effective Federal Funds Rate [FEDFUNDS]". To convert real GDP in per capita terms we use population from the U.S. Bureau of Labor Statistics (retrieved from FRED), "Population Level [CNP16OV]". Regarding the latter, we have employed the trailing moving average (4 quarters) of the series, which is robust to the critique in Edge et al. (2013).

[17] As usual, the posterior distribution can be approximated by the product of the prior and the likelihood function.

stickiness, θ, and price indexation, ω, we also choose a Beta distribution with mean 0.5 and standard deviation 0.10 (0.15 for ω).[18] For habit persistence, h, we set a Beta distribution with mean 0.70 and standard deviation 0.10. We also employ the same prior distribution as in Smets and Wouters (2007) to estimate σ and φ. For the parameters entering the Taylor rule, we use a Normal distribution with mean 1.5 and standard deviation 0.15 for the response to changes in inflation, ϕ_π. Likewise, for the response to deviations from potential output, ϕ_y, we set a normally distributed prior with mean 0.15 and standard deviation 0.10. As in Smets and Wouters (2007), we use a Beta distribution for the persistence parameters using a mean of 0.5 and a standard deviation of 0.2. Finally, for the standard deviation of the shocks we use an Inverse Gamma Distribution with a mean of 0.1 and infinite standard deviation.

Bayesian Inference. We solve the model and estimate the remaining parameters for each specification using Dynare.[19] We use the CMA-ES algorithm for computing the mode which is robust to multiple local maxima (Hansen et al. 2003). To sample and estimate the moments of the posterior distributions, we use a Markov Chain Monte Carlo with 500.000 draws from the Metropolis-Hastings algorithm and burn-in the first 125.000 (25%). The acceptance rate was of 23%. Since we only employ one chain for the Metropolis-Hastings algorithm to reduce estimation time, convergence is checked using the test proposed by Geweke (1991).

Estimation Algorithm and Belief Formation Frictions. We give a belief formation interpretation to $\overline{m}$ by matching the forecast underrevision coefficient in Coibion and Gorodnichenko (2015). Starting from a linear grid of $\overline{m}$ in the closed unit interval, we estimate the rest of the parameters for each guessed value of $\overline{m}$. We then simulate the theoretical framework using the point estimates of the parameters, and construct two time series of the model-implied ex-ante average forecast error, forecast $\text{error}_t = \pi_{t+3,t} - \mathbb{E}_t^B \pi_{t+3,t}$ and the average forecast revision, $\text{revision}_t = \mathbb{E}_t^B \pi_{t+3,t} - \mathbb{E}_{t-1}^B \pi_{t+3,t}$, where $\pi_{t+3,t}$ is the price level growth rate between period $t+3$ and $t-1$. We then compute the Coibion and Gorodnichenko (2015) estimate following Angeletos et al. (2021); Gallegos (2023), $\beta_{CG} = \frac{\mathbb{C}(\text{forecast error}_t, \text{revision}_t)}{\mathbb{V}(\text{revision}_t)}$. Finally, we select the $\overline{m}$ that minimizes the distance between the theoretical coefficient, and the estimated coefficient reported by Coibion and Gorodnichenko (2015), 1.193. Additionally, we perform an exercise in which we

[18]This prior would imply that the average length of price contracts is 6 months.
[19]See Adjemian et al. (2011) for more details.

directly estimate $\overline{m}$ together with the rest of the model parameters.

4. Findings

This section discusses the estimation results for our parameters of interest and their implications for the business cycle. First, we examine the ability of our model specifications to reconcile previous empirical estimates and compare the fit to the data through their log data densities calculated using the Laplace approximation as in Geweke (1991). Second, we consider whether our analytical models can replicate the responses of output gap and inflation to a monetary policy shock estimated by means of a BVAR using narrative sign restrictions.

4.1. Posterior Distributions and Moments

Standard New Keynesian Model. Table 1 displays the main results. We report the posterior mean and the 90% error bands. We begin by estimating an otherwise standard New Keynesian model using the first sample that goes from 1960:I to 1997:IV. The first column reports the estimation of the standard NK model. That is, we estimate the system (23) restricting $h = \omega = 0$ and $\overline{m} = 1$. In this standard framework there is no aggregate intrinsic persistence in the system since $\lambda_b = \gamma_b = \alpha_b = 0$, and the model exhibits an extreme forward-looking behavior. We find that this extreme forward-looking behavior produces the smallest log data density. As a result, this basic model is the least preferred among the four considered.

Hybrid New Keynesian Model. Our estimate of $h = 0.761$ is on the upper bound of the estimated values in the literature. In a meta-analysis, Havranek et al. (2017) find that the standard value of external habits in the macro literature is around 0.7, while the micro-consistent estimate is 0.4. There is less micro-empirical evidence on the true value of ω, which we estimate to be 0.199, since this form of indexation is a model artifact. This value is excessively small from the standard assumed value of 1 in the literature (see e.g., Christiano et al. 2005; Auclert et al. 2020). On top of these, our estimate of θ is in the upper range in the micro literature, although aligned with the macro literature. Bils and Klenow (2004); Nakamura and Steinsson (2008) find a median price duration of 4.5-11 months in US micro data. Galí (2008) sets $\theta = 0.75$ to match an implied duration of 1 year. Auclert et al. (2020) estimate θ between 0.88 and 0.93 from macro data, implying

	Prior Distribution	Mean (S.d)	Posterior Distribution 1960:I-1997:IV NK	HNK	BNK	BHNK
β	*Beta*	0.99	0.990	0.990	0.990	0.990
		(0.001)	(0.988, 0.992)	(0.988, 0.992)	(0.988, 0.992)	(0.988, 0.992)
σ	*Normal*	1.5	2.272	1.593	1.865	1.134
		(0.37)	(1.817, 2.723)	(1.075, 2.128)	(1.413, 2.320)	(0.537, 1.627)
φ	*Normal*	2	1.485	1.578	1.426	1.401
		(0.75)	(0.500, 2.344)	(0.500, 2.450)	(0.500, 2.286)	(0.500, 2.259)
ϕ_π	*Normal*	1.50	1.493	1.384	1.354	1.350
		(0.15)	(1.287, 1.691)	(1.178, 1.578)	(1.151, 1.560)	(1.133, 1.565)
ϕ_y	*Normal*	0.15	0.323	0.290	0.317	0.273
		(0.10)	(0.215, 0.439)	(0.181, 0.397)	(0.205, 0.424)	(0.158, 0.388)
θ	*Beta*	0.50	0.863	0.907	0.856	0.878
		(0.10)	(0.817, 0.911)	(0.877, 0.938)	(0.814, 0.899)	(0.839, 0.920)
h	*Beta*	0.70	—	0.761	—	0.647
		(0.10)	(—)	(0.657, 0.866)	(—)	(0.510, 0.790)
ω	*Beta*	0.50	—	0.199	—	0.681
		(0.15)	(—)	(0.053, 0.307)	(—)	(0.155, 0.902)
$\overline{m}$	*Implied*	—	1	1	0.445	0.340
		—	(—)	(—)	(—)	(—)
ρ_i	*Beta*	0.50	0.766	0.803	0.818	0.841
		(0.20)	(0.717, 0.819)	(0.756, 0.849)	(0.771, 0.867)	(0.795, 0.888)
ρ_d	*Beta*	0.50	0.807	0.419	0.860	0.717
		(0.20)	(0.748, 0.865)	(0.254, 0.588)	(0.801, 0.920)	(0.622, 0.812)
ρ_s	*Beta*	0.50	0.884	0.781	0.855	0.250
		(0.20)	(0.815, 0.949)	(0.702, 0.901)	(0.793, 0.917)	(0.016, 0.785)
ρ_{e_i}	*Beta*	0.50	0.191	0.203	0.184	0.183
		(0.20)	(0.058, 0.321)	(0.080, 0.325)	(0.064, 0.300)	(0.065, 0.291)
σ_d	*Inv. gamma*	0.10	0.242	0.270	0.498	0.541
		(∞)	(0.181, 0.302)	(0.210, 0.327)	(0.442, 0.554)	(0.487, 0.595)
σ_s	*Inv. gamma*	0.10	0.093	0.099	0.281	0.328
		(∞)	(0.062, 0.124)	(0.059, 0.131)	(0.250, 0.311)	(0.294, 0.362)
σ_i	*Inv. gamma*	0.10	0.256	0.242	0.240	0.237
		(∞)	(0.226, 0.285)	(0.217, 0.265)	(0.215, 0.262)	(0.214, 0.260)
	Log data density		–293.056	–278.537	–282.352	–278.320

Note: Results are reported at the posterior mean. 90% confidence intervals in parenthesis. The model-implied forecast-underrevision coefficients are 1.191 (BNK) and 1.210 (BHNK). The baseline forecast-underrevision reported in Coibion and Gorodnichenko (2015) is 1.193. The log data density for the Altig et al. (2011) estimation is –585.657.

TABLE 1. Estimated Structural Parameters

a price duration of 12-14 quarters. The model is flexible to allow for intrinsic persistence, and produces a larger log data density relative to the standard New Keynesian model.

Behavioral Hybrid New Keynesian Model. The above results motivate our departure from full rationality. The cognitive discount factor $\overline{m}$ interacts with the degree of external habits h and the Calvo price rigidity parameter θ backward-, contemporaneous and forward-looking terms. As a result, relaxing the cognitive discount factor helps match the other parameters to their micro empirical estimates. We now find values of external habits and price frictions that are closer to their microfounded and standard values in the literature. We estimate $h = 0.647$, $\theta = 0.878$ and $\omega = 0.681$. Regarding the degree of inattention in the economy, we obtain $\overline{m} = 0.340$, which results in a model-implied forecast underrevision coefficient of 1.210 (compared to 1.193 in Coibion and Gorodnichenko 2015).[20] This extension yields the largest log data density, –278.320.

Robustness Checks. We conduct several robustness checks to the BHNK framework. First, we replace the CPI data for the GDP Deflator, given that the model is a closed one-good economy (see Table A1, column 4). We find a similar estimate of $\overline{m}$, with a smaller price indexation coefficient. The remaining estimates are stable and we do not observe any considerable differences. Second, we directly estimate $\overline{m}$ together with the rest of the model parameters (see Table A1, column 5). We find an $\overline{m}$ estimate of 0.435, which implies a forecast underrevision coefficient of 0.805, with other estimates being similar to our benchmark exercise.

Third, we modify our sample to the 1985:I-2007:III period. The literature has found evidence of the fall in the persistence of inflation (Fuhrer 2010), the flattening of the Phillips curve (Rubbo 2019; del Negro et al. 2020; Hazell et al. 2022), a fall in the volatility of macroeconomic variables (McConnell and Perez-Quiros 2000), and heterogeneous changes in belief formation frictions (Coibion and Gorodnichenko 2015; Gallegos 2023) in this period (see Table A1, column 6). As argued before, the empirical evidence suggests that the (intrinsic) persistence of inflation fell in the 1980s, which is reflected by the lower estimates of ω (which generates intrinsic persistence in our framework) and ρ_S (which generates extrinsic persistence). On top of this, Angeletos et al. (2021) and Gallegos (2023) find evidence of a fall in the forecast-underrevision coefficient since the mid-1980s. For this reason, we do not target any Coibion and Gorodnichenko (2015) estimate, and instead estimate directly $\overline{m}$. We obtain an estimate of 0.401, which together with the lack of intrinsic persistence of inflation, implies a forecast-underrevision coefficient of 0.287, consistent with the empirical fingings in Angeletos et al. (2021); Gallegos (2023).

[20] Cornea-Madeira et al. (2019) estimate a myopia coefficient of 0.353, Ilabaca et al. (2020) estimate a cognitive discount factor that is around 0.5, and Andrade et al. (2019) report a value of 0.67 using maximum likelihood inference.

We estimate the full model using a third sample that goes from 1955:I to 2007:III, right before the Great Recession (see Table A1, column 7). Our estimates are stable and we do not observe any considerable differences.

Lastly, we also estimate the medium-scale DSGE model in Altig et al. (2011), using the Dynare implementation of the model by Taylor and Wieland (2012). We use Bayesian techniques instead of matching VAR-based impulse-response functions. We use the same data series as in our benchmark model, and calculate the marginal data density. We find a log data density of –585.657.

4.2. Monetary Policy Shocks: Narrative BVAR vs. NK Models

Empirical Impulse Response Functions. In order to compare our set of models to the data, we also estimate a BVAR and reproduce the impulse responses after an expansionary 25 bp monetary policy shock. First, we identify the VAR monetary policy shock by means of sign restrictions. Table 2 displays the signs imposed for the standard sign restriction approach. Besides the monetary policy shock, we control for an aggregate demand shock and an aggregate supply shock. The table imposes well-known sign restrictions required to identify these three different shocks. We follow Uhlig (2005) and assume that an expansionary monetary policy shock is the one that reduces the nominal rate and rises output gap, inflation, non-borrowed reserves and total reserves for the first two quarters.[21]

In addition to pure sign restrictions, we impose narrative sign restrictions as in Antolín-Díaz and Rubio-Ramírez (2018). Therefore, it is required that the identified monetary policy shock series and the historical decomposition are constrained on particular dates. In particular, we consider the Volcker reform in 1979:IV as a period of an exogenous monetary policy change. For this event we impose the following restrictions:

- ***Narrative Restriction 1:*** The monetary policy shock must be positive for the observation in 1979:IV.
- ***Narrative Restriction 2:*** The monetary policy shock is the most important contributor to the observed changes in the federal funds rate in 1979:IV.

The VAR includes the same observables as in the theoretical model over the period 1960:I through 1997:IV. It features three lags (given the Akaike information criterion)

[21] We also include non-borrowed and total reserves for the sake of completeness, as Uhlig (2005) and Antolín-Díaz and Rubio-Ramírez (2018). The timing restriction is similar to the one in the aforementioned studies.

and is estimated by Bayesian methods under a conjugate normal inverse-wishart prior following Antolín-Díaz and Rubio-Ramírez (2018).

	MP shock	Demand shock	Supply shock
Output gap	+	+	+
Inflation	+	+	–
Nominal interest rate	–	+	–
Non-borrowed reserves	+	?	?
Total reserves	+	?	?

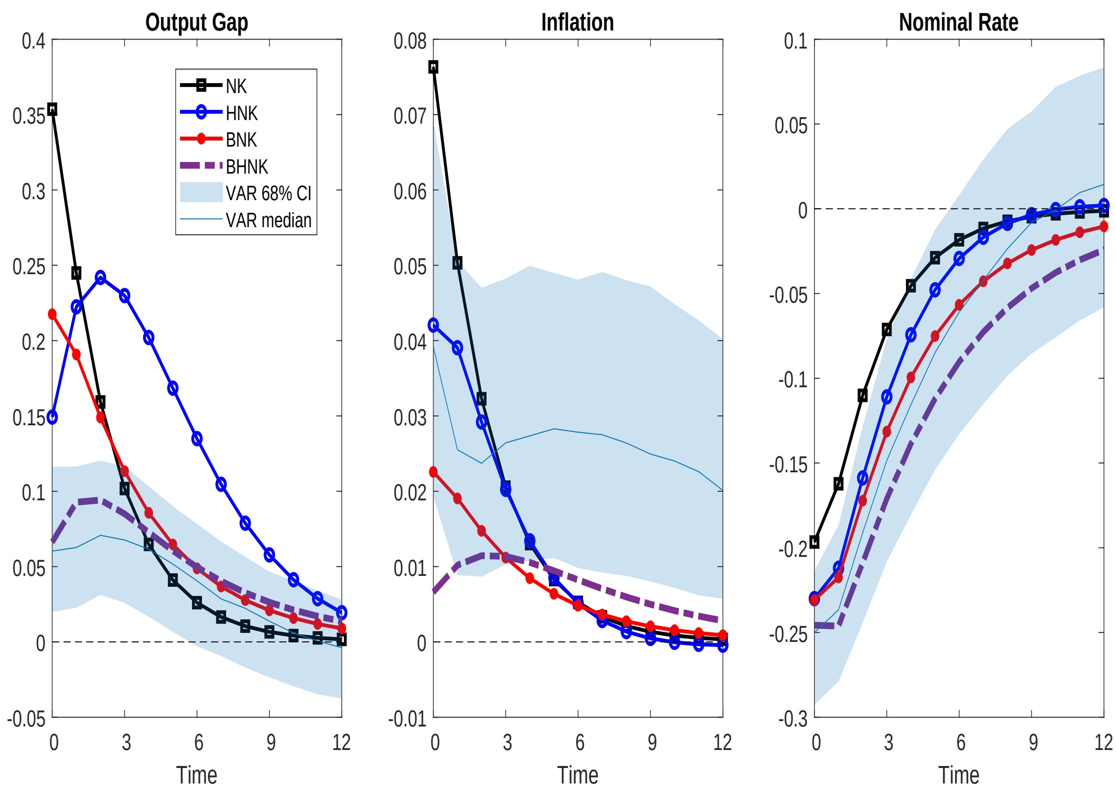
Note: Sign restrictions are imposed for the first two quarters. Symbols + and – refer to the direction of the response for the considered period of time. When agnostic about the sign, the symbol ? is employed.

TABLE 2. Sign Restrictions

Theoretical Impulse Response Functions. A failure of the standard model is that it does not produce hump-shaped impulse responses after an exogenous monetary policy shock, which is at odds with empirical macro evidence (see e.g., Christiano et al. 2005 and Altig et al. 2011). Applied macro studies generally find that the peak effect of output after a monetary policy shock occurs after 2-8 quarters, whereas in the standard model without intrinsic persistence the peak effect occurs instantaneously, and the impulse responses are monotonically decreasing over time. In Figure 1, we plot the impulse response functions (IRFs) of output, inflation and nominal interest rates after an expansionary monetary policy shock of 25 bp in the standard NK (black line) and their empirical counterparts (blue line, with an associated confidence interval). The counterfactual shape of the impulse responses motivate our departure from the benchmark model.

Figure 1 also displays the IRFs of the HNK framework (blue line). The inclusion intrinsic persistence leads to strong hump-shaped responses for output gap and inflation following an exogenous monetary policy shock, but larger in magnitude than what the empirical evidence suggests.

When we introduce behavioral features into the HNK model, labelled "BHNK" (violet and dashed line), we observe that cognitive discounting dampens the aggregate response to a monetary policy shock. Adding the backward-looking behavior together with cognitive discounting helps obtain impulse responses that are closer to their empirical counterpart — the reaction is smaller, more persistent and exhibit hump-shaped dynamics. Intuitively, because there is intrinsic persistence due to price indexation, less attentive firms' actions will be determined by past aggregates to a larger extent.



Note: The dynamic paths for the variables are reported under different model specifications after an expansionary 25 bp monetary policy shock: (i) a standard NK model in black lines (squares), (ii) a hybrid NK model in blue lines (circles), (iii) a behavioral NK model in red lines (asterisks), and (iv) a behavioral hybrid NK model in purple lines (dashed). The VAR-based monetary policy shock is identified by means of narrative sign restrictions as in Antolín-Díaz and Rubio-Ramírez (2018). The horizontal axis displays the time which is measured in quarters. Vertical axis values refer to deviations from steady state in percentage.

FIGURE 1. Dynamic Responses to a Monetary Policy Shock.

For completeness, we also report the results for the BNK model without intrinsic persistence (red line). We observe that the exclusion of external habits and inflation indexation implies that the IRFs are not hump-shaped. Therefore, we conclude that we need both cognitive discounting and intrinsic persistence, first to match the empirical estimates for certain parameters of interest, and second to obtain hump-shaped IRFs and initially muted responses for both output gap and inflation. In particular, the strong inflation persistence obtained in VAR frameworks is exclusively present in the BHNK model.

5. Conclusion

The benchmark NK model is purely forward looking and lacks the ability to capture the intrinsic persistence in output and inflation that we observe in the data. In order to avoid this, the literature has included backward-looking agents, either assuming a backward-looking utility function for households or sticky price indexation for firms. Unfortunately, the parameter values that characterize the frictions required to produce the degree of intrinsic persistence that the data suggests are at odds with empirical evidence. In this paper, we harmonize these discrepancies between empirics and theory by building and estimating a New Keynesian model augmented with backward-looking agents *and* cognitive discounting. We find strong evidence for aggregate myopia, with a cognitive discount factor estimate of 0.34 at a quarterly frequency, producing the largest log data density.

For the estimation of the structural parameters, we follow a Bayesian approach that allows a transparent comparison across models. We estimate four different models: the standard NK model, the hybrid NK model, the behavioral NK model, and the behavioral hybrid NK model. We show that cognitive discounting is successful in producing myopia but does not produce intrinsic persistence on its own. We find that the cognitive discount factor, *together* with habit persistence and price indexation, is key to obtain macro estimates that align better with their micro counterpart. Finally, in order to test the ability of our set of models to replicate empirical impulse-response functions, we compare them with an estimated monetary policy shock. We find that only our Behavioral NK model with both habit formation and backward-looking firms is able to generate, at the same time, hump-shaped responses and enough output and inflation persistence as we observe in the data.

References

Adjemian, Stéphane, Houtan Bastani, Michel Juillard, Frédéric Karamé, Junior Maih, Ferhat Mihoubi, George Perendia, Johannes Pfeifer, Marco Ratto, and Sébastien Villemot, "Dynare: Reference Manual, Version 4," 2011. Dynare Working Papers, 1, CEPREMAP.

Altig, David, Lawrence Christiano, Martin Eichenbaum, and Jesper Linde, "Firm-Specific Capital, Nominal Rigidities and the Business Cycle," *Review of Economic Dynamics*, April 2011, *14* (2), 225–247.

Andrade, Joaquim, Pedro Cordeiro, and Guilherme Lambais, "Estimating a Behavioral New Keynesian Model," 2019.

Angeletos, George-Marios, Zhen Huo, and Karthik A. Sastry, "Imperfect Macroeconomic Expectations: Evidence and Theory," *NBER Macroeconomics Annual*, 2021, *35*, 1–86.

Antolín-Díaz, Juan and Juan F. Rubio-Ramírez, "Narrative Sign Restrictions for SVARs," *American Economic Review*, October 2018, *108* (10), 2802–29.

Auclert, Adrien, Matthew Rognlie, and Ludwig Straub, "Micro Jumps, Macro Humps: Monetary Policy and Business Cycles in an Estimated HANK Model," 2020.

Bils, Mark and Peter J. Klenow, "Some Evidence on the Importance of Sticky Prices," *Journal of Political Economy*, October 2004, *112* (5), 947–985.

Blanchard, Olivier, Christopher J Erceg, and Olivier Blanchard, "Jump Starting the Euro Area Recovery: Would a Rise in Core Fiscal Spending Help the Recovery?," 2015.

Boehl, Gregor, Gavin Goy, and Felix Strobel, "A structural investigation of quantitative easing," *Review of Economics and Statistics*, 2022. forthcoming.

Bouakez, Hafedh, Emanuela Cardia, and Francisco J. Ruge-Murcia, "Habit formation and the persistence of monetary shocks," *Journal of Monetary Economics*, September 2005, *52* (6), 1073–1088.

Christiano, Lawrence J, Martin Eichenbaum, and Charles L Evans, "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, 2005, *113* (1), 1–45.

Coibion, Olivier and Yuriy Gorodnichenko, "Information rigidity and the expectations formation process: A simple framework and new facts," *American Economic Review*, 2015.

Cornea-Madeira, Adriana, Cars Hommes, and Domenico Massaro, "Behavioral Heterogeneity in U.S. Inflation Dynamics," *Journal of Business & Economic Statistics*, 2019, *37* (2), 288–300.

del Negro, Marco, Michele Lenza, Giorgio Primiceri, and Andrea Tambalotti, "What's up with the Phillips Curve?," *National Bureau of Economic Research*, 2020.

Edge, Rochelle M, Refet Gürkaynak, and Burcin Kisacikoglu, "Judging the DSGE Model by Its Forecast," 2013.

Fernández-Villaverde, Jesus and Juan F. Rubio-Ramírez, "Comparing dynamic equilibrium models to data: a Bayesian approach," *Journal of Econometrics*, November 2004, *123* (1), 153–187.

Fuhrer, Jeff and George Moore, "Inflation Persistence," *The Quarterly Journal of Economics*, 1995, *110* (1), 127–159.

Fuhrer, Jeffrey C., "Inflation Persistence," *Handbook of Monetary Economics*, 2010, *2*.

Gabaix, Xavier, "A sparsity-based model of bounded rationality," *Quarterly Journal of Economics*, 2014, *129* (4), 1661–1710.

___ , "Behavioral Macroeconomics Via Sparse Dynamic Programming," Working Paper 21848, National Bureau of Economic Research January 2016.

___ , "A Behavioral New Keynesian Model," *American Economic Review*, 2020, *110* (8), 2271–2327.

Galí, Jordi, "Introduction to Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework," in "Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework" Introductory Chapters, Princeton University Press, 2008.

Galí, Jordi and Mark Gertler, "Inflation dynamics: A structural econometric analysis," *Journal of Monetary Economics*, 1999, *44* (2), 195–222.

Gallegos, José-Elías, "Inflation Persistence, Noisy Information and the Phillips Curve," Technical Report 2023.

Geweke, John F., "Evaluating the accuracy of sampling-based approaches to the calculation of posterior moments," 1991.

Grauwe, Paul De, "Animal spirits and monetary policy," *Economic Theory*, 2011, *47* (2/3), 423–457.

Hajdini, Ina, "Mis-specified Forecasts and Myopia in an Estimated New Keynesian Model," 2022.

Hansen, Nikolaus, Sibylle D. Müller, and Petros Koumoutsakos, "Reducing the Time Complexity of the Derandomized Evolution Strategy with Covariance Matrix Adaptation (CMA-ES)," *Evolutionary Computation*, 2003, *11* (1), 1–18.

Havranek, Tomas, Marek Rusnak, and Anna Sokolova, "Habit formation in consumption: A meta-analysis," *European Economic Review*, 2017, *95* (C), 142–167.

Hazell, Jonathon, Juan Herreño, Emi Nakamura, and Jón Steinsson, "The Slope of the Phillips Curve: Evidence from U.S. States*," *The Quarterly Journal of Economics*, 02 2022, *137* (3), 1299–1344.

Hommes, Cars and Joep Lustenhouwer, "Inflation targeting and liquidity traps under endogenous credibility," *Journal of Monetary Economics*, 2019, *107*, 48–62.

Ilabaca, Francisco, Greta Meggiorini, and Fabio Milani, "Bounded rationality, monetary policy, and macroeconomic stability," *Economics Letters*, 2020, *186* (C).

Kulish, Mariano, James Morley, and Tim Robinson, "Estimating DSGE models with zero interest rate policy," *Journal of Monetary Economics*, 2017, *88*, 35–49.

Madeira, Joao, "Overtime Labor, Employment Frictions, and the New Keynesian Phillips Curve," *Review of Economics and Statistics*, 2014, *96* (4), 767–778.

Mankiw, N. Gregory and Ricardo Reis, "Sticky information versus sticky prices: A proposal to replace the new Keynesian Phillips curve," *Quarterly Journal of Economics*, 2002, *117* (4), 1295–1328.

Mavroeidis, Sophocles, Mikkel Plagborg-Moller, and James H. Stock, "Empirical Evidence on Inflation Expectations in the New Keynesian Phillips Curve," *Journal of Economic Literature*, March 2014, *52* (1), 124–188.

McConnell, Margaret M. and Gabriel Perez-Quiros, "Output Fluctuations in the United States: What Has Changed Since the Early 1980's?," *The American Economic Review*, 2000, *90* (5), 1464–1476.

Milani, Fabio, "Expectations, learning and macroeconomic persistence," *Journal of Monetary Economics*, October 2007, *54* (7), 2065–2082.

Nakamura, Emi and Jón Steinsson, "Five Facts about Prices: A Reevaluation of Menu Cost Models," *Quarterly Journal of Economics*, 2008, *123* (4), 1415–1464.

Rabanal, Pau and Juan F. Rubio-Ramírez, "Comparing New Keynesian models of the business cycle: A Bayesian approach," *Journal of Monetary Economics*, September 2005, *52* (6), 1151–1166.

Roberts, John M., "Is inflation sticky?," *Journal of Monetary Economics*, July 1997, *39* (2), 173–196.

Rubbo, Elisa, "Networks, Phillips Curves, and Monetary Policy," *Working Paper*, 2019.

Sheedy, Kevin D., "Intrinsic inflation persistence," *Journal of Monetary Economics*, November 2010, *57* (8), 1049–1061.

Smets, Frank and Rafael Wouters, "Shocks and Frictions in US Business Cycles: A Bayesian

DSGE Approach," *American Economic Review*, June 2007, *97* (3), 586–606.

Taylor, John B. and Volker Wieland, "Surprising comparative properties of monetary models: results from a new model database," *Review of Economics and Statistics*, 2012, *94* (3), 800–816.

Uhlig, Harald, "What are the effects of monetary policy on output? Results from an agnostic identification procedure," *Journal of Monetary Economics*, 2005, *52* (2), 381 – 419.

Woodford, M, "Imperfect Common Knowledge and the Effects of Monetary Policy," *Information and Expectations in Modern Macroeconomics*, 2003.

Appendix A. Appendix for "online" publication

A.1. Demand for good i, Aggregate Price Index and Optimality Conditions

The representative household derives utility from consumption of different goods, indexed $i \in I = [0, 1]$, according to the consumption index. Let $\mathcal{C} = \{C_t \in \mathcal{L}^1 \colon C_t \colon I \to \mathbb{R}$ is quasi-concave and Borel measurable, $t \in \mathbb{Z}_+\}$ be the set of consumption choice functions over the set of goods I in the economy at a given period t.

Given the price function $P_t \colon I \to \mathbb{R}_+$ with $\|P_t\|_\infty < \infty$, and for a fixed endowment $Z_t \in \mathbb{R}_+$, the representative household's maximization problem at period t is:

$$\widetilde{C}_t = \max_{C_t \in \mathcal{C}} \left[\int_0^1 C_t(i)^{\frac{\epsilon_t - 1}{\epsilon_t}} \, di \right]^{\frac{\epsilon_t}{\epsilon_t - 1}} \tag{A1}$$

subject to the budget constraint:

$$\int_0^1 P_t(i) C_t(i) \;\, di \leq Z_t \tag{A2}$$

which will be satisfied with equality in the optimum. The derivative of the Lagrangian with respect to $C_t(i)$, the consumption level of good i, yields:

$$\left[\int_0^1 C_t(i)^{\frac{\epsilon_t - 1}{\epsilon_t}} di \right]^{\frac{1}{\epsilon_t - 1}} C_t(i)^{-\frac{1}{\epsilon_t}} - \lambda_t P_t(i) = 0 \implies \widetilde{C}_t^{\frac{1}{\epsilon_t}} C_t(i)^{-\frac{1}{\epsilon_t}} = \lambda_t P_t(i)$$

where λ_t is the sequence of Lagrange multipliers attached to the sequence of restrictions (A2). By dividing the last expression for two different goods $i, j \in I$, we find relation between the optimal consumption levels of two different goods:

$$C_t(i) = \left[\frac{P_t(j)}{P_t(i)} \right]^{\epsilon_t} C_t(j) \tag{A3}$$

and inserting (A3) into (A2),

$$Z_t = \int_0^1 P_t(i) \left[\frac{P_t(j)}{P_t(i)} \right]^{\epsilon_t} C_t(j) \;\, di \implies C_t(j) = \frac{Z_t P_t(j)^{-\epsilon_t}}{\int_0^1 P_t(i)^{1-\epsilon_t} di} \tag{A4}$$

we obtain an expression for the optimal consumption levels of almost all goods in terms of prices and the initial endowment. Integrating the last equation over all goods gives the optimal aggregate consumption level:

$$\widetilde{C}_t = \left[\int_0^1 \left(\frac{Z_t P_t(i)^{-\epsilon_t}}{\int_0^1 P_t(i)^{1-\epsilon_t} di} \right)^{\frac{\epsilon_t - 1}{\epsilon_t}} di \right]^{\frac{\epsilon_t}{\epsilon_t - 1}} = Z_t \left[\int_0^1 P_t(i)^{1-\epsilon_t} di \right]^{\frac{1}{\epsilon_t - 1}}$$

Now, let's define $\widetilde{P}_t$ as the unit cost of the aggregate consumption level $\widetilde{C}_t$ at endowment level Z, $\widetilde{P}_t \widetilde{C}_t = Z_t$. Hence,

$$\widetilde{P}_t Z_t \left[\int_0^1 P_t(i)^{1-\epsilon_t} di \right]^{\frac{1}{\epsilon_t - 1}} = Z_t \implies \widetilde{P}_t = \left[\int_0^1 P_t(i)^{1-\epsilon_t} di \right]^{\frac{1}{1-\epsilon_t}} \tag{A5}$$

where (A5) is the price index. Inserting (A5) into (A4)

$$C_t(j) = \frac{Z_t P_t(j)^{-\epsilon_t}}{\widetilde{P}_t^{\,1-\epsilon_t}} = \frac{Z_t}{\widetilde{P}_t} \left[\frac{\widetilde{P}_t}{P_t(i)} \right]^{\epsilon_t} \tag{A6}$$

And finally, replacing Z_t we find the desired optimal consumption for good i in terms of the aggregate good and the aggregate price:

$$C_t(i) = \left[\frac{P_t(i)}{\widetilde{P}_t} \right]^{-\epsilon_t} \widetilde{C}_t \tag{A7}$$

With market clearing and a representative household setting, $C_t(i) = Y_t(i)$ and $\widetilde{C}_t = \widetilde{Y}_t$, and we obtain expression (3). Since we deal with the aggregate quantities in the rest of the paper, with a slight abuse of notation we drop the tilde from the aggregate terms.

Finally, in order to obtain the optimality conditions we form the Lagrangian,

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(C_t - h\overline{C}_{t-1})^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} + \lambda_t \left[B_{t-1} + W_t N_t + T_t - P_t C_t - Q_t B_t \right] \right]$$

The FOCs with respect to C_t, B_t and N_t yield,

$$\begin{aligned} C_t &: \quad \lambda_t P_t = (C_t - h\overline{C}_{t-1})^{-\sigma} \\ N_t &: \quad \lambda_t W_t = N_t^{\varphi} \\ B_t &: \quad \lambda_t Q_t = \lambda_{t+1} \end{aligned}$$

Combining them and cancelling the lagrange multiplier λ_t we obtain the optimality conditions

$$\frac{W_t}{P_t} = \frac{N_t^{\varphi}}{(C_t - h\overline{C}_{t-1})^{-\sigma}}, \qquad Q_t = \beta \mathbb{E}_t \left[\left(\frac{C_{t+1} - h\overline{C}_t}{C_t - h\overline{C}_{t-1}} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right]$$

A.2. Log-linearization of Behavioural Household's Optimality Conditions

We now proceed to log-linearize (6)-(7). Starting with (6), taking a first order Taylor approximation around a zero-inflation steady state, we obtain, $\widehat{w}_t - \widehat{p}_t = \varphi\widehat{n}_t + \frac{\sigma}{1-h}\widehat{c}_t - \frac{\sigma h}{1-h}\widehat{c}_{t-1}$. Turning to (7), we first take logs, $\log Q_t = \log\beta + \mathbb{E}_t^B\{-\sigma\log(C_{t+1} - hC_t) + \sigma\log(C_t - hC_{t-1}) + \log P_t - \log P_{t+1}\}$. Since $Q_t = 1/(1+i_t)$, one can show that $i_t \approx -\log Q_t$. We then write $\rho = -\log\beta$ and $\pi_{t+1} = \log\frac{P_{t+1}}{P_t}$. Let us now log-linearize the terms that include consumption,

$$\begin{aligned}\log[C_{t+1} - hC_t] &\approx \log[(1-h)C] + \frac{1}{(1-h)C}[C_{t+1} - C] - \frac{h}{(1-h)C}[C_t - C] \\ &= \log[(1-h)C] + \frac{1}{1-h}\widehat{c}_{t+1} - \frac{h}{1-h}\widehat{c}_t\end{aligned}$$

proceeding in a similar manner with the other consumption term, and plugging into the above expression leads to

$$0 = \mathbb{E}_t^B\left\{i_t - \rho - \frac{\sigma}{1-h}\left[\widehat{c}_{t+1} - (1+h)\widehat{c}_t + h\widehat{c}_{t-1}\right] - \pi_{t+1}\right\} \tag{A8}$$

Under cognitive discounting, $\mathbb{E}_t^B x_{t+k} = \overline{m}^k\mathbb{E}_t x_{t+k}$ for any variable x. Hence,

$$0 = \widehat{i}_t - \frac{\sigma}{1-h}\overline{m}\mathbb{E}_t\widehat{c}_{t+1} - \frac{(1+h)\sigma}{1-h}c_t - \frac{h\sigma}{1-h}c_{t-1} - \overline{m}\mathbb{E}_t\pi_{t+1}$$

where we have defined $\widehat{i}_t = i_t - i = i_t - \rho$. Rewritting this last expression leads to (9). Written in natural terms and denoting the real interest rate as $r_t = \widehat{i}_t - \overline{m}\mathbb{E}_t\pi_{t+1}$, the previous equation yields

$$\widehat{c}_t^n = \frac{h}{1+h}\widehat{c}_{t-1}^n + \frac{1}{1+h}\overline{m}\mathbb{E}_t\widehat{c}_{t+1}^n - \frac{1-h}{\sigma(1+h)}r_t^n$$

Since $\widehat{c}_t = \widehat{y}_t = \widehat{c}_t^n = \widehat{y}_t^n$, we can rewrite it in terms of the output gap $\widetilde{y}_t = y_t - y_t^n$ and it yields (10).

A.3. Solving the Firm Problem

We can rewrite condition (14) as

$$
\begin{aligned}
P_t^*(i) &= \frac{\mathbb{E}_t \sum_{k=0}^{\infty}(\theta\beta)^k (C_{t+k} - hC_{t+k-1})^{-\sigma} C_{t+k} P_{t+k}^{\epsilon_{t+k}-1} P_{t+k-1}^{-\omega\epsilon_{t+k}} \frac{W_{t+k}}{A_{t+k}} \mathcal{M}_{t+k}}{\mathbb{E}_t \sum_{k=0}^{\infty}(\theta\beta)^k (C_{t+k} - hC_{t+k-1})^{-\sigma} C_{t+k} P_{t+k}^{\epsilon_{t+k}-1} P_{t+k-1}^{\omega(1-\epsilon_{t+k})}} P_{t-1}^{\omega} = \\
&= \frac{\mathbb{E}_t \sum_{k=0}^{\infty}(\theta\beta)^k (C_{t+k} - hC_{t+k-1})^{-\sigma} C_{t+k} P_{t+k}^{\epsilon_{t+k}} P_{t+k-1}^{-\omega\epsilon_{t+k}} MC_{t+k} \mathcal{M}_{t+k}}{\mathbb{E}_t \sum_{k=0}^{\infty}(\theta\beta)^k (C_{t+k} - hC_{t+k-1})^{-\sigma} C_{t+k} P_{t+k}^{\epsilon_{t}-1} P_{t+k-1}^{\omega(1-\epsilon_{t+k})}} P_{t-1}^{\omega}
\end{aligned}
\tag{A9}
$$

where we have used $MC_{t+k} = \frac{W_{t+k}}{A_{t+k}P_{t+k}}$. With flexible prices, (A9) collapses to

$$
P_t^*(i) = \mathcal{M}_t \frac{(C_t - hC_{t-1})^{-\sigma} C_t P_t^{\epsilon_t} P_{t-1}^{-\omega\epsilon_t} MC_t}{(C_t - hC_{t-1})^{-\sigma} C_t P_t^{\epsilon_t - 1} P_{t-1}^{\omega(1-\epsilon_t)}} P_{t-1}^{\omega} = \mathcal{M}_t P_t MC_t \tag{A10}
$$

where (A10) is the frictionless mark-up. To simplify computation, we now log-linearize (A9). Separating both sides,

$$
\begin{aligned}
P_t^* \mathbb{E}_t \sum_{k=0}^{\infty}(\theta\beta)^k (C_{t+k} - hC_{t+k-1})^{-\sigma} C_{t+k} P_{t+k}^{\epsilon_{t+k}-1} P_{t+k-1}^{\omega(1-\epsilon_{t+k})} &= \\
= \mathbb{E}_t \sum_{k=0}^{\infty}(\theta\beta)^k (C_{t+k} - hC_{t+k-1})^{-\sigma} C_{t+k} P_{t+k}^{\epsilon_{t+k}} P_{t+k-1}^{-\omega\epsilon_{t+k}} MC_{t+k} P_{t-1}^{\omega} \mathcal{M}_{t+k}
\end{aligned}
\tag{A11}
$$

We know that, in steady-state, $P_t^* = P_t = P_{t-1} = P$, $\Pi_t = \Pi = 1$, $C_t = C$, $Q_{t,t+k} = \beta^k$ and $MC_t = MC$. It lasts to find MC. To obtain it, we can write (A9) in steady-state and solve for MC,

$$
P = \mathcal{M}P\text{MC}
$$

Hence, $MC = \frac{1}{\mathcal{M}}$. Before log-linearizing, divide (A11) by P_{t-1},

$$
\begin{aligned}
\frac{P_t^*}{P_{t-1}} \mathbb{E}_t \sum_{k=0}^{\infty}(\theta\beta)^k (C_{t+k} - hC_{t+k-1})^{-\sigma} C_{t+k} P_{t+k}^{\epsilon_{t+k}-1} P_{t+k-1}^{\omega(1-\epsilon_{t+k})} &= \\
= \mathbb{E}_t \sum_{k=0}^{\infty}(\theta\beta)^k (C_{t+k} - hC_{t+k-1})^{-\sigma} C_{t+k} P_{t+k}^{\epsilon_{t+k}} P_{t+k-1}^{-\omega\epsilon_{t+k}} MC_{t+k} P_{t-1}^{\omega-1} \mathcal{M}_{t+k}
\end{aligned}
\tag{A12}
$$

Log-linearizing the LHS,

$$\frac{P_t^*}{P_{t-1}}\mathbb{E}_t\sum_{k=0}^{\infty}(\theta\beta)^k(C_{t+k}-hC_{t+k-1})^{-\sigma}C_{t+k}P_{t+k}^{\epsilon-1}P_{t+k-1}^{\omega(1-\epsilon)}\simeq$$

$$\begin{aligned}
&\simeq\sum_{k=0}^{\infty}(\theta\beta)^kC^{1-\sigma}(1-h)^{-\sigma}P^{-(1-\epsilon)(1-\omega)}+\\
&\quad+\frac{1}{P}\sum_{k=0}^{\infty}(\theta\beta)^kC^{1-\sigma}(1-h)^{-\sigma}P^{-(1-\epsilon)(1-\omega)}(P_t^*-P)-\\
&\quad-\frac{P}{P^2}\sum_{k=0}^{\infty}(\theta\beta)^kC^{1-\sigma}(1-h)^{-\sigma}P^{-(1-\epsilon)(1-\omega)}(P_{t-1}-P)+\\
&\quad+\sum_{k=0}^{\infty}(\theta\beta)^kC^{1-\sigma}(1-h)^{-\sigma}(\epsilon-1)P^{\epsilon-2}P^{\omega(1-\epsilon)}(P_{t+k}-P)+\\
&\quad+\sum_{k=0}^{\infty}(\theta\beta)^kC^{1-\sigma}(1-h)^{-\sigma}P^{\epsilon-1}\omega(1-\epsilon)P^{\omega(1-\epsilon)-1}(P_{t+k-1}-P)+\\
&\quad+\sum_{k=0}^{\infty}(\theta\beta)^k\{\underbrace{(-\sigma)[C(1-h)]^{-\sigma-1}C+[C(1-h)]^{-\sigma}}_{C^{-\sigma}(1-h)^{-\sigma-1}(1-h-\sigma)}\}P^{-(1-\epsilon)(1-\omega)}(C_{t+k}-C)+\\
&\quad+\sum_{k=0}^{\infty}(\theta\beta)^k\underbrace{(-\sigma)[C(1-h)]^{-\sigma-1}(-h)C}_{\sigma hC^{-\sigma}(1-h)^{-\sigma-1}}P^{-(1-\epsilon)(1-\omega)}(C_{t+k-1}-C)+\\
&\quad+\sum_{k=0}^{\infty}(\theta\beta)^kC^{1-\sigma}(1-h)^{-\sigma}P^{-(1-\epsilon)(1-\omega)}\ln(P^{1-\omega})(\epsilon_{t+k}-\epsilon)=\\
&=\sum_{k=0}^{\infty}(\theta\beta)^kC^{1-\sigma}(1-h)^{-\sigma}P^{-(1-\epsilon)(1-\omega)}\Big\{1+p_t^*-p-p_{t-1}+p-(1-\epsilon)(p_{t+k}-p)+\\
&\quad+\omega(1-\epsilon)(p_{t+k-1}-p)+\left(1-\frac{\sigma}{1-h}\right)(c_{t+k}-c)+\frac{\sigma h}{1-h}(c_{t+k-1}-c)+\ln(P^{1-\omega})(\epsilon_{t+k}-\epsilon)\Big\}
\end{aligned}$$

Log-linearizing the RHS,

$$\mathbb{E}_t\sum_{k=0}^{\infty}(\theta\beta)^k(C_{t+k}-hC_{t+k-1})^{-\sigma}C_{t+k}P_{t+k}^{\epsilon_{t+k}}P_{t+k-1}^{-\omega\epsilon_{t+k}}MC_{t+k}P_{t-1}^{\omega-1}\mathcal{M}_{t+k}\simeq$$

$$\simeq\mathcal{M}\sum_{k=0}^{\infty}(\theta\beta)^kC^{1-\sigma}(1-h)^{-\sigma}P^{-(1-\epsilon)(1-\omega)}MC+$$

$$+\mathcal{M}\sum_{k=0}^{\infty}(\theta\beta)^k C^{1-\sigma}(1-h)^{-\sigma}P^{\epsilon}P^{-\omega\epsilon}MC(\omega-1)P^{\omega-2}(P_{t-1}-P)+$$

$$+\mathcal{M}\sum_{k=0}^{\infty}(\theta\beta)^k C^{1-\sigma}(1-h)^{-\sigma}\epsilon P^{\epsilon-1}P^{-\omega\epsilon}MCP^{\omega-1}(P_{t+k}-P)+$$

$$+\mathcal{M}\sum_{k=0}^{\infty}(\theta\beta)^k C^{1-\sigma}(1-h)^{-\sigma}P^{\epsilon}(-\omega\epsilon)P^{-\omega\epsilon-1}MCP^{\omega-1}(P_{t+k-1}-P)+$$

$$+\mathcal{M}\sum_{k=0}^{\infty}(\theta\beta)^k C^{-\sigma}(1-h)^{-\sigma-1}(1-h-\sigma)P^{-(1-\epsilon)(1-\omega)}MC(C_{t+k}-C)+$$

$$+\mathcal{M}\sum_{k=0}^{\infty}(\theta\beta)^k C^{-\sigma}(1-h)^{-\sigma-1}\sigma h P^{-(1-\epsilon)(1-\omega)}MC(C_{t+k-1}-C)+$$

$$+\mathcal{M}\sum_{k=0}^{\infty}(\theta\beta)^k C^{1-\sigma}(1-h)^{-\sigma}P^{-(1-\epsilon)(1-\omega)}(MC_{t+k}-MC)+$$

$$+\mathcal{M}\sum_{k=0}^{\infty}(\theta\beta)^k C^{1-\sigma}(1-h)^{-\sigma}P^{-(1-\epsilon)(1-\omega)}\ln(P^{1-\omega})\mathrm{MC}(\epsilon_{t+k}-\epsilon)+$$

$$+\sum_{k=0}^{\infty}(\theta\beta)^k C^{1-\sigma}(1-h)^{-\sigma}P^{-(1-\epsilon)(1-\omega)}\mathrm{MC}(\mathcal{M}_{t+k}-\mathcal{M})=$$

$$=\sum_{k=0}^{\infty}(\theta\beta)^k C^{1-\sigma}(1-h)^{-\sigma}P^{-(1-\epsilon)(1-\omega)}\Big\{1-(1-\omega)(p_{t-1}-p)+\epsilon(p_{t+k}-p)-$$

$$-\omega\epsilon(p_{t+k-1}-p)+\left(1-\frac{\sigma}{1-h}\right)(c_{t+k}-c)+\frac{\sigma h}{1-h}(c_{t+k-1}-c)+mc_{t+k}-mc+$$

$$+\ln(P^{1-\omega})(\epsilon_{t+k}-\epsilon)+(\mu_{t+k}-\mu)\Big\}$$

Noticing that in steady state $P=1$, and setting LHS=RHS, eliminating $C^{1-\sigma}(1-h)^{-\sigma}P^{-(1-\epsilon)(1-\omega)}$ on both sides,

$$\sum_{k=0}^{\infty}(\theta\beta)^k\Big\{1+p_t^*-p-p_{t-1}+p-(1-\epsilon)(p_{t+k}-p)+\omega(1-\epsilon)(p_{t+k-1}-p)+$$

$$+\left(1-\frac{\sigma}{1-h}\right)(c_{t+k}-c)+\frac{\sigma h}{1-h}(c_{t+k-1}-c)+(\mu_{t+k}-\mu)\Big\}=$$

$$=\sum_{k=0}^{\infty}(\theta\beta)^k\Big\{1-(1-\omega)(p_{t-1}-p)+\epsilon(p_{t+k}-p)-\omega\epsilon(p_{t+k-1}-p)+$$

$$+\left(1-\frac{\sigma}{1-h}\right)(c_{t+k}-c)+\frac{\sigma h}{1-h}(c_{t+k-1}-c)+mc_{t+k}-mc+\mu_{t+k}-\mu\Big\}$$

Rearranging and cancelling terms, we end up with

$$\begin{aligned}
p_t^* &= (1-\theta\beta)\sum_{k=0}^{\infty}(\theta\beta)^k\mathbb{E}_t\left[p_{t+k}-\omega(\omega\, p_{t+k-1}-p_{t-1})+mc_{t+k}-mc+\mu_{t+k}-\mu\right]\\
&= (1-\theta\beta)\sum_{k=0}^{\infty}(\theta\beta)^k\mathbb{E}_t\left[p_{t+k}-\omega(\omega\, p_{t+k-1}-p_{t-1})+\widehat{mc}_{t+k}+\widehat{\mu}_{t+k}\right]\\
&= p_t+(1-\theta\beta)\sum_{k=0}^{\infty}(\theta\beta)^k\mathbb{E}_t\left[(p_{t+k}-p_t)-\omega(\omega\, p_{t+k-1}-p_{t-1})+\widehat{mc}_{t+k}+\widehat{\mu}_{t+k}\right]
\end{aligned}$$

which can be rewritten as (16).

A.4. Aggregate Price Dynamics

Let S_t denote the subset of firms not reoptimizing at time t,

$$\begin{aligned}
P_t &= \left[\int_0^1 P_t(i)^{1-\epsilon_t}di\right]^{\frac{1}{1-\epsilon_t}} =\\
&= \Bigg\{\underbrace{\int_{S_t}[P_{t-1}(i)\Pi_{t-1}^{\omega}]^{1-\epsilon_t}di}_{\Pi_{t-1}^{\omega(1-\epsilon_t)}\int_{S_t}P_{t-1}(i)^{1-\epsilon_t}di}+\int_{S_t^C}(P_t^*)^{1-\epsilon_t}di\Bigg\}^{\frac{1}{1-\epsilon_t}} =\\
&= \left[\Pi_{t-1}^{\omega(1-\epsilon_t)}\theta\int_0^1 P_{t-1}(i)^{1-\epsilon_t}di+(1-\theta)\int_0^1(P_t^*)^{1-\epsilon_t}di\right]^{\frac{1}{1-\epsilon_t}} =\\
&= \left[\Pi_{t-1}^{\omega(1-\epsilon_t)}\theta P_{t-1}^{1-\epsilon_t}+(1-\theta)(P_t^*)^{1-\epsilon_t}\right]^{\frac{1}{1-\epsilon_t}}
\end{aligned}$$

Moving the exponent from the RHS to the LHS, and dividing in both sides by $P_{t-1}^{1-\epsilon_t}$,

$$\left(\frac{P_t}{P_{t-1}}\right)^{1-\epsilon_t}=\Pi_{t-1}^{\omega(1-\epsilon_t)}\theta+(1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\epsilon_t}\implies$$

$$\implies \Pi_t^{1-\epsilon_t}=\left(\frac{P_{t-1}}{P_{t-2}}\right)^{\omega(1-\epsilon_t)}\theta+(1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\epsilon_t} \tag{A13}$$

To simplify computation, I now log-linearize the left-hand side of (A13),

$$
\begin{aligned}
\Pi_t^{1-\epsilon_t} &\simeq \Pi^{1-\epsilon} + (1-\epsilon)\Pi^{-\epsilon}\underbrace{(\Pi_t - \Pi)}_{\pi_t} = \\
&= 1 + (1-\epsilon)\pi_t
\end{aligned}
$$

since $\Pi = \frac{P}{P} = 1$. A log-linearization of the right-hand side around a zero-inflation steady-state yields

$$
\begin{aligned}
\left(\frac{P_{t-1}}{P_{t-2}}\right)^{\omega(1-\epsilon_t)} \theta + (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\epsilon_t} \simeq & \left(\frac{P}{P}\right)^{\omega(1-\epsilon)} \theta + (1-\theta)\left(\frac{P^*}{P}\right)^{1-\epsilon} + \\
& + (1-\theta)(1-\epsilon)P^{-\epsilon}P^{\epsilon-1}(P_t^* - P) + \\
& + \left[\theta\omega(1-\epsilon)P^{\omega(1-\epsilon)-1}P^{-\omega(1-\epsilon)} - (1-\theta)(1-\epsilon)P^{1-\epsilon}P^{2-\epsilon}\right] \times \\
& \times (P_{t-1} - P) - \theta\omega(1-\epsilon)P^{\omega(1-\epsilon)}P^{-\omega(1-\epsilon)-1}(P_{t-2} - P) = \\
= & \ \theta + 1 - \theta + (1-\theta)(1-\epsilon)\widehat{p}_t^* + \\
& + [\theta\omega(1-\epsilon) - (1-\theta)(1-\epsilon)]\widehat{p}_{t-1} - \theta\omega(1-\epsilon)\widehat{p}_{t-2} = \\
= & \ 1 + (1-\theta)(1-\epsilon)\widehat{p}_t^* - (1-\epsilon)[1-\theta(1+\omega)]\widehat{p}_{t-1} - \\
& - \theta\omega(1-\epsilon)\widehat{p}_{t-2} = \\
= & \ 1 + (1-\theta)(1-\epsilon)\, p_t^* - (1-\epsilon)[1-\theta(1+\omega)]\, p_{t-1} - \\
& - \theta\omega(1-\epsilon)\, p_{t-2}
\end{aligned}
$$

Writing $\widehat{x}_t = x_t - x$, all log prices are cancelled out.

$$
\begin{aligned}
\text{LHS=RHS:}\quad & 1+(1-\epsilon)\pi_t = 1 + (1-\theta)(1-\epsilon)\, p_t^* - (1-\epsilon)[1-\theta(1+\omega)]\, p_{t-1} - \theta\omega(1-\epsilon)\, p_{t-2} \implies \\
& \implies \pi_t = (1-\theta)\, p_t^* - [1-\theta(1+\omega)]\, p_{t-1} - \theta\omega\, p_{t-2} \\
& \qquad = \theta\omega\pi_{t-1} + (1-\theta)(p_t^* - p_{t-1})
\end{aligned}
$$

A.5. Deriving the Behavioural Hybrid New Keynesian Phillips Curve

Rewriting $\theta\beta\overline{m} = \delta$ and $\widetilde{\mathrm{mc}}_t = \widehat{\mathrm{mc}}_t + \widehat{\mu}_t$, the firm's problem optimality condition (17) reads

$$
p_t^* = p_t + (1-\theta\beta)\sum_{k=0}^{\infty}\delta^k\mathbb{E}_t\left[\overline{m}(\pi_{t+1} + \ldots + \pi_{t+k}) - \omega\overline{m}(\pi_t + \ldots + \pi_{t+k-1}) + \overline{m}\widetilde{mc}_{t+k}\right]
$$

(A14)

$$= p_t + (1-\theta\beta)\mathbb{E}_t\left[\overline{m}\sum_{k=0}^{\infty}\delta^k(\pi_{t+1}+\ldots+\pi_{t+k}) - \omega\overline{m}\sum_{k=0}^{\infty}\delta^k(\pi_t+\ldots+\pi_{t+k-1}) + \overline{m}\sum_{k=0}^{\infty}\delta^k\widetilde{mc}_{t+k}\right]$$

We can calculate the following

$$H_t = \sum_{k=1}^{\infty}\delta^k(\pi_{t+1}+\ldots+\pi_{t+k}) = \sum_{j=1}^{\infty}\pi_{t+j}\sum_{k=j}^{\infty}\delta^k = \sum_{j=1}^{\infty}\pi_{t+j}\frac{\delta^j}{1-\delta} = \frac{1}{1-\delta}\sum_{j=1}^{\infty}\pi_{t+j}\delta^j =$$

$$= \frac{1}{1-\delta}\sum_{k=0}^{\infty}\pi_{t+k}\delta^k 1_{\{k>0\}}$$

$$\widetilde{H}_t = \sum_{k=1}^{\infty}\delta^k(\pi_t+\ldots+\pi_{t+k-1}) = \sum_{j=1}^{\infty}\pi_{t+j-1}\sum_{k=j}^{\infty}\delta^k = \sum_{j=1}^{\infty}\pi_{t+j-1}\frac{\delta^j}{1-\delta} = \frac{1}{1-\delta}\sum_{j=1}^{\infty}\pi_{t+j-1}\delta^j =$$

$$= \frac{1}{1-\delta}\sum_{k=0}^{\infty}\pi_{t+k-1}\delta^k 1_{\{k>0\}}$$

Rewriting (A14),

$$p_t^* - p_t = (1-\theta\beta)\mathbb{E}_t\left[\overline{m}H_t - \omega\overline{m}\widetilde{H}_t + \overline{m}\sum_{k=0}^{\infty}\delta^k\widetilde{mc}_{t+k}\right]$$

$$= (1-\theta\beta)\mathbb{E}_t\left[\overline{m}\frac{1}{1-\delta}\sum_{k=0}^{\infty}\pi_{t+k}\delta^k 1_{\{k>0\}} - \omega\overline{m}\frac{1}{1-\delta}\sum_{k=0}^{\infty}\pi_{t+k-1}\delta^k 1_{\{k>0\}} + \overline{m}\sum_{k=0}^{\infty}\delta^k\widetilde{mc}_{t+k}\right]$$

$$= (1-\theta\beta)\mathbb{E}_t\sum_{k=0}^{\infty}\delta^k\left[\overline{m}\frac{1}{1-\delta}\pi_{t+k}1_{\{k>0\}} - \omega\overline{m}\frac{1}{1-\delta}\pi_{t+k-1}1_{\{k>0\}} + \overline{m}\widetilde{mc}_{t+k}\right]$$

$$= \mathbb{E}_t\sum_{k=0}^{\infty}\delta^k\left[\overline{m}\frac{1-\theta\beta}{1-\delta}\pi_{t+k}1_{\{k>0\}} - \omega\overline{m}\frac{1-\theta\beta}{1-\delta}\pi_{t+k-1}1_{\{k>0\}} + \overline{m}(1-\theta\beta)\widetilde{mc}_{t+k}\right]$$

(A15)

$$= \mathbb{E}_t\sum_{k=0}^{\infty}\delta^k\left[\widetilde{m}_\pi\pi_{t+k}1_{\{k>0\}} - \omega\widetilde{m}_\pi\pi_{t+k-1}1_{\{k>0\}} + \widetilde{m}_\mu\widetilde{mc}_{t+k}\right]$$

where $\widetilde{m}_\pi = \overline{m}\frac{1-\theta\beta}{1-\delta}$ and $\widetilde{m}_\mu = \overline{m}(1-\theta\beta)$. Rewriting the price evolution expression (18),

$$p_t^* - p_{t-1} + p_t - p_t = \frac{\pi_t - \theta\omega\pi_{t-1}}{1-\theta} \implies p_t^* - p_t = \frac{\theta}{1-\theta}(\pi_t - \omega\pi_{t-1})$$

Hence, we can rewrite (A15) as

$$\frac{\theta}{1-\theta}(\pi_t - \omega\pi_{t-1}) = \mathbb{E}_t \sum_{k=0}^{\infty} \delta^k \left[\widetilde{m}_\pi \pi_{t+k} 1_{\{k>0\}} - \omega \widetilde{m}_\pi \pi_{t+k-1} 1_{\{k>0\}} + \widetilde{m}_\mu \widetilde{mc}_{t+k}\right] \tag{A16}$$

Let us now introduce the forward operator F such that $F^k x_t = x_{t+k}$. Using the forward operator, we can write

$$\sum_{k=0}^{\infty} \delta^k x_{t+k} = \sum_{k=0}^{\infty} \delta^k F^k x_t = \sum_{k=0}^{\infty} (\delta F)^k x_t = \frac{x_t}{1-\delta F} \tag{A17}$$

Rewriting (A16) using (A17)

$$\begin{aligned}
\frac{\theta}{1-\theta}(\pi_t - \omega\pi_{t-1}) &= \widetilde{m}_\pi \mathbb{E}_t \left[\sum_{k=0}^{\infty} \delta^k \pi_{t+k} 1_{\{k>0\}}\right] - \omega \widetilde{m}_\pi \mathbb{E}_t \left[\sum_{k=0}^{\infty} \delta^k \pi_{t+k-1} 1_{\{k>0\}}\right] + \widetilde{m}_\mu \mathbb{E}_t \left[\sum_{k=0}^{\infty} \delta^k \widetilde{mc}_{t+k}\right] \\
&= \widetilde{m}_\pi \mathbb{E}_t \left[\sum_{k=0}^{\infty} (\delta F)^k \pi_t 1_{\{k>0\}}\right] - \omega \widetilde{m}_\pi \mathbb{E}_t \left[\sum_{k=0}^{\infty} (\delta F)^k \pi_{t-1} 1_{\{k>0\}}\right] + \widetilde{m}_\mu \mathbb{E}_t \left[\sum_{k=0}^{\infty} (\delta F)^k \widetilde{mc}_t\right] \\
&= \widetilde{m}_\pi \mathbb{E}_t \left[\sum_{k=0}^{\infty} (\delta F)^k \pi_t - \pi_t\right] - \omega \widetilde{m}_\pi \mathbb{E}_t \left[\sum_{k=0}^{\infty} (\delta F)^k \pi_{t-1} - \pi_{t-1}\right] + \widetilde{m}_\mu \mathbb{E}_t \left[\sum_{k=0}^{\infty} (\delta F)^k \widetilde{mc}_t\right] \\
&= \widetilde{m}_\pi \mathbb{E}_t \left[\frac{\pi_t}{1-\delta F} - \pi_t\right] - \omega \widetilde{m}_\pi \mathbb{E}_t \left[\frac{\pi_{t-1}}{1-\delta F} - \pi_{t-1}\right] + \widetilde{m}_\mu \mathbb{E}_t \left[\frac{\widetilde{mc}_t}{1-\delta F}\right] \\
&= \widetilde{m}_\pi \mathbb{E}_t \left[\frac{\delta F \pi_t}{1-\delta F}\right] - \omega \widetilde{m}_\pi \mathbb{E}_t \left[\frac{\delta F \pi_{t-1}}{1-\delta F}\right] + \widetilde{m}_\mu \mathbb{E}_t \left[\frac{\widetilde{mc}_t}{1-\delta F}\right]
\end{aligned}$$

Premultiplying by $(1-\delta F)$,

$$\frac{\theta}{1-\theta}(1-\delta F)(\pi_t - \omega\pi_{t-1}) = \widetilde{m}_\pi \mathbb{E}_t \left[\delta F \pi_t\right] - \omega \widetilde{m}_\pi \mathbb{E}_t \left[\delta F \pi_{t-1}\right] + \widetilde{m}_\mu \mathbb{E}_t \left[\widetilde{mc}_t\right]$$

which can be rearranged to (19). Let us now derive the Behavioural Hybrid New Keynesian Phillips curve. We have the following expressions

$$mc_t = w_t - p_t - a_t \tag{A18}$$

$$y_t = a_t + n_t \tag{A19}$$

$$w_t - p_t = \varphi n_t + \frac{\sigma}{1-h} c_t - \frac{\sigma h}{1-h} c_{t-1} \tag{A20}$$

$$c_t = y_t \tag{A21}$$

Hence, we can write

$$
\begin{aligned}
mc_t &= w_t - p_t - a_t \\
&= \varphi n_t + \frac{\sigma}{1-h} c_t - \frac{\sigma h}{1-h} c_{t-1} - a_t \\
&= \varphi(y_t - a_t) + \frac{\sigma}{1-h} c_t - \frac{\sigma h}{1-h} c_{t-1} - a_t \\
&= \varphi(y_t - a_t) + \frac{\sigma}{1-h} y_t - \frac{\sigma h}{1-h} y_{t-1} - a_t \\
&= \left(\varphi + \frac{\sigma}{1-h}\right) y_t - \frac{\sigma h}{1-h} y_{t-1} - (1+\varphi) a_t
\end{aligned}
$$

In the natural equilibrium (with no price frictions), the marginal cost is constant at its steady-state level

$$
mc_t^r = mc = -\mu = \left(\varphi + \frac{\sigma}{1-h}\right) y_t^n - \frac{\sigma h}{1-h} y_{t-1}^n - (1+\varphi) a_t
$$

hence, we can write

$$
\widehat{mc}_t = mc_t - mc = \left(\varphi + \frac{\sigma}{1-h}\right) \tilde{y}_t - \frac{\sigma h}{1-h} \tilde{y}_{t-1}
$$

which, inserted into the (19), yields the Behavioural Hybrid New Keynesian Phillips curve (21).

A.6. Intrinsic Myopia

Consider instead a framework without habit formation nor price indexation, but with the following belief formation process. For any variable $\widehat{x}_t$, the BR forecast of such variable at horizon h is instead given by

$$
\mathbb{E}_t^B \widehat{x}_{t+h} = \overline{m}^h \mathbb{E}_t \widehat{x}_{t+h} + (1 - \overline{m}^h) \widehat{x}_{t-1}
$$

for $h > 0$, which implies that expectations of objects more in the future are more anchored to the past.

Let us first derive the DIS curve. Starting from (A8) without habit formation, we can write

$$
\widehat{c}_t = -\frac{1}{\sigma}(i_t - \rho - \mathbb{E}_t^B \pi_{t+1}) + \mathbb{E}_t^B \widehat{c}_{t+1} = -\frac{1}{\sigma}\left[i_t - \rho - \overline{m}\mathbb{E}_t \pi_{t+1} - (1-\overline{m})\pi_{t-1}\right] + \overline{m}\mathbb{E}_t \widehat{c}_{t+1} + (1-\overline{m})\widehat{c}_{t-1}
$$

and we can finally write the intrinsic myopia DIS curve as

$$\widetilde{y}_t = -\frac{1}{\sigma}\left[\widehat{i}_t - \overline{m}\mathbb{E}_t\pi_{t+1} - (1-\overline{m})\pi_{t-1} - r_t^n\right] + \overline{m}\mathbb{E}_t\widetilde{y}_{t+1} + (1-\overline{m})\widetilde{y}_{t-1}.$$

Let us move to the supply side. Rewriting condition (16) without price indexation, but with intrinsic myopia,

$$\begin{aligned}
p_t^* - p_{t-1} &= (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k\mathbb{E}_t^B\widetilde{\mathrm{mc}}_{t+k} + \sum_{k=0}^{\infty}(\beta\theta)^k\mathbb{E}_t^B\pi_{t+k} + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k\mathbb{E}_t^B\widehat{\mu}_{t+k} \\
&= \frac{\beta\theta(1-\overline{m})}{1-\beta\theta\overline{m}}\widetilde{\mathrm{mc}}_{t-1} + \frac{\beta\theta(1-\overline{m})}{(1-\beta\theta)(1-\beta\theta\overline{m})}\pi_{t-1} + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta\overline{m})^k\mathbb{E}_t\widetilde{\mathrm{mc}}_{t+k} + \sum_{k=0}^{\infty}(\beta\theta\overline{m})^k\mathbb{E}_t\pi_{t+k}
\end{aligned}$$

where we have used $\sum_{k=0}^{\infty}(\beta\theta)^k\mathbb{E}_t^B\widehat{x}_{t+h} = \sum_{k=0}^{\infty}(\beta\theta\overline{m})^k\mathbb{E}_t\widehat{x}_{t+k} + \frac{\beta\theta(1-\overline{m})}{(1-\beta\theta)(1-\beta\theta\overline{m})}\widehat{x}_{t-1}$. If we take out past and present elements of each summation operator, the equation can be written more compactly as a difference equation,

$$\begin{aligned}
p_t^* - p_{t-1} &= \frac{\beta\theta(1-\overline{m})}{1-\beta\theta\overline{m}}\widetilde{\mathrm{mc}}_{t-1} + \frac{\beta\theta(1-\overline{m})}{(1-\beta\theta)(1-\beta\theta\overline{m})}\pi_{t-1} + (1-\beta\theta)\widetilde{\mathrm{mc}}_t + \pi_t \\
&+ (1-\beta\theta)\sum_{k=1}^{\infty}(\beta\theta\overline{m})^k\mathbb{E}_t\widetilde{\mathrm{mc}}_{t+k} + \sum_{k=1}^{\infty}(\beta\theta\overline{m})^k\mathbb{E}_t\pi_{t+k} \\
&= \frac{\beta\theta(1-\overline{m})}{1-\beta\theta\overline{m}}\widetilde{\mathrm{mc}}_{t-1} + \frac{\beta\theta(1-\overline{m})}{(1-\beta\theta)(1-\beta\theta\overline{m})}\pi_{t-1} + (1-\beta\theta)\widetilde{\mathrm{mc}}_t + \pi_t \\
&+ \beta\theta\overline{m}\left[(1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta\overline{m})^k\mathbb{E}_t\widetilde{\mathrm{mc}}_{t+k+1} + \sum_{k=0}^{\infty}(\beta\theta\overline{m})^k\mathbb{E}_t\pi_{t+k+1}\right] \\
&= \frac{\beta\theta(1-\overline{m})}{1-\beta\theta\overline{m}}\widetilde{\mathrm{mc}}_{t-1} + \frac{\beta\theta(1-\overline{m})}{(1-\beta\theta)(1-\beta\theta\overline{m})}\pi_{t-1} + (1-\beta\theta)\widetilde{\mathrm{mc}}_t + \pi_t \\
&+ \beta\theta\overline{m}\left[\mathbb{E}_t(p_{t+1}^* - p_t) - \frac{\beta\theta(1-\overline{m})}{1-\beta\theta\overline{m}}\widetilde{\mathrm{mc}}_t - \frac{\beta\theta(1-\overline{m})}{(1-\beta\theta)(1-\beta\theta\overline{m})}\pi_t\right] \\
&= \beta\theta\overline{m}\mathbb{E}_t(p_{t+1}^* - p_t) + \left[1-\beta\theta-\beta\theta\overline{m}\frac{\beta\theta(1-\overline{m})}{1-\beta\theta\overline{m}}\right]\widetilde{\mathrm{mc}}_t + \left[1-\beta\theta\overline{m}\frac{\beta\theta(1-\overline{m})}{(1-\beta\theta)(1-\beta\theta\overline{m})}\right]\pi_t \\
&+ \frac{\beta\theta(1-\overline{m})}{1-\beta\theta\overline{m}}\widetilde{\mathrm{mc}}_{t-1} + \frac{\beta\theta(1-\overline{m})}{(1-\beta\theta)(1-\beta\theta\overline{m})}\pi_{t-1}
\end{aligned}$$

Inserting the aggregate price dynamics (18), we can write

$$\frac{1}{1-\theta}\pi_t = \frac{\beta\theta\overline{m}}{1-\theta}\mathbb{E}_t\pi_{t+1} + \left[1-\beta\theta-\beta\theta\overline{m}\frac{\beta\theta(1-\overline{m})}{1-\beta\theta\overline{m}}\right]\widetilde{\mathrm{mc}}_t + \left[1-\beta\theta\overline{m}\frac{\beta\theta(1-\overline{m})}{(1-\beta\theta)(1-\beta\theta\overline{m})}\right]\pi_t$$

$$+\frac{\beta\theta(1-\overline{m})}{1-\beta\theta\overline{m}}\widetilde{\mathrm{mc}}_{t-1}+\frac{\beta\theta(1-\overline{m})}{(1-\beta\theta)(1-\beta\theta\overline{m})}\pi_{t-1}$$

rearranging elements,

$$\begin{aligned}
\pi_t &= \frac{\overline{m}\beta(1-\beta\theta)(1-\beta\theta\overline{m})}{1-\beta\theta\{1+\overline{m}[1-\beta+\beta\overline{m}(1-\theta)]\}}\mathbb{E}_t\pi_{t+1}+\frac{(1-\theta)(1-\beta\theta)\{1-\beta\theta[1+\overline{m}(1-\beta\theta\overline{m})]\}}{\theta\{1-\beta\theta[1+\overline{m}-\beta\overline{m}(1-\overline{m})]+\overline{m}^2\beta^2\theta^2\}}\widetilde{\mathrm{mc}}_t\\
&+\frac{(1-\overline{m})\beta(1-\beta\theta)(1-\theta)}{1-\beta\theta\{1+\overline{m}[1-\beta+\beta\overline{m}(1-\theta)]\}}\widetilde{\mathrm{mc}}_{t-1}+\frac{(1-\overline{m})\beta(1-\theta)}{1-\beta\theta\{1+\overline{m}[1-\beta+\beta\overline{m}(1-\theta)]\}}\pi_{t-1}\\
&= \frac{\overline{m}\beta(1-\beta\theta)(1-\beta\theta\overline{m})}{1-\beta\theta\{1+\overline{m}[1-\beta+\beta\overline{m}(1-\theta)]\}}\mathbb{E}_t\pi_{t+1}+\frac{(1-\theta)(1-\beta\theta)\{1-\beta\theta[1+\overline{m}(1-\beta\theta\overline{m})]\}}{\theta\{1-\beta\theta[1+\overline{m}-\beta\overline{m}(1-\overline{m})]+\overline{m}^2\beta^2\theta^2\}}(\widehat{\mathrm{mc}}_t+\widehat{\mu}_t)\\
&+\frac{(1-\overline{m})\beta(1-\beta\theta)(1-\theta)}{1-\beta\theta\{1+\overline{m}[1-\beta+\beta\overline{m}(1-\theta)]\}}(\widehat{\mathrm{mc}}_{t-1}+\widehat{\mu}_{t-1})+\frac{(1-\overline{m})\beta(1-\theta)}{1-\beta\theta\{1+\overline{m}[1-\beta+\beta\overline{m}(1-\theta)]\}}\pi_{t-1}
\end{aligned}$$

Finally, introducing (20) without habit formation, we can write the intrinsic myopia NK Phillips curve,

$$\begin{aligned}
\pi_t &= \frac{\overline{m}\beta(1-\beta\theta)(1-\beta\theta\overline{m})}{1-\beta\theta\{1+\overline{m}[1-\beta+\beta\overline{m}(1-\theta)]\}}\mathbb{E}_t\pi_{t+1}+\frac{(1-\theta)(1-\beta\theta)\{1-\beta\theta[1+\overline{m}(1-\beta\theta\overline{m})]\}}{\theta\{1-\beta\theta[1+\overline{m}-\beta\overline{m}(1-\overline{m})]+\overline{m}^2\beta^2\theta^2\}}[(\sigma+\varphi)\widetilde{y}_t+\widehat{\mu}_t]\\
&+\frac{(1-\overline{m})\beta(1-\beta\theta)(1-\theta)}{1-\beta\theta\{1+\overline{m}[1-\beta+\beta\overline{m}(1-\theta)]\}}[(\sigma+\varphi)\widetilde{y}_{t-1}+\widehat{\mu}_{t-1}]+\frac{(1-\overline{m})\beta(1-\theta)}{1-\beta\theta\{1+\overline{m}[1-\beta+\beta\overline{m}(1-\theta)]\}}\pi_{t-1}
\end{aligned}$$

A.7. Robustness Checks

	Prior Distribution		Posterior Distribution			
		Mean (S.d)	**GDP Deflator**	**Estimating $\overline{m}$**	**1985:I-2007:III**	**1955:I-2007:III**
β	*Beta*	0.99	0.990	0.990	0.990	0.990
		(0.001)	(0.989, 0.992)	(0.988, 0.992)	(0.988, 0.992)	(0.989, 0.992)
σ	*Normal*	1.50	1.171	1.151	1.251	1.249
		(0.37)	(0.599, 1.703)	(0.578, 1.697)	(0.652, 1.809)	(0.678, 1.789)
φ	*Normal*	2	1.435	1.423	1.419	1.442
		(0.75)	(0.501, 2.285)	(0.500, 2.278)	(0.500, 2.275)	(0.500, 2.289)
ϕ_π	*Normal*	1.50	1.426	1.355	1.392	1.346
		(0.15)	(1.203, 1.644)	(1.136, 1.569)	(1.145, 1.646)	(1.140, 1.545)
ϕ_y	*Normal*	0.15	0.282	0.276	0.286	0.333
		(0.10)	(0.165, 0.398)	(0.159, 0.391)	(0.128, 0.450)	(0.222, 0.443)
θ	*Beta*	0.50	0.913	0.883	0.875	0.909
		(0.10)	(0.883, 0.944)	(0.841, 0.927)	(0.827, 0.920)	(0.879, 0.939)
h	*Beta*	0.70	0.650	0.651	0.656	0.665
		(0.10)	(0.518, 0.787)	(0.512, 0.790)	(0.508, 0.807)	(0.534, 0.795)
ω	*Beta*	0.50	0.140	0.569	0.269	0.781
		(0.15)	(0.048, 0.232)	(0.108, 0.880)	(0.114, 0.416)	(0.708, 0.857)
$\overline{m}$	*Implied*	—	0.410	0.435	0.401	0.440
		—	(—)	(0.208, 0.669)	(0.166, 0.608)	(—)
ρ_i	*Beta*	0.50	0.829	0.839	0.934	0.861
		(0.20)	(0.778, 0.886)	(0.794, 0.886)	(0.906, 0.962)	(0.827, 0.894)
ρ_d	*Beta*	0.50	0.704	0.702	0.742	0.696
		(0.20)	(0.608, 0.802)	(0.598, 0.810)	(0.626, 0.861)	(0.608, 0.785)
ρ_s	*Beta*	0.50	0.835	0.368	0.190	0.068
		(0.20)	(0.759, 0.913)	(0.032, 0.828)	(0.044, 0.333)	(0.010, 0.121)
ρ_{e_i}	*Beta*	0.50	0.223	0.182	0.573	0.167
		(0.20)	(0.095, 0.349)	(0.067, 0.293)	(0.421, 0.724)	(0.069, 0.263)
σ_d	*Inv. gamma*	0.10	0.514	0.496	0.292	0.489
		(∞)	(0.459, 0.566)	(0.374, 0.621)	(0.218, 0.366)	(0.445, 0.533)
σ_s	*Inv. gamma*	0.10	0.193	0.301	0.328	0.323
		(∞)	(0.172, 0.214)	(0.228, 0.375)	(0.274, 0.378)	(0.294, 0.349)
σ_i	*Inv. gamma*	0.10	0.243	0.237	0.081	0.207
		(∞)	(0.219, 0.267)	(0.215, 0.261)	(0.071, 0.091)	(0.189, 0.223)
	Log data density		–230.971	–278.413	–24.124	–354.726

Note: Results are reported at the posterior mean. 90% confidence intervals in parenthesis. The model-implied forecast-underrevision coefficient is 1.288 (column 4), 0.805 (column 5), 0.287 (column 6) and 1.182 (column 7). In columns 5 and 6, which directly estimate $\overline{m}$, we assume a prior Beta distribution with mean (S.d.) of 0.50 (0.15).

TABLE A1. Estimated Structural Parameters: Robustness Checks