Reconciling Empirics and Theory: The Behavioural New Keynesian model*

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November 12, 2018

Abstract

Structural estimates of the standard New Keynesian model are at odds with microeconomic estimates. To reconcile these findings, we develop and estimate a behavioral New Keynesian model that includes *keeping up with the Joneses* households and backward-looking firms. We find (i) strong evidence for bounded rationality, which motivates our departure from full rationality; (ii) that the behavioral setting reconciles the New Keynesian theory with empirical studies; (iii) we estimate the cognitive discount factor to be between 0.97 and 0.99 at a quarterly frequency; and (iv) consistent with the theory, the degree of myopia increases when we drop the additional behavioral assumptions, yet remains significantly different from the rational case.

Keywords: New Keynesian, bounded rationality, GMM.
JEL Classifications: E27, E52, E71.

*We would like to thank Xavier Gabaix, Mark Gertler, Per Krusell, Jesper Lindé, Lars E.O. Svensson, Jörgen Weibull and seminar participants at IIES and the Stockholm School of Economics for useful feedback and comments.

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1 Introduction

> *"Despite the advances in theoretical modeling, accompanying econometric analysis of the 'new Phillips curve' has been rather limiting [...]. The work to date has generated some useful findings, but these findings have raised some troubling questions about the existing theory."*
>
> J. Galí and M. Gertler, *Inflation dynamics: A structural econometric analysis (1999).*

The most convenient aspect of the standard New Keynesian (NK) model is that it can be summarized in a system of three difference equations: the Dynamic IS curve, the Phillips curve and a reaction function that governs the interest rate path. Every slope in these curves is a combination of different parameters in the model, namely the discount factor, the degree of risk aversion, the Frisch elasticity, the Calvo-style probability of being able to reset prices, etc. As a result, by estimating the slopes of the final system of equations one can retrieve the structural parameters of the model. However, when the monetary economics literature performed such analyses, estimated parameters were at odds with microeconometric studies. We try to overcome this conflict using a Behavioral NK model inspired by the work of Gabaix (2016a).

Reconciling the NK theory with the data has proved to be a difficult exercise. Gali and Gertler (1999) focus only on the supply side of the model and estimate two different (micro-founded) NK Phillips curves: the standard curve (NKPC) and the Hybrid curve (HNKPC). Each coefficient in the NKPC and HNKPC is a combination of parameters in the model: the subjective discount factor, the Calvo probability of changing prices and (in the case of the HNKPC) the share of backward-looking firms. However, they find a subjective discount factor estimate that is excessively low (0.90 at a quarterly frequency) and a price change frequency estimate that is excessively low compared to survey evidence (Nakamura and Steinsson 2008).

To reconcile these differences between empirics and theory, we build a Behavioral NK model, similar in spirit to the one described in Gabaix (2016a), that additionally includes *habit persistent* households and backward-looking firms.[1] A key feature of our model is that it can be easily reduced to the one described in Gabaix (2016a) by turning off two key parameters, the degree of habit persistence and the share of backward-looking firms. As a result, our model nests Gabaix (2016a) and allows us to estimate Gabaix (2016a) Behavioral NK setting. Importantly, because the behavioral model now includes an additional parameter with respect to the standard NK setting, the cognitive discount factor, we cannot only estimate the supply side

[1]These ingredients are necessary to obtain hump-shaped IRFs in the theoretical model, which is what we observe in empirical research (see (Christiano et al. 2005)).

of the economy as Gali and Gertler (1999) do (or, to be concise, we would benefit from a degree of freedom.) Therefore, we contribute to the literature by additionally estimating the Behavioral Dynamic IS (DIS) curve.

Our contribution to the literature is twofold. First, we extend the standard behavioral NK setting to allow for habit persistence and backward-looking firms. Second, we identify all the structural parameters behind the coefficients in the behavioral DIS and hybrid NK Phillips curves. In doing so, we reconcile two key parameters in the theory that were at odds with empirical evidence: the subjective discount factor and the average duration that an individual price is fixed (degree of 'stickyness'). Additionally, we are the first up to our knowledge to estimate the cognitive discount factor (the key bounded rationality (BR) parameter) in a NK setting. Importantly, we find statistical evidence for rational inattentive behavior, deviating from commonly assumed fully rational behavior.

We find that all key variables (marginal costs, output gap and inflation) are statistically significant and quantitatively important, as the standad and behavioral versions of the theory suggest. We also find that, although forward-looking behavior is the main driver of the model, backward-looking household and firm behavior is non negligible. In the household case, this behavior drives up to 46% of the fluctuations in the output gap.

We include backward-looking firms in the light of Gali and Gertler (1999). We do so in order to obtain a hybrid NK Phillips curve that is closer to empirical evidence, in the sense that it also includes lags of inflation. The motivation for this is mostly empirical, since previous studies have found the inflation equation to be largely inertial.

We include household habit persistence in the light of Christiano et al. (2005) and Blanchard et al. (2015). Christiano et al. (2005) find a quantitatively important degree of household habit persistence for the US. Most importantly, they show that including habit persistence is critical to obtain hump-shaped impulse responses in the model, as the empirical VAR literature has observed. Given that our intention is to build a model that is close to data in order to consistently estimate the behavioral DIS curve, we follow their approach. To further motivate the addition of behavioral households, Blanchard et al. (2015) find that the degree of habit persistence is significant and even greater in Europe.

The paper proceeds as follows. In section 2 we cover a literature review on behavioral monetary economics. In sections 3 and 4 we set up the model, which derivation is delegated to the Appendix due to its algebraic complexity. In section 5 we estimate the model structural parameters using the full model. Interestingly for the rational inattention literature, we estimate the BR parameter. Section 7 concludes.

2 Literature Review

The literature on the New Keynesian monetary policy analysis can be categorized under four approaches. The first approach is to use the new Keynesian Phillips curve, which is based on the dynamic price setting under rational expectation models of Taylor (1980), Rotemberg (1982), and Calvo (1983). Under this approach, new Keynesian Phillips curve is used to understand the effects of monetary policy shocks on the firm side. Goodfriend and King (1997) and Clarida et al. (1999) can be seen as examples of this approach. The major drawback of this model is that it does not explain the inertia in inflation that we observe empirically.

Several approaches have been since proposed to match with the empirics better. Another common approach is to use the accelerationist Phillips curve, where firms are assumed to be backward-looking when forming expectations. Typical examples of this strand are Svensson (1997) and Ball (1999).

The third approach, which is one of the inspirations for this paper, can be seen as the hybrid Phillips curve of Gali and Gertler (1999), where a combination of backward-looking and rational expectation forming firms are assumed. Besides failing to address the problems of the previous two approaches, this model yields parameter value predictions that differ from what microeconomic.

The behavioral approach is the fourth and the final category of this literature. The main idea of this approach is that both the households and the firms can be less than fully rational or fully informed, which might explain the observed features of the economic conditions with solid microfoundations instead of high level adhoc assumptions of the previous models. The main strand, the one that our paper belongs to, of the behavioral approach is to use information as the limited resource that keeps agents from the fully rational path. Although information economics has a rich history, Sims (2003) can be seen as one of the earlier seminal works that takes this approach in the context of monetary politics, and there has been a rich literature built since then. Our work directly builds upon Gabaix (2016b), where he sets up a behavioral new Keynesian economy by endowing agents the sparsity-based model of bounded rationality of Gabaix (2014), which is basically a myopia for the information of the past. The details of the setup is explained in the following chapters.

Another theoretical paper that assumes limited information acquisition in order to explain the real effect of monetary policy shocks is Maćkowiak and Wiederholt (2009). In this model of firm side economy the firms have limited access to available information, which is modeled as a constraint on the information flow. Given the exogenous stochastic paths of nominal aggregate demand and idiosyncratic state variables, which reflects firm-specific conditions, firms face a trade-off and endogenously choose how to allocate their limited attention between these two. Different from the Calvo pricing of our model, they allow firms to set prices at every period.

Assuming that the idiosyncratic conditions are more volatile, the model posits that firms optimally pay more attention to them instead of the macroeconomic conditions, and as a result of this optimizing behavior, prices react slower and by smaller amounts to the nominal shocks. This model has three predictions: impulse response of prices are sticky, real volatility increases as nominal shocks become larger, and the Phillips curve, consistent with Lucas (1973), becomes steeper as the variance of nominal aggregate demand increases. The main shortcoming of this work is that it fails to explain why prices might be fixed for some period, as the model predicts that prices change in every period.

Mankiw and Reis (2002) and Ball et al. (2005) also suggest to replace the new Keynesian Phillips curve by positing a model of slow information dissemination. Similar to Maćkowiak and Wiederholt (2009), they also assume fully flexible price adjustments, but now instead of sticky prices they suggest to use sticky information. In a similar fashion with Calvo pricing, in each period with some probability a firm gets perfect information about the current state of the economy and sets the optimal prices based on this information, while with the residual probability the firm uses old information and thus sets the prices based on the expectations formed in whenever they had access to the information last. As a result, current price levels in the sticky-information Phillips curve depends on the expectation of current price levels formed in the past, as opposed to the current expectations of the future variables in the standard new Keynesian Phillips curve. The model displays three properties of how the monetary policy works. Firstly disinflations are always contractionary, although surprise ones have larger effects. Secondly, monetary policy shocks have their maximum impact on inflation with a substantial delay. And lastly the change in inflation is positively correlated with the level of economic activity.

Woodford (2003) also has a similar approach, but instead of probabilistic access to perfect information, the firms receives noisy signals about the current state of the economy in each period. For a model of costly information acquisition, see Moscarini (2004). Lastly Ball (2000) suggest an ad hoc behavioral approach in order to capture different inflation inertia properties under different monetary regimes. He suggests a model where agents optimally use all the past information for a variable to form expectations, such that the expectations are based on optimal univariate forecasts, and they fail to incorporate information from other variables.

3 Bounded Rationality Basics

Before setting the model up, let us briefly mention the type of BR that both households and firms exhibit in our model. We follow Gabaix (2016b), which itself follows the sparsity-based model of BR first developed for static setting (Gabaix (2014)), and then extended for dynamic settings (Gabaix (2016a)). In the model, agents have some ex-ante perception of their environment, modeled as default values for each parameter that constitutes the environment. Upon facing an optimization problem, the agents first adjust their perception of the world by endogenously allocating their limited attention to the dimensions of the world that they expect to be most relevant and impactful for their decision in the first order. As a result of this limited attention, they partially adjust the true values of the parameters, proportional to the limited attention for those parameters. Finally the agents solve the usual maximization problem under this simplified perception of the environment, which is defined as the smax operator. In a dynamic macroeconomic setting, this induces myopia to the changes in the stochastic variables from their default values, which might be thought of as their steady state values.

The model is flexible in the sense that by assuming different default values and using different cost of attention it is possible to capture wide variety of behavioral phenomena. Gabaix (2017) provides an extensive review of different behavioral inattention models and various behavioral phenomena which might be explained by using these models. We have explicitly made two additional behavioral assumptions that can be explained through informational frictions: habit formation on the demand side and backward-looking firms on the supply side. These assumptions in our canonical model are made for two reasons. The first one, as mentioned above, is to reconcile empirics with theory. The second reason is to capture the effects of household habit formation and backward-looking firms on the estimated cognitive discount factor, which is the key variable that shows the degree of myopia. In section 5 we show that, consistent with the theory, the degree of myopia increases when we drop the additional behavioral assumptions, yet remains significantly different than full rational case.

The following are the assumptions that we use to apply behavioral inattention à la Gabaix. There is a state vector X_t (comprising TFP shocks A_t as well as announced actions in monetary policy), that will evolve in equilibrium as

$$X_{t+1} = G^X(X_t, \epsilon_{t+1})$$

for some equilibrium transition function G^X and mean zero innovations ϵ_{t+1}.

We assume that X_t has mean zero. Linearizing, the law of motion becomes

$$X_{t+1} = \Gamma X_t + \epsilon_{t+1}$$

for some matrix Γ.

The main assumption is the following (as in Gabaix (2016a)):

Assumption 1. *The agent perceives that the state vector evolves as*

$$X_{t+1} = \overline{m} G^X(X_t, \epsilon_{t+1})$$

where $\overline{m} \in [0,1]$ *is a cognitive discounting parameter measuring the attention to the future.*

Linearizing,

$$X_{t+1} = \overline{m}(\Gamma X_t + \epsilon_{t+1})$$

Hence, the expectation of the behavioural agent is $\mathbb{E}_t^{BR}(X_{t+1}) = \overline{m}\Gamma X_t$, and iterating, $\mathbb{E}_t^{BR}(X_{t+k}) = \overline{m}^k \Gamma^k X_t$. The rational expectation is $\mathbb{E}_t(X_{t+1}) = \Gamma X_t$, and iterating, $\mathbb{E}_t(X_{t+1}) = \Gamma^k X_t$. Hence, we can make use of the following equivalence

$$\mathbb{E}_t^{BR}(X_{t+k}) = \overline{m}^k \mathbb{E}_t(X_{t+k})$$

Lemma 1. *For any variable* $z(X_t)$ *with* $z(0) = 0$*, the beliefs of the behavioural agent satisfy, for all* $k \geq 0$*, and linearizing*

$$\mathbb{E}_t^{BR}[z(X_{t+k})] = \overline{m}^k \mathbb{E}_t[z(X_{t+k})]$$

4 The Behavioural Agents and Firms Setting

4.1 Households

There is a continuum of atomistic households with identical preferences over expected lifetime utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{(C_t - h\overline{C}_{t-1})^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right\}$$

where C_t is consumption, N_t is labor supply, $\overline{C}_{t-1}$ denotes the average consumption level in the economy (which is taken as given by the individual household), σ is the intertemporal elasticity of substitution and $1/\varphi$ is the Frisch elasticity. When $h > 0$ the household derives utility from consumption relative to some reference level. Here, it is assumed that the reference level is (a fraction of) the past average consumption level in the economy, i.e., the utility function exhibits a *keeping up with the Joneses* element.[2] The household's budget constraint is given by

$$P_t C_t + Q_t B_t = B_{t-1} + W_t N_t + T_t$$

where P_t is the price of the consumption good, B_t stands for bond holdings at the household, Q_t is the price of each bond, W_t is the wage rate for each unit of labor

[2]The consequences of such assumption are similar to assuming habit persistence, albeit simplifying computation.

supply and T_t are transfers to households. We show in Appendix A that the demand for good i is given by

$$Y_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\epsilon} Y_t \tag{1}$$

where $Y_t = C_t$ (since we are in a representative household economy), and the aggregate price index P_t is given by

$$P_t = \left[\int_0^1 P_t(i)^{1-\epsilon} di\right]^{\frac{1}{1-\epsilon}}$$

which is also derived in Appendix A. The rational household optimizing conditions are

$$\frac{W_t}{P_t} = \frac{N_t^{\varphi}}{(C_t - h\overline{C}_{t-1})^{-\sigma}} \tag{2}$$

$$Q_t = \beta \mathbb{E}_t \left[\left(\frac{C_{t+1} - h\overline{C}_t}{C_t - h\overline{C}_{t-1}}\right)^{-\sigma} \frac{P_t}{P_{t+1}}\right] \tag{3}$$

Notice that, since households are identical, we can impose $C_t = \overline{C}_t \ \forall t$.[3] Let us now focus on the behavioural household. We can write the conditions as

$$\frac{W_t}{P_t} = \frac{N_t^{\varphi}}{(C_t - hC_{t-1})^{-\sigma}} \tag{4}$$

$$Q_t = \beta \mathbb{E}_t^{BR} \left\{\left[\frac{C(X_{t+1}) - hC(X_t)}{C(X_t) - hC(X_{t-1})}\right]^{-\sigma} \frac{P(X_t)}{P(X_{t+1})}\right\} \tag{5}$$

As one can see, the labor supply condition is kept unchanged: since it is an intratemporal condition, expectations (and thus BR) plays no role here. On the other hand, the Euler condition now has a different expectation operator. Importantly, fully rational and BR households do not differ in intratemporal actions, but in their perception of the future. The log-linearized version of both optimality conditions, derived in Appendix B, is

$$\hat{w}_t - \hat{p}_t = \varphi \hat{n}_t + \frac{\sigma}{1-h}\hat{c}_t - \frac{\sigma h}{1-h}\hat{c}_{t-1} \tag{6}$$

$$\hat{c}_t = \frac{h}{1+h}\hat{c}_{t-1} + \frac{1}{1+h}\overline{m}\mathbb{E}_t\hat{c}_{t+1} - \frac{1-h}{\sigma(1+h)}\left(\hat{i}_t - \overline{m}\mathbb{E}_t\pi_{t+1}\right) \tag{7}$$

where a hat on top of a variable denotes the log deviation from steady-state. Here we have made use of the BR assumptions described in the previous section, setting

[3]This will prove to be of use in the log-linearization of the model.

$\mathbb{E}_t^{BR}\widehat{c}_{t+1} = \overline{m}\mathbb{E}_t\widehat{c}_{t+1}$ and $\mathbb{E}_t^{BR}\pi_{t+1} = \overline{m}\mathbb{E}_t\pi_{t+1}$. The Euler condition (7) can be rewritten in terms of the output gap as

$$\tilde{y}_t = \lambda_b \tilde{y}_{t-1} + \lambda_f \mathbb{E}_t \tilde{y}_{t+1} + \lambda_r \left(i_t - \overline{m}\mathbb{E}_t \pi_{t+1} - r_t^n\right) \tag{8}$$

where $\lambda_b = \frac{h}{1+h}$, $\lambda_f = \frac{1}{1+h}\overline{m}$, $\lambda_r = -\frac{1-h}{\sigma(1+h)}$; and r_t^n is the natural interest rate. The BRDIS departs from the standard DIS in several dimensions. First, $\lambda_b > 0$ accounts for habit persistence in the household side: household optimal consumption is anchored to a predetermined level. Second, $\lambda_b(\overline{m}) > 0$ reflects that household expectations are both anchored to the predetermined level and dampened by myopia. Finally, money illusion acts through the cognitive discount factor attached to expected inflation.

4.2 Firms

There is a continuum of firms, each producing a different type of good. Good i is produced by a monopolistic firm with technology

$$Y_t(i) = A_t N_t(i) \tag{9}$$

Given $Y_t = C_t$ and $Y_t(i) = C_t(i)$,[4] we know that the final good is produced competitively in quantity Y_t

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

and its price is

$$P_t = \left[\int_0^1 P_t(i)^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}}$$

Prices are set subject to Calvo-style friction, i.e., in each period a firm is only allowed to reset its price with probability $1-\theta$. However –and departing from the standard NK setting–, it is assumed that when a firm is unable to reoptimize, its price is partially indexed to past inflation, i.e.,

$$P_t(i) = P_{t-1}(i)\Pi_{t-1}^{\omega} \tag{10}$$

where $\Pi_t = \frac{P_t}{P_{t-1}}$ is the gross rate of inflation and ω is the elasticity of prices with respect to past inflation.[5] As a result, a firm that last reset its price in period t will

[4]No firm will choose to produce more that what is demanded.

[5]This assumption is equivalent to the more ad-hoc derivation of backward-looking firms in Gali and Gertler (1999). However, since firms are identical, its consequences are equivalent: ω could be interpreted as the share of backward-looking firms.

in period $t+k$ have a nominal price of $P_t^* \chi_{t,t+k}$, where

$$\chi_{t,t+k} = \begin{cases} \Pi_t^{\omega} \Pi_{t+1}^{\omega} \Pi_{t+2}^{\omega} \cdots \Pi_{t+k-1}^{\omega} & \text{if } k \geq 1 \\ 1 & \text{if } k = 0 \end{cases}$$

The (rational) firm's problem is to maximize its discounted profit stream

$$\mathbb{E}_0 \sum_{t=0}^{\infty} Q_t [P_t(i) Y_t(i) - W_t N_t(i)] \tag{11}$$

subject to the sequence of demand constraints (1) and technology constraints (9). We can rewrite the objective function (profits) as

$$\begin{aligned} P_t(i) Y_t(i) - W_t N_t(i) &= P_t(i) Y_t(i) - W_t \frac{Y_t(i)}{A_t} \\ &= \left[P_t(i) - \frac{W_t}{A_t} \right] Y_t(i) \\ &= [P_t(i) - P_t \text{MC}_t] \left[\frac{P_t(i)}{P_t} \right]^{-\varepsilon} Y_t \end{aligned}$$

where marginal costs are defined as $\text{MC}_t = (W_t/P_t)(\partial Y_t / \partial N_t) = W_t/(P_t A_t)$.

Consider a firm reoptimizing its price at time t. Let the firm's optimal price be denoted $P_t^*(i)$, such that in this setting at time $t+k$ its price will be $P(i)_t^* \chi_{t,t+k}$. Ignoring states in which reoptimization is allowed, its maximization program is

$$\max_{P_t^*(i)} \mathbb{E}_t \sum_{k=0}^{\infty} \theta^k Q_{t+k} [P_t^*(i) \chi_{t,t+k} - P_{t+k} \text{MC}_{t+k}] \left[\frac{P_t^*(i) \chi_{t,t+k}}{P_t} \right]^{-\epsilon} Y_{t+k}$$

which yields the first order condition,[6]

$$P_t^*(i) = \mathcal{M} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} (\theta\beta)^k (C_{t+k} - hC_{t+k-1})^{-\sigma} C_{t+k} P_{t+k}^{\epsilon} P_{t+k-1}^{-\omega\epsilon} \text{MC}_{t+k}}{\mathbb{E}_t \sum_{k=0}^{\infty} (\theta\beta)^k (C_{t+k} - hC_{t+k-1})^{-\sigma} C_{t+k} P_{t+k}^{\epsilon-1} P_{t+k-1}^{\omega(1-\epsilon)}} P_{t-1}^{\omega} \tag{12}$$

where we have used the Euler condition (5), $\chi_{t,t+k} = \left(\frac{P_{t+k-1}}{P_{t-1}} \right)^{\omega}$ and $\mathcal{M} = \frac{\epsilon}{\epsilon - 1}$, which stands for the mark-up. Notice that with flexible prices (i.e., $\theta = 0$) the optimal pricing condition (12) collapses to

$$P_t^*(i) = \mathcal{M} P_t \text{MC}_t \tag{13}$$

where (13) is the frictionless mark-up. Since all firms who get to reset are facing an identical environment (i.e., we can treat them as if they were a representative firm),

[6] A detailed derivation can be found in Appendix C.

they choose to set the same price: $P_t^*(i) = P_t^* \;\forall i$. The log-linearized version of the optimal pricing condition (12) is

$$p_t^* = p_t + (1-\theta\beta)\mathbb{E}_t \sum_{k=0}^{\infty} (\theta\beta)^k \left[(\pi_{t+1} + ... + \pi_{t+k}) - \omega(\pi_t + ... + \pi_{t+k-1}) + \widehat{\mathrm{mc}}_{t+k}\right] \tag{14}$$

where $\widehat{\mathrm{mc}}_t = \mathrm{mc}_t - \mathrm{mc} = \mathrm{mc}_t + \mu$ and $\mu = -\mathrm{mc} = -\log \mathrm{MC} = -\log \frac{1}{\mathcal{M}} = \log \mathcal{M}$. That is, a resetting firm will choose a price that corresponds to the desired mark-up over a convex combination of current and expected future prices and nominal marginal costs, in addition of prices in the previous period.

The behavioural firm faces the same problem, with a less accurate view of reality. Most importantly, the behavioural firm also perceives the future via the cognitive discounting mechanism discussed in Section 3. To be precise, we model that at time t the firm perceives the future inflation and marginal costs at date $t+k$ as

$$\mathbb{E}_t^{BR}[\pi(X_{t+k})] = \overline{m}^k \mathbb{E}_t[\pi(X_{t+k})]$$
$$\mathbb{E}_t^{BR}[\widehat{\mathrm{mc}}(X_{t+k})] = \overline{m}^k \mathbb{E}_t[\widehat{\mathrm{mc}}(X_{t+k})]$$

Hence, the equivalent condition of equation (14) for a behavioural firm is

$$\begin{aligned} p_t^* &= p_t + (1-\theta\beta) \sum_{k=0}^{\infty} (\theta\beta)^k \mathbb{E}_t^{BR} \left[(\pi_{t+1} + ... + \pi_{t+k}) - \omega(\pi_t + ... + \pi_{t+k-1}) + \widehat{\mathrm{mc}}_{t+k}\right] \\ &= p_t + (1-\theta\beta) \sum_{k=0}^{\infty} (\theta\beta\overline{m})^k \mathbb{E}_t \left[(\pi_{t+1} + ... + \pi_{t+k}) - \omega(\pi_t + ... + \pi_{t+k-1}) + \widehat{\mathrm{mc}}_{t+k}\right] \end{aligned} \tag{15}$$

4.3 Aggregate Price Dynamics

In this economy, at every period, there are two types of firms: those allowed to reset their price and those who are not, which price is updated with previous aggregate inflation. We can describe price dynamics as

$$P_t = \left[\Pi_{t-1}^{\omega(1-\epsilon)} \theta P_{t-1}^{1-\epsilon} + (1-\theta)(P_t^*)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}$$

which log-linearized version is,[7]

$$\pi_t = \theta\omega\pi_{t-1} + (1-\theta)(p_t^* - p_{t-1}) \tag{16}$$

[7] Derived in Appendix D.

4.4 The Behavioural Hybrid New Keynesian Curve

After some tedious algebra, relegated to Appendix E, a rearrangement of expressions (15) and (16) yields the Behavioral Hybrid NK Phillips curve in terms of marginal costs

$$\pi_t = \gamma_b \pi_{t-1} + \gamma_{\text{mc}} \widehat{\text{mc}}_t + \gamma_f \mathbb{E}_t \pi_{t+1} \tag{17}$$

where $\gamma_b = \frac{\omega}{1+\omega\beta\overline{m}\left[\theta+(1-\theta)\frac{1-\theta\beta}{1-\theta\beta\overline{m}}\right]}$, $\gamma_{\text{mc}} = \frac{(1-\theta)(1-\theta\beta)}{\theta\left\{1+\omega\beta\overline{m}\left[\theta+(1-\theta)\frac{1-\theta\beta}{1-\theta\beta\overline{m}}\right]\right\}}$ and $\gamma_f = \frac{\beta\overline{m}\left[\theta+(1-\theta)\frac{1-\theta\beta}{1-\theta\beta\overline{m}}\right]}{1+\omega\beta\overline{m}\left[\theta+(1-\theta)\frac{1-\theta\beta}{1-\theta\beta\overline{m}}\right]}$. As one can see, in this BR setting the effective discount factor is $\widehat{\beta} = \beta\overline{m}$. However, after a careful look at $(\gamma_b, \gamma_{\text{mc}}, \gamma_f)$ one can see that the subjective discount factor is not always accompanied by the cognitive discount factor. For this to happen, it is crucial that fully rational and BR agents are identical regarding intratemporal actions but different when taking intertemporal decisions.

In order to obtain the Behavioural Hybrid NK Phillips curve in terms of the output gap, recall that in this economy with technological progress $\text{MC}_t = W_t/(A_t P_t)$. Taking logs, we can write $\text{mc}_t = w_t - p_t$. Additionally, firm technology implies $y_t = a_t + n_t$ and the aggregate resource constraint implies $y_t = c_t$. Finally, thanks to the labor supply condition (4) we know $w_t - p_t = \varphi n_t + \frac{\sigma}{1-h} c_t - \frac{\sigma h}{1-h} c_{t-1}$. Using these four expressions together yields

$$\widehat{\text{mc}}_t = \left(\varphi + \frac{\sigma}{1-h}\right)\widetilde{y}_t - \frac{\sigma h}{1-h}\widetilde{y}_{t-1} \tag{18}$$

Introducing (18) into (17) leads to the Hybrid NK Phillips curve

$$\pi_t = \gamma_b \pi_{t-1} + \alpha_b \widetilde{y}_{t-1} + \alpha_c \widetilde{y}_t + \gamma_f \mathbb{E}_t \pi_{t+1} \tag{19}$$

where $\alpha_b = -\gamma_{\text{mc}} \frac{\sigma h}{1-h}$ and $\alpha_c = \gamma_{\text{mc}}\left(\varphi + \frac{\sigma}{1-h}\right)$. The Behavioral Hybrid NK Phillips curve (19), together with the Behavioral Dynamic IS curve (8) and a reaction function for the Central Bank (an ad-hoc Taylor rule)

$$i_t = \rho + (1-\rho_r)(\phi_\pi \pi_t + \phi_y \widetilde{y}_t) + \rho_r i_{t-1} + e_t \tag{20}$$

constitute the Behavioral New Keynesian framework with "keeping up with the Joneses" households and hybrid firms.

5 Estimation

5.1 Econometric specification

We follow Gali and Gertler (1999) and obtain our measure of real marginal cost from the labor share. In our setting, production technology is of the form

$$Y_t = A_t N_t \tag{21}$$

Real marginal costs are given by the ratio of the wage rate to the marginal product of labor: $\mathrm{MC}_t = (W_t/P_t)/(\partial Y_t/\partial N_t)$. Hence, using (21) we can write

$$\mathrm{MC}_t = \frac{W_t N_t}{P_t Y_t}$$

which is the labor income share (real unit costs). We define $\widehat{\mathrm{mc}}_t$ as the percentage deviation from steady state, $\widehat{\mathrm{mc}}_t = (\mathrm{MC}_t - \mathrm{MC})/\mathrm{MC}$, where MC is the average labor income share as in Lindé (2005).

We use quarterly data for the US in the 1960Q1-1997Q4 period. This particular sample period was chosen to be able to compare our estimates to the ones in Gali and Gertler (1999). Our key variables include the log labor income share in the non-farm business sector for $\widehat{\mathrm{mc}}_t$ (described above), the percentage change in the GDP deflator for inflation π_t, the log (quadratically) detrended GDP for our output gap measure $\widetilde{y}_t$, and the (quadratically) detrended real interest rate for $\widetilde{r}_t$. Finally, given that we will proceed using a GMM estimation, our instrument vector contains four lags of inflation, contemporaneous and four lags of wage inflation, four lags of commodity inflation, three lags of the marginal cost, four lags of output gap, contemporaneous and four lags of interest rate gap and the contemporaneous interest rate spread

We are not able to estimate directly every parameter in the Phillips curve since we have too many structural parameters. Therefore, our estimation strategy consists of two stages. We first estimate the demand side of the economy, the BRDIS curve, to obtain $\overline{m}$. Once we have estimated $\overline{m}$, we then turn to the BRNKP curve and estimate it restricting $\overline{m}$ to its estimated parameter $\widehat{\overline{m}}$.

5.2 Structural estimates

We discussed in section 1 that one advantage of our model is that both household habit persistence and firms backward-looking behavior can be shut down easily. Thanks to this feature we can estimate not only our BRDIS and HNKP curves, but also the ones presented in the benchmark setting in Gabaix (2016a).

We begin with the most complete setting, allowing for household habit persistence and backward-looking firms. Following the estimation strategy discussed above, we first estimate the BRDIS curve (8). In our behavioural setting, the real interest rate is defined as $r_t = i_t - \overline{m}\mathbb{E}_t\pi_{t+1}$, differently from Gabaix (2016a)[8]. We define the interest rate gap as $\widetilde{r}_t = r_t - r_t^n$. Making use of a rational expectations assumption, we can write $\mathbb{E}_t\widetilde{y}_{t+1} = \widetilde{y}_{t+1} + \varepsilon_{t+1}^y$ where ε_{t+1}^y is the error forecast. Assuming that the error is uncorrelated with information dated at t (hence, rational expectations hold), it follows from (8) that

[8] Gabaix (2016a) instead defines it as $r_t = i_t - \mathbb{E}_t\pi_{t+1}$. In our setting, $r_t = i_t - \mathbb{E}_t^{BR}\pi_{t+1} = i_t - \overline{m}\mathbb{E}_t\pi_{t+1}$.

$$\mathbb{E}_t(\varepsilon^y_{t+1} z_t) = \mathbb{E}_t \left\{ \left[(1+h)\widetilde{y}_t - h\widetilde{y}_{t-1} - \overline{m}\widetilde{y}_{t+1} + \frac{1-h}{\sigma}\widetilde{r}_t \right] z_t \right\} = 0 \tag{22}$$

where z_t is a vector containing variables at t and earlier. The orthogonality condition in (22) is key to estimate (8) by making use of the generalized method of moments (GMM).

We then turn to the second stage of our estimation strategy. We estimate the supply side of the economy, the behavioral hybrid NK Phillips curve (17) implementing GMM. Again, making use of the rational expectations assumption we can write $\mathbb{E}_t \pi_{t+1} = \pi_{t+1} + \varepsilon^\pi_{t+1}$, where ε^π_{t+1} is the error forecast. Assuming that the error is uncorrelated with information dated at t (hence, rational expectations hold), it follows from (17) that

$$\begin{aligned} \mathbb{E}_t(\varepsilon^\pi_{t+1} z_t) = & \mathbb{E}_t \Bigg\{ \Bigg\{ \left\{ 1 + \omega\beta\overline{m} \left[\theta + (1-\theta)\frac{1-\theta\beta}{1-\theta\beta\overline{m}} \right] \right\} \pi_t - \omega\pi_{t-1} - \\ & - \frac{(1-\theta)(1-\theta\beta)}{\theta}\widehat{mc}_t - \beta\overline{m} \left[\theta + (1-\theta)\frac{1-\theta\beta}{1-\theta\beta\overline{m}} \right] \mathbb{E}_t \pi_{t+1} \Bigg\} z_t \Bigg\} = 0 \end{aligned} \tag{23}$$

where z_t is the same vector containing variables at t and earlier. The orthogonality condition in (23) is key to estimate (17) using the generalized method of moments (GMM).

We are interested in obtaining $\widehat{\overline{m}}^{GMM}$, $\widehat{h}^{GMM}$, $\widehat{\theta}^{GMM}$, $\widehat{\beta}^{GMM}$ and $\widehat{\omega}^{GMM}$. We first estimate the structural parameters in the demand side: the cognitive discount factor and the habit persistence, using a nonlinear instrumental variables estimator with the set of instruments described above.

The results are presented in Table 1. We will now restrict our attention (since we indeed are rationally inattentive) to the first row, which contains the estimates for the most complete model. The first two columns report the estimates for the (quarterly) cognitive discount factor $\overline{m}$ and the degree of habit persistence h, the demand side structural parameters. The parameter $\overline{m}$ is estimated to be around 0.99 with standard error 0.007. This implies that, at an annual frequency, $\overline{m}$ is around 0.97. This estimate is less than 2 standard errors from 1, a finding that weakly supports the idea of departing from full rationality (recall that households are fully rational when $\overline{m} = 1$).

We now turn to the estimate of the degree of habit persistence. The parameter h is estimated to be around 0.85 with standard error 0.07, implying that household habits are highly persistent. This estimate is in the range of previous findings in the literature, where Christiano et al. (2005) estimate it to be around 0.66 for the US and Blanchard et al. (2015) around 0.87 for Europe. In the household side, the

Table 1: Estimates of the Behavioural New Keynesian model.

	$\overline{m}$	h	λ_b	λ_f	θ	β	ω	γ_b	γ_f
Joneses & Hybrid	0.992	0.854	0.462	0.535	0.864	0.990	0.420	0.371	0.620
	(0.007)	(0.074)	(0.024)	(0.023)	(0.029)	(0.019)	(0.106)	(0.036)	(0.035)
Standard & Hybrid	0.970	0	0	0.959	0.846	1.034	0.420		
	(0.011)			(0.011)	(0.028)	(0.024)	(0.106)		
Joneses & Standard					0.913	0.978	0	0	0.980
					(0.038)	(0.010)			(0.010)
Standard & Standard					0.897	1.023	0		
					(0.037)	(0.013)			

quantitative importance of backward-looking behavior is large: the implied reduced-form coefficients on lagged values versus expected output gap are 0.46 (for λ_b in (8)) and 0.53 (for λ_f).

Stage 2 estimates are reported in columns 5-7. These structural parameters are all within the supply side of the economy: the average duration of prices $\frac{1}{1-\theta}$, the subjective discount factor β and the share of backward-looking firms ω. The parameter θ is estimated to be around 0.86 with standard error 0.03, implying that prices are fixed for 7 quarters on average. This length duration is somewhat large, on the right tail of survey evidence.[9] However, this estimate might be upward biased since our setting does not account for real rigidities, such as Kimball kinked demand curve, firm-specific capital and labor.

We now turn to the estimate of the subjective discount factor. The parameter β is estimated to be around 0.99 with standard error 0.02. We are thus able to successfully reconcile the theory with empirical evidence: previous studies that tried to do so, estimated a subjective discount factor of around 0.90, which is excessively low to be at a quarterly frequency. One interesting result is that, in the second row, when the habit formation dropped, the estimated cognitive discount factor decreases, and in turn θ decreases, which provides support for the proposal made in Gabaix (2017) that inertia that is caused by habit formation, at least partially, can be explained by the limited attention of the households.

Finally, the estimate of the share of backward-looking firms ω is around 0.42 with standard error 0.11. This implies that less than half of price-setters are backward-looking and is in line with the findings in Gali and Gertler (1999). In the firms side, the quantitative importance of backward-looking behavior is not as large as in the household side: the implied reduced-form coefficients on lagged values versus expected output gap are 0.37 (for γ_b in (17)) and 0.62 (for γ_f). This finding supports the intuition that firms' behavior is closer to rational than households.

6 Conclusion

In this paper we build and estimate a Behavioral (Bounded Rational) New Keynesian model that is proves to be successful in reconciling structural estimates on New Keynesian theory and microeconometric evidence. We include two sources of Bounded Rationality: habit persistence and myopia. We find empirical evidence for Bounded Rationality, and we show that (i) firms are more forward-looking than households; (ii) the cognitive discount factor is between 0.97 and 0.99 at a quarterly frequency (between 0.88 and 0.97 at yearly frequency); (iii) that, consistent with the theory, the degree of myopia increases when we drop the additional behavioral

[9] See **?** and Nakamura and Steinsson (2008).

assumptions (habit formation), yet remains significantly different from the rational case.

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Appendices

A Demand for good i and Aggregate Price Index

The representative household derives utility from consumption of different goods, indexed $i \in [0,1]$, according to the consumption index

$$C_t = \left[\int_0^1 C_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}} \tag{24}$$

which should be maximized for any given expenditure level Z_t

$$\int_0^1 P_t(i) C_t(i) \; di \equiv Z_t \tag{25}$$

where $P_t(i)$ is the price of variety i. The first-order condition wrt $C_t(i)$ yields

$$\left[\int_0^1 C_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{1}{\epsilon-1}} C_t(i)^{-\frac{1}{\epsilon}} - \lambda_t P_t(i) = 0 \implies C_t^{\frac{1}{\epsilon}} C_t(i)^{-\frac{1}{\epsilon}} = \lambda_t P_t(i)$$

where λ_t is the sequence of Lagrange multipliers attached to the sequence of restrictions (25). Since the last condition must hold for any two goods i and j, λ_t is cancelled out and we find the condition

$$C_t(i) = \left[\frac{P_t(j)}{P_t(i)} \right]^{\epsilon} C_t(j) \tag{26}$$

Inserting (26) into (25),

$$Z_t = \int_0^1 P_t(i) \left[\frac{P_t(j)}{P_t(i)} \right]^{\epsilon} C_t(j) \; di \implies C_t(j) = \frac{Z_t P_t(j)^{-\epsilon}}{\int_0^1 P_t(i)^{1-\epsilon} di}$$

Inserting this last result into (24), for good i instead of j

$$C_t = \left\{ \int_0^1 \left[\frac{Z_t P_t(i)^{-\epsilon}}{\int_0^1 P_t(i)^{1-\epsilon} di} \right]^{\frac{\epsilon-1}{\epsilon}} di \right\}^{\frac{\epsilon}{\epsilon-1}} = Z_t \left[\int_0^1 P_t(i)^{1-\epsilon} di \right]^{\frac{1}{\epsilon-1}}$$

Define P_t as the expenditure needed to purchase 1 unit of C_t (that is, $P_t = Z_t|_{C_t=1}$). Hence,

$$1 = Z_t \left[\int_0^1 P_t(i)^{1-\epsilon} di \right]^{\frac{1}{\epsilon-1}} \implies Z_t \equiv P_t = \left[\int_0^1 P_t(i)^{1-\epsilon} di \right]^{1-\epsilon} \tag{27}$$

where (27) is the price index. Inserting (26) and (27) into (25)

$$Z_t = \int_0^1 P_t(i)\left[\frac{P_t(j)}{P_t(i)}\right]^{\epsilon} C_t(j)\ di = C_t(j)P_t(j)^{\epsilon}\int_0^1 P_t(i)^{1-\epsilon}di = \left[\frac{P_t}{P_t(j)}\right]^{-\epsilon} P_tC_t(j) \implies$$

$$\implies C_t(i) = \frac{Z_t}{P_t}\left[\frac{P_t}{P_t(i)}\right]^{\epsilon} \tag{28}$$

Inserting this last identity into (24),

$$C_t = \left\{\int_0^1 \left[\frac{Z_t}{P_t}\left(\frac{P_t}{P_t(i)}\right)^{\epsilon}\right]^{\frac{\epsilon-1}{\epsilon}} di\right\}^{\frac{\epsilon}{\epsilon-1}} = \frac{Z_t}{P_t} \implies Z_t = P_tC_t \tag{29}$$

Inserting the last expression into Z_t we find the desired optimal consumption for good i,

$$C_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\epsilon} C_t \tag{30}$$

With market clearing and a representative household setting, $C_t(i) = Y_t(i)$ and $C_t = Y_t$, and we obtain expression (1).

B Log-linearization of Behavioural Household's Optimality Conditions

We now proceed to log-linearize (4)-(5). Starting with (4), taking a first order Taylor approximation,

$$\frac{W}{P} + \frac{1}{P}(W_t - W) - \frac{W}{P^2}(P_t - P) = \frac{N^{\varphi}}{[C(1-h)]^{-\sigma}} + \varphi\frac{N^{\varphi-1}}{[C(1-h)]^{-\sigma}}(N_t - N) +$$

$$+ \sigma\frac{N^{\varphi}}{[C(1-h)]^{1-\sigma}}(C_t - C) - \sigma h\frac{N^{\varphi}}{[C(1-h)]^{1-\sigma}}(C_{t-1} - C)$$

$$\implies \frac{W}{P}\left\{1 + \frac{W_t - W}{W} - \frac{P_t - P}{P}\right\} = \frac{N^{\varphi}}{[C(1-h)]^{-\sigma}}\times$$

$$\times\left\{1 + \varphi\frac{N_t - N}{N} + \frac{\sigma}{1-h}\frac{C_t - C}{C} - \frac{\sigma h}{1-h}\frac{C_{t-1} - C}{C}\right\} \implies$$

$$\implies \hat{w}_t - \hat{p}_t = \varphi\hat{n}_t + \frac{\sigma}{1-h}\hat{c}_t - \frac{\sigma h}{1-h}\hat{c}_{t-1}$$

where any variable x satisfies $\hat{x}_t = x_t - x = \log X_t - \log X = \frac{X_t - X}{X}$. Turning to (5). Taking logs

$$\log Q_t = \log\beta + \mathbb{E}_t^{BR}\big\{-\sigma\log[C(X_{t+1}) - hC(X_t)] + \sigma\log[C(X_t) - hC(X_{t-1})] +$$

$$+ \log P(X_t) - \log P(X_{t+1})\big\}$$

Since $Q_t = 1/(1+i_t)$, one can show that $i_t \approx -\log Q_t$. We then set $\rho = -\log \beta$ and $\pi(X_{t+1}) = \log P(X_{t+1})/P(X_t)$. Let us now log-linearize the terms that include consumption

$$\begin{aligned}\log[C(X_{t+1}) - hC(X_t)] &\approx \log[(1-h)C] + \frac{1}{(1-h)C}[C(X_{t+1}) - C] - \frac{h}{(1-h)C}[C(X_t) - C] \\ &= \log[(1-h)C] + \frac{1}{1-h}[c(X_{t+1}) - c] - \frac{h}{1-h}[c(X_t) - c] \\ &= \log[(1-h)C] + \frac{1}{1-h}\widehat{c}(X_{t+1}) - \frac{h}{1-h}\widehat{c}(X_t)\end{aligned}$$

proceeding in a similar manner with the other consumption term, and plugging into the above expression leads to

$$-i_t = -\rho + \mathbb{E}_t^{BR}\left\{-\frac{\sigma}{1-h}\left[\widehat{c}(X_{t+1}) - h\widehat{c}(X_t) - \widehat{c}(X_t) + h\widehat{c}(X_{t-1})\right] - \pi(X_{t+1})\right\}$$

By Lemma 1, $\mathbb{E}_t^{BR}[z(X_{t+k})] = \overline{m}^k\mathbb{E}_t[z(X_{t+k})]$. Hence,

$$-i_t = -\rho - \frac{\sigma}{1-h}\overline{m}\mathbb{E}_t\left[\widehat{c}(X_{t+1})\right] + \frac{\sigma}{1-h}\left[(1+h)\widehat{c}(X_t) - h\widehat{c}(X_{t-1})\right] - \overline{m}\mathbb{E}_t\left[\pi(X_{t+1})\right]$$

which, denoting $z_t \equiv z(X_t)$ and $\widehat{i}_t = i_t - i = i_t - \rho$, leads to (7). Written in natural terms and denoting $r_t = i_t - \overline{m}\mathbb{E}_t\pi_{t+1}$ yields

$$\hat{c}_t^n = \frac{h}{1+h}\hat{c}_{t-1}^n + \frac{1}{1+h}\overline{m}\mathbb{E}_t\hat{c}_{t+1}^n - \frac{1-h}{\sigma(1+h)}\left(r_t^n - \rho\right)$$

Since $\widehat{c}_t = \widehat{y}_t$, we can rewrite it in terms of the output gap $\tilde{y}_t = y_t - y_t^n$,

$$\tilde{y}_t = \frac{h}{1+h}\tilde{y}_{t-1} + \frac{1}{1+h}\overline{m}\mathbb{E}_t\tilde{y}_{t+1} - \frac{1-h}{\sigma(1+h)}\left(i_t - \overline{m}\mathbb{E}_t\pi_{t+1} - r_t^n\right)$$

C Solving the Firm Problem

Taking the first-order condition of (4.2) with respect to $P_t^*(i)$ yields

$$\begin{aligned}P_t^*(i): \quad & \mathbb{E}_t\sum_{k=0}^{\infty}\theta^k Q_{t,t+k}\left[\frac{P_t^*(i)\chi_{t,t+k}}{P_{t+k}}\right]^{-\epsilon} C_{t+k}\left[\chi_{t,t+k}(1-\epsilon) + \epsilon\frac{W_{t+k}}{P_t^*(i)A_{t+k}}\right] = 0 \implies \\ & \underset{\times\frac{P_t^*(i)}{1-\epsilon}}{\implies} \mathbb{E}_t\sum_{k=0}^{\infty}\theta^k Q_{t,t+k}\left[\frac{P_t^*(i)\chi_{t,t+k}}{P_{t+k}}\right]^{-\epsilon} C_{t+k}\left[P_t^*(i)\chi_{t,t+k} - \mathcal{M}\frac{W_{t+k}}{A_{t+k}}\right] = 0\end{aligned}$$

where $\mathcal{M} = \frac{\epsilon}{\epsilon-1}$. Separating both sides,

$$\mathbb{E}_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} \left[P_t^*(i)\chi_{t,t+k}\right]^{1-\epsilon} P_{t+k}^{\epsilon} C_{t+k} = \mathbb{E}_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} \left[\frac{P_t^*(i)\chi_{t,t+k}}{P_{t+k}}\right]^{-\epsilon} C_{t+k} \mathcal{M} \frac{W_{t+k}}{A_{t+k}}$$

Inserting $Q_{t,t+k} = \beta^k \left(\frac{C_{t+k}-hC_{t+k-1}}{C_t-hC_{t-1}}\right)^{-\sigma} \frac{P_t}{P_{t+k}}$ and $\chi_{t,t+k} = \left(\frac{P_{t+k-1}}{P_{t-1}}\right)^{\omega}$, and solving for P_t^*,

$$\begin{aligned}
\text{LHS:}\ \ &\mathbb{E}_t \sum_{k=0}^{\infty} \theta^k \beta^k \left(\frac{C_{t+k}-hC_{t+k-1}}{C_t-hC_{t-1}}\right)^{-\sigma} \frac{P_t}{P_{t+k}} \left[P_t^*(i) \left(\frac{P_{t+k-1}}{P_{t-1}}\right)^{\omega}\right]^{1-\epsilon} P_{t+k}^{\epsilon} C_{t+k} = \\
&= \mathbb{E}_t \sum_{k=0}^{\infty} (\theta\beta)^k \left(\frac{C_{t+k}-hC_{t+k-1}}{C_t-hC_{t-1}}\right)^{-\sigma} C_{t+k} P_t P_{t+k}^{\epsilon-1} P_t^*(i)^{1-\epsilon} \left(\frac{P_{t+k-1}}{P_{t-1}}\right)^{\omega(1-\epsilon)} = \\
&= (C_t - hC_{t-1})^{\sigma} P_t P_t^*(i)^{1-\epsilon} P_{t-1}^{-\omega(1-\epsilon)} \mathbb{E}_t \sum_{k=0}^{\infty} (\theta\beta)^k (C_{t+k}-hC_{t+k-1})^{-\sigma} C_{t+k} P_{t+k}^{\epsilon-1} P_{t+k-1}^{\omega(1-\epsilon)}
\end{aligned}$$

$$\begin{aligned}
\text{RHS:}\ \ &\mathbb{E}_t \sum_{k=0}^{\infty} \theta^k \beta^k \left(\frac{C_{t+k}-hC_{t+k-1}}{C_t-hC_{t-1}}\right)^{-\sigma} \frac{P_t}{P_{t+k}} \left[\frac{P_t^*(i) \left(\frac{P_{t+k-1}}{P_{t-1}}\right)^{\omega}}{P_{t+k}}\right]^{-\epsilon} C_{t+k} \mathcal{M} \frac{W_{t+k}}{A_{t+k}} = \\
&= \mathcal{M} \mathbb{E}_t \sum_{k=0}^{\infty} (\theta\beta)^k \left(\frac{C_{t+k}-hC_{t+k-1}}{C_t-hC_{t-1}}\right)^{-\sigma} C_{t+k} P_t P_{t+k}^{\epsilon-1} P_t^*(i)^{-\epsilon} P_{t-1}^{\omega\epsilon} P_{t+k-1}^{-\omega\epsilon} \frac{W_{t+k}}{A_{t+k}} = \\
&= \mathcal{M} (C_t - hC_{t-1})^{\sigma} P_t P_t^*(i)^{-\epsilon} P_{t-1}^{\omega\epsilon} \mathbb{E}_t \sum_{k=0}^{\infty} (\theta\beta)^k (C_{t+k}-hC_{t+k-1})^{-\sigma} C_{t+k} P_{t+k}^{\epsilon-1} P_{t+k-1}^{-\omega\epsilon} \frac{W_{t+k}}{A_{t+k}}
\end{aligned}$$

$$\begin{aligned}
\text{LHS=RHS:}\ \ &(C_t - hC_{t-1})^{\sigma} P_t P_t^*(i)^{1-\epsilon} P_{t-1}^{-\omega(1-\epsilon)} \mathbb{E}_t \sum_{k=0}^{\infty} (\theta\beta)^k (C_{t+k}-hC_{t+k-1})^{-\sigma} C_{t+k} P_{t+k}^{\epsilon-1} P_{t+k-1}^{\omega(1-\epsilon)} = \\
&= \mathcal{M} (C_t - hC_{t-1})^{\sigma} P_t P_t^*(i)^{-\epsilon} P_{t-1}^{\omega\epsilon} \mathbb{E}_t \sum_{k=0}^{\infty} (\theta\beta)^k (C_{t+k}-hC_{t+k-1})^{-\sigma} C_{t+k} P_{t+k}^{\epsilon-1} P_{t+k-1}^{-\omega\epsilon} \frac{W_{t+k}}{A_{t+k}} \\
\implies P_t^*(i) &= \mathcal{M} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} (\theta\beta)^k (C_{t+k}-hC_{t+k-1})^{-\sigma} C_{t+k} P_{t+k}^{\epsilon-1} P_{t+k-1}^{-\omega\epsilon} \frac{W_{t+k}}{A_{t+k}}}{\mathbb{E}_t \sum_{k=0}^{\infty} (\theta\beta)^k (C_{t+k}-hC_{t+k-1})^{-\sigma} C_{t+k} P_{t+k}^{\epsilon-1} P_{t+k-1}^{\omega(1-\epsilon)}} P_{t-1}^{\omega} = \\
&= \mathcal{M} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} (\theta\beta)^k (C_{t+k}-hC_{t+k-1})^{-\sigma} C_{t+k} P_{t+k}^{\epsilon} P_{t+k-1}^{-\omega\epsilon} MC_{t+k}}{\mathbb{E}_t \sum_{k=0}^{\infty} (\theta\beta)^k (C_{t+k}-hC_{t+k-1})^{-\sigma} C_{t+k} P_{t+k}^{\epsilon-1} P_{t+k-1}^{\omega(1-\epsilon)}} P_{t-1}^{\omega}
\end{aligned} \tag{31}$$

where we have used $MC_{t+k} = \frac{W_{t+k}}{A_{t+k}P_{t+k}}$. With flexible prices, (31) collapses to

$$P_t^*(i) = \mathcal{M}\frac{(C_t - hC_{t-1})^{-\sigma}C_tP_t^{\epsilon}P_{t-1}^{-\omega\epsilon}MC_t}{(C_t - hC_{t-1})^{-\sigma}C_tP_t^{\epsilon-1}P_{t-1}^{\omega(1-\epsilon)}}P_{t-1}^{\omega} = \mathcal{M}P_tMC_t \tag{32}$$

where (32) is the frictionless mark-up. To simplify computation, I now log-linearize (31): separating (back) both sides,

$$P_t^*\mathbb{E}_t\sum_{k=0}^{\infty}(\theta\beta)^k(C_{t+k} - hC_{t+k-1})^{-\sigma}C_{t+k}P_{t+k}^{\epsilon-1}P_{t+k-1}^{\omega(1-\epsilon)} = \\ = \mathcal{M}\mathbb{E}_t\sum_{k=0}^{\infty}(\theta\beta)^k(C_{t+k} - hC_{t+k-1})^{-\sigma}C_{t+k}P_{t+k}^{\epsilon}P_{t+k-1}^{-\omega\epsilon}MC_{t+k}P_{t-1}^{\omega} \tag{33}$$

We know that, in steady-state, $P_t^* = P_t = P_{t-1} = P$, $\Pi_t = \Pi = 1$, $C_t = C$, $Q_{t,t+k} = \beta^k$ and $MC_t = MC$. It lasts to find MC. To obtain it, write (31) in steady-state and solve for MC,

$$\begin{aligned} P &= \mathcal{M}\frac{\sum_{k=0}^{\infty}(\theta\beta)^kC^{1-\sigma}(1-h)^{-\sigma}P^{\epsilon(1-\omega)}MC}{\sum_{k=0}^{\infty}(\theta\beta)^kC^{1-\sigma}(1-h)^{-\sigma}P^{-(1-\epsilon)(1-\omega)}}P^{\omega} = \\ &= \mathcal{M}\frac{C^{1-\sigma}(1-h)^{-\sigma}P^{\epsilon(1-\omega)}MC\frac{1}{1-\theta\beta}}{C^{1-\sigma}(1-h)^{-\sigma}P^{-(1-\epsilon)(1-\omega)}\frac{1}{1-\theta\beta}}P^{\omega} = \\ &= \mathcal{M}P^{1-\omega}MCP^{\omega} \end{aligned}$$

Hence, $MC = \frac{1}{\mathcal{M}}$. Before log-linearizing, divide (34) by P_{t-1},

$$\frac{P_t^*}{P_{t-1}}\mathbb{E}_t\sum_{k=0}^{\infty}(\theta\beta)^k(C_{t+k} - hC_{t+k-1})^{-\sigma}C_{t+k}P_{t+k}^{\epsilon-1}P_{t+k-1}^{\omega(1-\epsilon)} = \\ = \mathcal{M}\mathbb{E}_t\sum_{k=0}^{\infty}(\theta\beta)^k(C_{t+k} - hC_{t+k-1})^{-\sigma}C_{t+k}P_{t+k}^{\epsilon}P_{t+k-1}^{-\omega\epsilon}MC_{t+k}P_{t-1}^{\omega-1} \tag{34}$$

Log-linearizing the LHS,

$$\frac{P_t^*}{P_{t-1}}\mathbb{E}_t\sum_{k=0}^{\infty}(\theta\beta)^k(C_{t+k}-hC_{t+k-1})^{-\sigma}C_{t+k}P_{t+k}^{\epsilon-1}P_{t+k-1}^{\omega(1-\epsilon)}\simeq$$

$$\begin{aligned}
&\simeq\sum_{k=0}^{\infty}(\theta\beta)^kC^{1-\sigma}(1-h)^{-\sigma}P^{-(1-\epsilon)(1-\omega)}+\\
&\quad+\frac{1}{P}\sum_{k=0}^{\infty}(\theta\beta)^kC^{1-\sigma}(1-h)^{-\sigma}P^{-(1-\epsilon)(1-\omega)}(P_t^*-P)-\\
&\quad-\frac{P}{P^2}\sum_{k=0}^{\infty}(\theta\beta)^kC^{1-\sigma}(1-h)^{-\sigma}P^{-(1-\epsilon)(1-\omega)}(P_{t-1}-P)+\\
&\quad+\sum_{k=0}^{\infty}(\theta\beta)^kC^{1-\sigma}(1-h)^{-\sigma}(\epsilon-1)P^{\epsilon-2}P^{\omega(1-\epsilon)}(P_{t+k}-P)+\\
&\quad+\sum_{k=0}^{\infty}(\theta\beta)^kC^{1-\sigma}(1-h)^{-\sigma}P^{\epsilon-1}\omega(1-\epsilon)P^{\omega(1-\epsilon)-1}(P_{t+k-1}-P)+\\
&\quad+\sum_{k=0}^{\infty}(\theta\beta)^k\{\underbrace{(-\sigma)[C(1-h)]^{-\sigma-1}C+[C(1-h)]^{-\sigma}}_{C^{-\sigma}(1-h)^{-\sigma-1}(1-h-\sigma)}\}P^{-(1-\epsilon)(1-\omega)}(C_{t+k}-C)+\\
&\quad+\sum_{k=0}^{\infty}(\theta\beta)^k\underbrace{(-\sigma)[C(1-h)]^{-\sigma-1}(-h)C}_{\sigma hC^{-\sigma}(1-h)^{-\sigma-1}}P^{-(1-\epsilon)(1-\omega)}(C_{t+k-1}-C)=\\
&=\sum_{k=0}^{\infty}(\theta\beta)^kC^{1-\sigma}(1-h)^{-\sigma}P^{-(1-\epsilon)(1-\omega)}\Bigg\{1+p_t^*-p-p_{t-1}+p-(1-\epsilon)(p_{t+k}-p)+\\
&\qquad+\omega(1-\epsilon)(p_{t+k-1}-p)+\left(1-\frac{\sigma}{1-h}\right)(c_{t+k}-c)+\frac{\sigma h}{1-h}(c_{t+k-1}-c)\Bigg\}
\end{aligned}$$

Log-linearizing the RHS,

$$\mathcal{M}\mathbb{E}_t\sum_{k=0}^{\infty}(\theta\beta)^k(C_{t+k}-hC_{t+k-1})^{-\sigma}C_{t+k}P_{t+k}^{\epsilon}P_{t+k-1}^{-\omega\epsilon}MC_{t+k}P_{t-1}^{\omega-1}\simeq$$

$$\begin{aligned}
&\simeq\mathcal{M}\sum_{k=0}^{\infty}(\theta\beta)^kC^{1-\sigma}(1-h)^{-\sigma}P^{-(1-\epsilon)(1-\omega)}MC+\\
&\quad+\mathcal{M}\sum_{k=0}^{\infty}(\theta\beta)^kC^{1-\sigma}(1-h)^{-\sigma}P^{\epsilon}P^{-\omega\epsilon}MC(\omega-1)P^{\omega-2}(P_{t-1}-P)+\\
&\quad+\mathcal{M}\sum_{k=0}^{\infty}(\theta\beta)^kC^{1-\sigma}(1-h)^{-\sigma}\epsilon P^{\epsilon-1}P^{-\omega\epsilon}MCP^{\omega-1}(P_{t+k}-P)+\\
&\quad+\mathcal{M}\sum_{k=0}^{\infty}(\theta\beta)^kC^{1-\sigma}(1-h)^{-\sigma}P^{\epsilon}(-\omega\epsilon)P^{-\omega\epsilon-1}MCP^{\omega-1}(P_{t+k-1}-P)+\\
&\quad+\mathcal{M}\sum_{k=0}^{\infty}(\theta\beta)^kC^{-\sigma}(1-h)^{-\sigma-1}(1-h-\sigma)P^{-(1-\epsilon)(1-\omega)}MC(C_{t+k}-C)+\\
&\quad+\mathcal{M}\sum_{k=0}^{\infty}(\theta\beta)^kC^{-\sigma}(1-h)^{-\sigma-1}\sigma hP^{-(1-\epsilon)(1-\omega)}MC(C_{t+k-1}-C)+\\
&\quad+\mathcal{M}\sum_{k=0}^{\infty}(\theta\beta)^kC^{1-\sigma}(1-h)^{-\sigma}P^{-(1-\epsilon)(1-\omega)}(MC_{t+k}-MC)=\\
&=\sum_{k=0}^{\infty}(\theta\beta)^kC^{1-\sigma}(1-h)^{-\sigma}P^{-(1-\epsilon)(1-\omega)}\Big\{1-(1-\omega)(p_{t-1}-p)+\epsilon(p_{t+k}-p)-\\
&\quad-\omega\epsilon(p_{t+k-1}-p)+\left(1-\frac{\sigma}{1-h}\right)(c_{t+k}-c)+\frac{\sigma h}{1-h}(c_{t+k-1}-c)+mc_{t+k}-mc\Big\}
\end{aligned}$$

Set LHS=RHS, eliminating $C^{1-\sigma}(1-h)^{-\sigma}P^{-(1-\epsilon)(1-\omega)}$ on both sides,

$$\begin{aligned}
&\sum_{k=0}^{\infty}(\theta\beta)^k\Big\{1+p_t^*-p-p_{t-1}+p-(1-\epsilon)(p_{t+k}-p)+\omega(1-\epsilon)(p_{t+k-1}-p)+\\
&\qquad+\left(1-\frac{\sigma}{1-h}\right)(c_{t+k}-c)+\frac{\sigma h}{1-h}(c_{t+k-1}-c)\Big\}=\\
&=\sum_{k=0}^{\infty}(\theta\beta)^k\Big\{1-(1-\omega)(p_{t-1}-p)+\epsilon(p_{t+k}-p)-\omega\epsilon(p_{t+k-1}-p)+\\
&\qquad+\left(1-\frac{\sigma}{1-h}\right)(c_{t+k}-c)+\frac{\sigma h}{1-h}(c_{t+k-1}-c)+mc_{t+k}-mc\Big\}
\end{aligned}$$

Rearranging and cancelling terms, we end up with

$$\mathbb{E}_t \sum_{k=0}^{\infty} (\theta\beta)^k \left[p_t^* - \omega p_{t-1} - p_{t+k} + \omega p_{t+k-1}\right] = \mathbb{E}_t \sum_{k=0}^{\infty} (\theta\beta)^k \left[mc_{t+k} - mc\right] \implies$$

$$\begin{aligned}
\implies p_t^* &= (1-\theta\beta) \sum_{k=0}^{\infty} (\theta\beta)^k \left[p_{t+k} - \omega(\omega p_{t+k-1} - p_{t-1}) + mc_{t+k} - mc\right] \\
&= (1-\theta\beta) \sum_{k=0}^{\infty} (\theta\beta)^k \left[p_{t+k} - \omega(\omega p_{t+k-1} - p_{t-1}) + \widehat{mc}_{t+k}\right] \\
&= p_t + (1-\theta\beta) \sum_{k=0}^{\infty} (\theta\beta)^k \left[(p_{t+k} - p_t) - \omega(\omega p_{t+k-1} - p_{t-1}) + \widehat{mc}_{t+k}\right]
\end{aligned}$$

which can be rewritten as (14)

D Aggregate Price Dynamics

Let S_t denote the subset of firms not reoptimizing at time t,

$$\begin{aligned}
P_t &= \left[\int_0^1 P_t(i)^{1-\epsilon} di\right]^{\frac{1}{1-\epsilon}} = \\
&= \left\{ \underbrace{\int_{S_t} \left[P_{t-1}(i)\Pi_{t-1}^{\omega}\right]^{1-\epsilon} di}_{\Pi_{t-1}^{\omega(1-\epsilon)} \int_{S_t} P_{t-1}(i)^{1-\epsilon} di} + \int_{S_t^C} (P_t^*)^{1-\epsilon} di \right\}^{\frac{1}{1-\epsilon}} = \\
&= \left[\Pi_{t-1}^{\omega(1-\epsilon)} \theta \int_0^1 P_{t-1}(i)^{1-\epsilon} di + (1-\theta) \int_0^1 (P_t^*)^{1-\epsilon} di\right]^{\frac{1}{1-\epsilon}} = \\
&= \left[\Pi_{t-1}^{\omega(1-\epsilon)} \theta P_{t-1}^{1-\epsilon} + (1-\theta)(P_t^*)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}
\end{aligned}$$

Moving the exponent from the RHS to the LHS, and dividing in both sides by $P_{t-1}^{1-\epsilon}$,

$$\begin{aligned}
\left(\frac{P_t}{P_{t-1}}\right)^{1-\epsilon} &= \Pi_{t-1}^{\omega(1-\epsilon)} \theta + (1-\theta) \left(\frac{P_t^*}{P_{t-1}}\right)^{1-\epsilon} \implies \\
\implies \Pi_t^{1-\epsilon} &= \left(\frac{P_{t-1}}{P_{t-2}}\right)^{\omega(1-\epsilon)} \theta + (1-\theta) \left(\frac{P_t^*}{P_{t-1}}\right)^{1-\epsilon} \qquad (35)
\end{aligned}$$

To simplify computation, I now log-linearize (35),

$$\text{LHS:}\quad \Pi_t^{1-\epsilon} \simeq \Pi^{1-\epsilon} + (1-\epsilon)\Pi^{-\epsilon}\underbrace{(\Pi_t - \Pi)}_{\pi_t} =$$
$$= 1 + (1-\epsilon)\pi_t$$

since $\Pi = \frac{P}{P} = 1$.

$$\begin{aligned}
\text{RHS:}\quad \left(\frac{P_{t-1}}{P_{t-2}}\right)^{\omega(1-\epsilon)}\theta + (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\epsilon} \simeq\ & \left(\frac{P}{P}\right)^{\omega(1-\epsilon)}\theta + (1-\theta)\left(\frac{P^*}{P}\right)^{1-\epsilon} + \\
& + (1-\theta)(1-\epsilon)P^{-\epsilon}P^{\epsilon-1}(P_t^* - P) + \\
& + \left[\theta\omega(1-\epsilon)P^{\omega(1-\epsilon)-1}P^{-\omega(1-\epsilon)} - (1-\theta)(1-\epsilon)P^{1-\epsilon}P^{2-\epsilon}\right] \times \\
& \times (P_{t-1} - P) - \theta\omega(1-\epsilon)P^{\omega(1-\epsilon)}P^{-\omega(1-\epsilon)-1}(P_{t-2} - P) = \\
=\ & \theta + 1 - \theta + (1-\theta)(1-\epsilon)\hat{p}_t^* + \\
& + [\theta\omega(1-\epsilon) - (1-\theta)(1-\epsilon)]\hat{p}_{t-1} - \theta\omega(1-\epsilon)\hat{p}_{t-2} = \\
=\ & 1 + (1-\theta)(1-\epsilon)\hat{p}_t^* - (1-\epsilon)[1 - \theta(1+\omega)]\hat{p}_{t-1} - \\
& - \theta\omega(1-\epsilon)\hat{p}_{t-2} = \\
=\ & 1 + (1-\theta)(1-\epsilon)p_t^* - (1-\epsilon)[1 - \theta(1+\omega)]p_{t-1} - \\
& - \theta\omega(1-\epsilon)p_{t-2}
\end{aligned}$$

Writing $\hat{x}_t = x_t - x$, luckily happens that all p's are cancelled.

$$\begin{aligned}
\text{LHS=RHS:}\quad & 1+(1-\epsilon)\pi_t = 1 + (1-\theta)(1-\epsilon)p_t^* - (1-\epsilon)[1-\theta(1+\omega)]p_{t-1} - \theta\omega(1-\epsilon)p_{t-2} \implies \\
& \implies \pi_t = (1-\theta)p_t^* - [1-\theta(1+\omega)]p_{t-1} - \theta\omega p_{t-2} \\
& \qquad = \theta\omega\pi_{t-1} + (1-\theta)(p_t^* - p_{t-1})
\end{aligned}$$

E Deriving the Behavioural Hybrid New Keynesian Phillips Curve

Rewriting $\theta\beta\overline{m} = \delta$, the firm's problem optimality condition (15) reads

$$\begin{aligned}
p_t^* &= p_t + (1-\theta\beta)\sum_{k=0}^{\infty}\delta^k\mathbb{E}_t\left[m_\pi^f(\pi_{t+1} + ... + \pi_{t+k}) - \omega m_\pi^f(\pi_t + ... + \pi_{t+k-1}) + m_\mu^f\widehat{mc}_{t+k}\right] \\
&= p_t + (1-\theta\beta)\mathbb{E}_t\left[m_\pi^f\sum_{k=0}^{\infty}\delta^k(\pi_{t+1} + ... + \pi_{t+k}) - \omega m_\pi^f\sum_{k=0}^{\infty}\delta^k(\pi_t + ... + \pi_{t+k-1}) + m_\mu^f\sum_{k=0}^{\infty}\delta^k\widehat{mc}_{t+k}\right]
\end{aligned} \tag{36}$$

We can calculate the following

$$H_t = \sum_{k=1}^{\infty} \delta^k (\pi_{t+1} + ... + \pi_{t+k}) = \sum_{j=1}^{\infty} \pi_{t+j} \sum_{k=j}^{\infty} \delta^k = \sum_{j=1}^{\infty} \pi_{t+j} \frac{\delta^j}{1-\delta} = \frac{1}{1-\delta} \sum_{j=1}^{\infty} \pi_{t+j} \delta^j =$$
$$= \frac{1}{1-\delta} \sum_{k=0}^{\infty} \pi_{t+k} \delta^k 1_{\{k>0\}}$$
$$\widetilde{H}_t = \sum_{k=1}^{\infty} \delta^k (\pi_t + ... + \pi_{t+k-1}) = \sum_{j=1}^{\infty} \pi_{t+j-1} \sum_{k=j}^{\infty} \delta^k = \sum_{j=1}^{\infty} \pi_{t+j-1} \frac{\delta^j}{1-\delta} = \frac{1}{1-\delta} \sum_{j=1}^{\infty} \pi_{t+j-1} \delta^j =$$
$$= \frac{1}{1-\delta} \sum_{k=0}^{\infty} \pi_{t+k-1} \delta^k 1_{\{k>0\}}$$

Rewriting (36),

$$\begin{aligned}
p_t^* - p_t &= (1-\theta\beta)\mathbb{E}_t \left[m_\pi^f H_t - \omega m_\pi^f \widetilde{H}_t + m_\mu^f \sum_{k=0}^{\infty} \delta^k \widehat{mc}_{t+k} \right] \\
&= (1-\theta\beta)\mathbb{E}_t \left[m_\pi^f \frac{1}{1-\delta} \sum_{k=0}^{\infty} \pi_{t+k} \delta^k 1_{\{k>0\}} - \omega m_\pi^f \frac{1}{1-\delta} \sum_{k=0}^{\infty} \pi_{t+k-1} \delta^k 1_{\{k>0\}} + m_\mu^f \sum_{k=0}^{\infty} \delta^k \widehat{mc}_{t+k} \right] \\
&= (1-\theta\beta)\mathbb{E}_t \sum_{k=0}^{\infty} \delta^k \left[m_\pi^f \frac{1}{1-\delta} \pi_{t+k} 1_{\{k>0\}} - \omega m_\pi^f \frac{1}{1-\delta} \pi_{t+k-1} 1_{\{k>0\}} + m_\mu^f \widehat{mc}_{t+k} \right] \\
&= \mathbb{E}_t \sum_{k=0}^{\infty} \delta^k \left[m_\pi^f \frac{1-\theta\beta}{1-\delta} \pi_{t+k} 1_{\{k>0\}} - \omega m_\pi^f \frac{1-\theta\beta}{1-\delta} \pi_{t+k-1} 1_{\{k>0\}} + m_\mu^f (1-\theta\beta) \widehat{mc}_{t+k} \right] \\
&= \mathbb{E}_t \sum_{k=0}^{\infty} \delta^k \left[\widetilde{m}_\pi^f \pi_{t+k} 1_{\{k>0\}} - \omega \widetilde{m}_\pi^f \pi_{t+k-1} 1_{\{k>0\}} + \widetilde{m}_\mu^f \widehat{mc}_{t+k} \right] \qquad (37)
\end{aligned}$$

where $\widetilde{m}_\pi^f = m_\pi^f \frac{1-\theta\beta}{1-\delta}$ and $\widetilde{m}_\mu^f = m_\mu^f (1-\theta\beta)$. Rewriting the price evolution expression (16),

$$p_t^* - p_{t-1} + p_t - p_t = \frac{\pi_t - \theta\omega\pi_{t-1}}{1-\theta} \implies p_t^* - p_t = \frac{\theta}{1-\theta}(\pi_t - \omega\pi_{t-1})$$

Hence, we can rewrite (37) as

$$\frac{\theta}{1-\theta}(\pi_t - \omega\pi_{t-1}) = \mathbb{E}_t \sum_{k=0}^{\infty} \delta^k \left[\widetilde{m}_\pi^f \pi_{t+k} 1_{\{k>0\}} - \omega \widetilde{m}_\pi^f \pi_{t+k-1} 1_{\{k>0\}} + \widetilde{m}_\mu^f \widehat{mc}_{t+k} \right] \quad (38)$$

Let us now introduce the forward operator F such that $F^k y_t = y_{t+k}$. Using the forward operator, we can write

$$\sum_{k=0}^{\infty} \delta^k y_{t+k} = \sum_{k=0}^{\infty} \delta^k F^k y_t = \sum_{k=0}^{\infty} (\delta F)^k y_t = \frac{y_t}{1-\delta F} \tag{39}$$

Rewriting (38) using (39)

$$\begin{aligned}
\frac{\theta}{1-\theta}(\pi_t - \omega\pi_{t-1}) &= \widetilde{m}_\pi^f \mathbb{E}_t \left[\sum_{k=0}^{\infty} \delta^k \pi_{t+k} 1_{\{k>0\}}\right] - \omega \widetilde{m}_\pi^f \mathbb{E}_t \left[\sum_{k=0}^{\infty} \delta^k \pi_{t+k-1} 1_{\{k>0\}}\right] + \widetilde{m}_\mu^f \mathbb{E}_t \left[\sum_{k=0}^{\infty} \delta^k \widehat{mc}_{t+k}\right] \\
&= \widetilde{m}_\pi^f \mathbb{E}_t \left[\sum_{k=0}^{\infty} (\delta F)^k \pi_t 1_{\{k>0\}}\right] - \omega \widetilde{m}_\pi^f \mathbb{E}_t \left[\sum_{k=0}^{\infty} (\delta F)^k \pi_{t-1} 1_{\{k>0\}}\right] + \widetilde{m}_\mu^f \mathbb{E}_t \left[\sum_{k=0}^{\infty} (\delta F)^k \widehat{mc}_t\right] \\
&= \widetilde{m}_\pi^f \mathbb{E}_t \left[\sum_{k=0}^{\infty} (\delta F)^k \pi_t - \pi_t\right] - \omega \widetilde{m}_\pi^f \mathbb{E}_t \left[\sum_{k=0}^{\infty} (\delta F)^k \pi_{t-1} - \pi_{t-1}\right] + \widetilde{m}_\mu^f \mathbb{E}_t \left[\sum_{k=0}^{\infty} (\delta F)^k \widehat{mc}_t\right] \\
&= \widetilde{m}_\pi^f \mathbb{E}_t \left[\frac{\pi_t}{1-\delta F} - \pi_t\right] - \omega \widetilde{m}_\pi^f \mathbb{E}_t \left[\frac{\pi_{t-1}}{1-\delta F} - \pi_{t-1}\right] + \widetilde{m}_\mu^f \mathbb{E}_t \left[\frac{\widehat{mc}_t}{1-\delta F}\right] \\
&= \widetilde{m}_\pi^f \mathbb{E}_t \left[\frac{\delta F \pi_t}{1-\delta F}\right] - \omega \widetilde{m}_\pi^f \mathbb{E}_t \left[\frac{\delta F \pi_{t-1}}{1-\delta F}\right] + \widetilde{m}_\mu^f \mathbb{E}_t \left[\frac{\widehat{mc}_t}{1-\delta F}\right]
\end{aligned}$$

Premultiplying by $(1-\delta F)$,

$$\frac{\theta}{1-\theta}(1-\delta F)(\pi_t - \omega\pi_{t-1}) = \widetilde{m}_\pi^f \mathbb{E}_t [\delta F \pi_t] - \omega \widetilde{m}_\pi^f \mathbb{E}_t [\delta F \pi_{t-1}] + \widetilde{m}_\mu^f \mathbb{E}_t [\widehat{mc}_t]$$

which can be rearranged to (17). Let us now derive the Behavioural Hybrid New Keynesian Phillips curve. We have the following expressions

$$mc_t = w_t - p_t - a_t \tag{40}$$

$$y_t = a_t + n_t \tag{41}$$

$$w_t - p_t = \varphi n_t + \frac{\sigma}{1-h} c_t - \frac{\sigma h}{1-h} c_{t-1} \tag{42}$$

$$c_t = y_t \tag{43}$$

Hence, we can write

$$
\begin{aligned}
mc_t &= w_t - p_t - a_t \\
&= \varphi n_t + \frac{\sigma}{1-h} c_t - \frac{\sigma h}{1-h} c_{t-1} - a_t \\
&= \varphi(y_t - a_t) + \frac{\sigma}{1-h} c_t - \frac{\sigma h}{1-h} c_{t-1} - a_t \\
&= \varphi(y_t - a_t) + \frac{\sigma}{1-h} y_t - \frac{\sigma h}{1-h} y_{t-1} - a_t \\
&= \left(\varphi + \frac{\sigma}{1-h}\right) y_t - \frac{\sigma h}{1-h} y_{t-1} - (1+\varphi) a_t
\end{aligned}
$$

In the natural equilibrium (with no price frictions), the marginal cost is constant at its steady-state level

$$
mc_t^r = mc = -\mu = \left(\varphi + \frac{\sigma}{1-h}\right) y_t^n - \frac{\sigma h}{1-h} y_{t-1}^n - (1+\varphi) a_t
$$

hence, we can write

$$
\widehat{mc}_t = mc_t - mc = \left(\varphi + \frac{\sigma}{1-h}\right) \widetilde{y}_t - \frac{\sigma h}{1-h} \widetilde{y}_{t-1}
$$

which, inserted into the (17) yields the Behavioural Hybrid New Keynesian Phillips curve (19).