Macroeconomics II
Problem Set 5

Jose E. Gallegos

April 29, 2020

Science: This last problem set contains two exercises. The first considers a small open economy model with complete asset markets. The second exercise considers a quantitative model of sovereign default à la Eaton & Gersovitz. You can solve this model by using value function iteration as in the Matlab scripts grid.m, solveEGVFI.m and simulateEG.m uploaded on Mondo.

Exercise 1: Complete Asset Markets and a Discrete Endowment Process

The economy is populated by a large number of identical agents (countries) with preferences described by the utility function

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t\right)$$

where u is a strictly increasing, concave and differentiable function. Each agent has initial financial wealth bo that is exogenous and measured in terms of the consumption good c. In each period, there are two possible states of nature, H and L, with transition probability matrix

$$\pi = \begin{bmatrix} \pi(H,H) & \pi(H,L) \\ \pi(L,H) & \pi(L,L) \end{bmatrix}$$

where $\pi(i,j)$ denotes the probability that the state of nature in period $t+1$ is $j \in \{H,L\}$ conditional on the state of nature in t being i $\in \{H,L\}$. Each agent is endowed with $y_t = \{y_H, y_L\}$ units of consumption goods in t where $y_H > y_L$. They also have access to the world financial market which offers a complete set of state-contingent claims. Let $p_{t+1}^t(i,j)$ be the period t price of a statecontingent claim that pays a unit of consumption in $t+1$ if the state of nature in $t+1$ is $j \in \{H,L\}$ conditional on the state of nature in t being $i \in \{H,L\}$. Suppose that the state of nature in period 0 is H and that $p_{t+1}^t(i,j) = \beta\pi(i,j) \forall t$

(a) State the agent's optimization problem.

(b) Characterize the equilibrium process of consumption and the trade balance.

(c) Derive an expression for the rate of return on a risk-free bond r_t

(d) In the standard SOE-RBC model agents can only trade in a single asset, namely a riskfree bond. In that model, the marginal utility of consumption follows a random walk when $(1 + r_t)\,\beta = 1 \forall t$ which in turn leads to a random walk in optimal consumption and the trade balance. In the light of this result, explain why complete asset markets induce a stationary solution.

Exercise 2: The Eaton-Gersovitz Model

Consider the standard Eaton-Gersovitz model where preferences are captured by the instantaneous utility function

$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}$$

and countries receive a stochastic and exogenous endowment $y \in Y = [y_{\min}, y_{\max}]$. In each period a country that defaults or that has defaulted previously is in bad financial standing, and a country that so far never defaulted is in good financial standing. A country that is in good financial standing faces a budget constraint $c + d = y + q(d')\, d'$ and a country that is in bad financial standing consumes its endowment ($c = y$). The default set ($D(d)$) contains all endowment levels at which a country chooses to default given a particular level of debt (d)

$$D(d) = \left\{ y \in Y : v^b(y) > v^c(d, y) \right\}$$

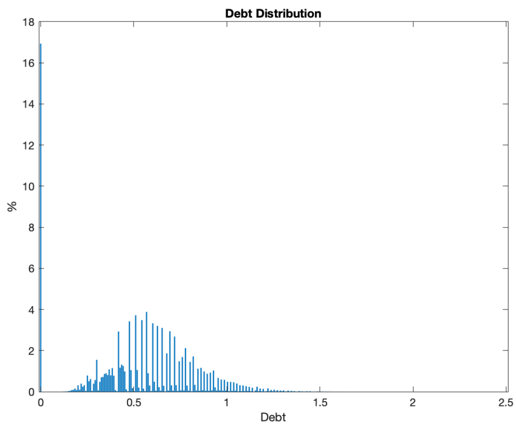
and provided that this is not an empty set, $D(d) = [y_{\min}, y^*(d)]$ where $y^*(d)$ is increasing in d if $y^*(d) < y_{\max}$. Foreign lenders are risk-neutral and perfectly competitive and the net world interest rate is $r^* > 0$

(a) State the value functions associated with bad financial standing, continuing to participate in capital markets and good financial standing. Also state the trade balance and the current account.

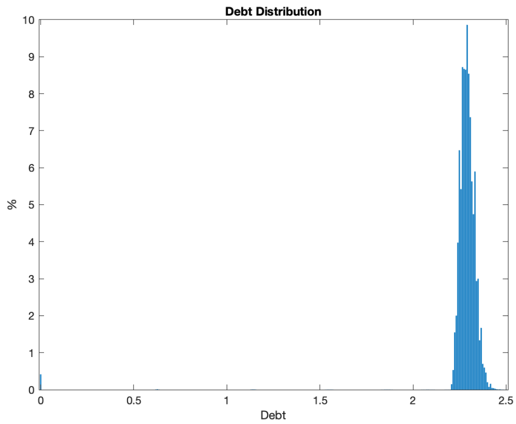
(b) What is the relation between the gross country interest rate premium and the probability of repayment (or an approximation of the probability of default)? Explain how a positively serially correlated endowment process affects this relationship.

Suppose now that the endowment process is serially correlated according to $\ln y_t = \rho \ln y_{t-1} + \sigma_\varepsilon \varepsilon_t, \varepsilon_t \sim N(0,1)$ and assume that a country that is in bad financial standing faces the probability θ of transiting to good financial standing in each period. Moreover, let countries in bad financial standing incur the loss $L(y_t) = \max\left\{0, a_0 + a_1 y_t + a_2 y_t^2\right\}$ in each period t, i.e. so that the net endowment is $\tilde{y}_t = y_t - L(y_t)$

(c) Use the the Matlab script grid.m that generates a grid for $\ln y$ to get a grid for y when $a_0 = 0, a_1 = -0.35$ and $a_2 = \frac{1-a_1}{2}\frac{1}{y_{\max}}$ and solve the model with value function iteration by using solveEGVFI.m. Simulate the model by using simulateEG.m. What is the average per period output loss of being in bad financial standing as share of total output, conditional on being in bad financial standing? Present the following quantitative predictions: The default frequency (per century), $\mathbb{E}[d/y], \mathbb{E}\left[r - r^*\right], \sigma\left(r - r^*\right), \operatorname{corr}\left(r - r^*, y\right)$ and $\operatorname{corr}\left(r - r^*, tb/y\right)$.[1] Show a graph of the distribution of debt in this economy and discuss how the output loss function affects the results. "Note that these are quarterly values, except for the annual country premium $r - r^*$



(d) Set $a_0 = a_2 = 0$ and a_1 to match the conditional average output loss of being in bad financial standing calculated in (c). Discuss how the quantitative predictions change. In particular, how does the modification of the loss function affect the model's ability to predict that countries default in bad times?



```
% Code adapted from the work of Gualtiero Azzalini (IIES)

clear
close all
clc

%% Part (c)

% generate the grid
run grid

% solve the model
outputloss = 'quadratic'; %the options are 'constant' or 'quadratic'
save loss_spec outputloss

run solveEGVFI

% simulate the model
run simulateEG

% statistics
run statistics

% debt distribution
Figure1 = figure;
bar(d,Ddistr);
xlabel('Debt');
ylabel('%');
title('Debt Distribution')
saveas(Figure1,'dist_quadratic.png')

Figure2 = figure;
plot(ya)
hold on
plot(y,'--')
hold off
xlabel('Grid point (y)')
ylabel('Output')
legend('Autarky', 'Good financial standing', 'Location', 'NorthWest')
saveas(Figure2,'loss_quadratic.png')

% save conditional average to calibrate the constant case
save cond_av loss

%% Part (d)

close all

```

```
% solve the model
outputloss = 'constant';  %the options are 'constant' or 'quadratic'
save loss_spec outputloss

run solveEGVFI

% simulate the model
run simulateEG

% statistics
run statistics

% debt distribution
Figure3 = figure;
bar(d,Ddistr);
xlabel('Debt');
ylabel('%');
title('Debt Distribution')
saveas(Figure3,'dist_constant.png')

Figure4 = figure;
plot(ya)
hold on
plot(y,'--')
hold off
xlabel('Grid point (y)')
ylabel('Output')
legend('Autarky', 'Good financial standing', 'Location', 'NorthWest')
saveas(Figure4,'loss_constant.png')

close all
```

```matlab
%grid.m
% discretizes the process y_t = rho * y_t-1 + sigma_epsilon * epsilon_t,
% where epsilon_t follows a standard normal distribution.
% Output:
% ygrid is the grid  of y_t values
% pai  transition probability matrix,

clear all

rho              = 0.931654648380119;
sigma_epsilon    = 0.036960096814455;
W                = 4.2; % width of y grid around its mean (0) in terms of standrad devs.

N1               = 200;      % initial number of grid  points.
                             % May be reduced if some points are never visited.

T = 1e7;     % Length of the time series of simulated values of output
             % (used for estimating the transition probability matrix)

% set randomization seed
% randn('state',0);
rng('default')
stdy= sigma_epsilon/sqrt(1-rho^2); % unconditional variance of y_t

UB = W*stdy;                        % highest value of y grid
ygrid = linspace(-UB,UB,N1);        % linearly spaced grid for y

PAI = zeros(N1); % initialize of transition probability matrix

% Simulate time series for log(gdp_traded)
% initialize simulation
[~,i0] = min(abs(ygrid));
y0 = ygrid(i0); % starting point = value of y in middle of grid

fprintf('Creating probability matrix...\n');
% Drawing
for t=2:T
y1 = y0*rho + randn*sigma_epsilon;
[~,i1] = min(abs(y1-ygrid));

% assign draw to transition probability matrix
PAI(i0,i1) = PAI(i0,i1) + 1;
y0 = y1;
i0 = i1;
end
fprintf('Done!\n');
```

```
% eliminate all rows and columns with all elements equal to zero
pai = PAI(sum(PAI,2)~=0,sum(PAI,1)~=0);
pai = pai ./ repmat(sum(pai,2),[1,size(pai,2)]);
keep = find(sum(PAI,2)~=0);
ygrid = ygrid(keep);
ygrid = ygrid(:);
N = length(ygrid);

save grid.mat pai ygrid
```

```
%solveEGVFI.m
% Use a value-function-iteration procedure to obtain the policy function of  the
% Eaton and Gersovitz default model.
% output:
% dpc is the policy function for debt under continuation. It is an ny-by-nd matrix, where ny is the number of output grid points and nd is the number of debt grid points
% dpix is the policy function for debt under continuation (dpc) but expressed in terms of the index in the debt grid. That is, dpc=d(dpix).
% vc is the value function under continuation, an ny-by-nd matrix.
% vb value function under bad standing, an ny-by-nd matrix.
% vg value function under good financial standing, an ny-by-nd matrix.
% q is the price of debt, an ny-by-nd matrix.  Note, the rows of q indicate y_t and the columns d_{t+1} (not d_t).
% tauc capital control tax under continuation, an ny-by-nd matrix. This fiscal instrument is the one that decentralizes the Eaton-Gersovitz model.
% lac, lag, marginal utility of consumption under continuation and good standing, respectively, both  ny-by-nd matrices.
% Elagp  the expected value of  next period's marginal utility of consumption, la_{t+1}, conditional on  continuation in t.
% This object is useful to compute the capital control tax rate, tauc, in period t.

 clear all

% outputloss = 'quadratic';  %the options are 'constant' or 'quadratic'

% load specification of the function
load loss_spec

%% Parameters & Grids
%Exogenous Process for Endowment:
load grid.mat ygrid pai     % ygrid=endowment grid; associated pai=transition probability matrix
ny = numel(ygrid);          % number of grid points for log of ouput
y = exp(ygrid(:));          % level of output

%Calibrated parameters
rstar = 0.01;     % quarterly risk-free interest rate (Chatterjee and Eyigungor, AER 2012).
theta = 0.0385;   % probability of reentry (USG  and Chatterjee and Eyigungor, AER 2012).
sig   = 2;        % intertemporal elasticity of consumption substitution
beta  = 0.85;     % discount factor, from Na, Schmitt-Grohe, Uribe, and Yue (2014)

%Output loss function
if strcmp(outputloss,'constant')
    load cond_av
```

```
    a0 =     0;
    a1 = loss; % set it to match conditional average
    a2 = 0;
    clear loss
end
if strcmp(outputloss,'quadratic')
    a0 = 0;
    a1 = -0.35;
    a2 = ((1-a1)/2)*(1/max(y));
end

yl = max(0,a0+ a1*y + a2*y.^2); % output loss
ya =  y - yl;                   % output in autarky

%debt grid
dupper  = 2.5;
dlower  = 0;
nd      = 300; % # of grid points for debt
d       = linspace(dlower,dupper,nd);
d       = d';

%force the element closest to zero to be exactly zero
[~,nd0] = min(abs(d));
d(nd0)  = 0;

n = ny*nd; % total number of states

% matrix for  indices of output as a function of the current state (size ny-by-nd)
yix = repmat((1:ny)',1,nd);

% matrix for  indices of debt as a function of the current state (size ny-by-nd)
dix = repmat(1:nd,ny,1);

% Consider a generic n-by-nd matrix Xtry. Each row of Xtry indicates one of
% the n possible states in the current period  and columns correspond to all
% (nd) possible values for debt assumed in the current peirod and due in
% the next period under continuation.
% The variable X could be d_t, y_t, d_{t+1}, etc.
dtry = repmat(d',[ny 1  nd]);
dtry = reshape(dtry,n,nd);

dptryix = repmat(1:nd,n,1);
dptry = d(dptryix);

ytryix = repmat(yix,nd,1);
ytry = y(ytryix);

% AUTARKY
```

```
% Consumption under autarky
ca = repmat(ya, [1 nd]);             % consumption of under bad standing
ua = ( ca.^(1-sig)-1)  / (1-sig);   % period utility under autarky

%Initialize the Value functions
vc       = zeros(ny,nd);          % continue repaying
vcnew    = vc;
vg       = zeros(ny,nd);          % good standing
vb       = zeros(ny,nd);          % bad standing
vr       = zeros(ny,nd);          % reentry
dpix     = zeros(ny,nd);          % debt policy function (expressed in indices)
dpixnew  = zeros(ny,nd);
f        = zeros(ny,nd);          % default indicator 1 if default, 0 otherwise; f
    maps (y_t,d_t)->{1,0}
q        = ones(ny,nd)/(1+rstar);% q is price of debt; it is a function of  (y_t,
    d_{t+1})

dist = 1;
while dist>1e-8

qtry = repmat(q,[nd 1]);
ctry =  dptry .* qtry - dtry + ytry;
utry = (ctry.^(1-sig) -1)  / (1-sig);
utry(ctry<=0) = -inf;

evgptry = pai *  vg;
Evgptry = repmat(evgptry,nd,1);

[vcnew(:), dpixnew(:)] = max(utry+beta*Evgptry,[],2); % maximum value for each row

vbnew = ua+beta*pai*(theta*vr + (1-theta) * vb);

f = vc<vb;

qnew = (1- pai*f)/(1+rstar);

dist = max(abs(qnew(:)-q(:))) + max(abs(vcnew(:)-vc(:))) + max(abs(vbnew(:)-vb(:))
    ) + max(abs(dpixnew(:)-dpix(:)));
distance = ['dist = ', num2str(dist)];
disp(distance);

q = qnew;
vc = vcnew;
vb = vbnew;
vg = max(vc,vb);
vr = repmat(vg(:,nd0),1,nd);
dpix = dpixnew;

```

```
end %while dist >...


% Policy functions under continuation;

% debt choice under continuation
dpc = d(dpix);

% consumption of tradables  under continuation
I    = sub2ind([ny nd],yix,dpix);
cc   = q(I).*dpc+y(yix)-d(dix);

% consumption  under good standing
cg = ca.*f + cc.*(1-f);

% marginal utility of consumption under continuation
lac = cc.^(-sig);

% marginal utility of consumption  in good standing
lag = cg.^(-sig);

% marginal utility of consumption in autarky
laa = ca.^(-sig);

% marginal utility of consumption under reentry;
lar = repmat(lag(:,nd0),1,nd);

% Elagp=E_tla_{t+1} if continuation in t
pai*lag;
Elagp =  ans(I);

% capital controls under continuation
beta* Elagp ./ lac ./q(I);
tauc = 1-ans;

% price of debt under autarky
beta*theta*pai*lar + beta*(1-theta)*pai*laa;
qa = ans./laa;

clear ans *try *new *tryix I

eval(['save eg_' outputloss])
```

```
%simulateEG.m
% produces time series implied by the Eaton-Gersovitz  model.

clear all, format compact
load loss_spec
rng('default')

eval(['load eg_' outputloss]) %load solution results

cpai = cumsum(pai,2); %cumulative prob distribution

%Initial conditions
[~,i] = min(abs(y-1)); %average endowment (index)
j = floor(nd/2); %middle of debt grid (index)
b = 0;  %b=0==>bad standing at the end of prevous period;
        %b=1==>good standing at the end of previous period

%Length of simulation and burning period
T = 1e6;
Tburn = 1e5;

Y = zeros(Tburn+T,1);
Ytilde = zeros(Tburn+T,1);
YA = zeros(Tburn+T,1);
B = zeros(Tburn+T,1);
D = zeros(Tburn+T,1);
C = zeros(Tburn+T,1);
Q = zeros(Tburn+T,1);
F = zeros(Tburn+T,1);
TAU = zeros(Tburn+T,1);
STATE = zeros(Tburn+T,1);
R = zeros(Tburn+1,1);
PM = zeros(Tburn+1,1);
rstar_annual = ((1+rstar)^4-1)*100; %world interest rate (annualized)

fprintf('Simulating...\n');
for t=1:T+Tburn

    STATE(t,1) = sub2ind([ny nd ],i,j);

    rr = rand; %random number  determining reentry if applicable
    F(t,1) = f(i,j); %F(t)=1 if default event in period t, 0 otherwise
    D(t,1) = d(j);
    Y(t,1) = y(i);
    YA(t,1) = ca(i,j);  %output in autarky

    if (b==0) & (F(t)==0) ; %choose to continue
        B(t,1) = 0; %B=0==>good standing at the beginning of current period, after
```

```
                    %receving reentry signal if applicable, but before making a
                    %default decision. .
                    %B==1==>bad  standing at the beginning of current period, after
                    %receving reentry signal
        Ytilde(t,1) = y(i);
        C(t,1) = cc(i,j);
        Q(t,1) = q(i,dpix(i,j));
        TAU(t,1) = tauc(i,j)*100;
        jp  = dpix(i,j); %update debt state
    end

    if (b==0) & (F(t) ==1); %choose to default
        B(t,1) = 0;
        Ytilde(t,1) = YA(t);
        C(t,1) = YA(t);
        Q(t,1) = qa(i,j);
        TAU(t,1) = 0;
        jp = nd0;
    end

    if (b==1) & (rr>theta);  %==> autarky (bad standing and did not get to re-enter)
        B(t,1) = 1;
        Ytilde(t,1) = YA(t);
        C(t,1) = YA(t);
        Q(t,1) = qa(i,j);
        TAU(t,1) = 0;
        jp = nd0;
    end

    if (b==1) & (rr<=theta) ; %reentry after having been in bad standing
        B(t,1) = 0;
        Ytilde(t,1) = Y(t);
        C(t,1) = cc(i,nd0);
        Q(t,1) = q(i,dpix(i,nd0));
        TAU(t,1) = tauc(i,nd0)*100;
        jp = dpix(i,nd0);
    end

    R(t,1) = ((1/Q(t,1))^4-1)*100; %country interest rate
    PM(t,1) = R(t,1)-rstar_annual;%country risk premium

    %update output state
    find(cpai(i,:)>rand);
    i = ans(1);

    b = B(t) + F(t);
```

```
    %update debt state
    j=jp;

end %for t=1:T+Tburn
fprintf('Done!\n');

%eliminate burning period
STATE = STATE(Tburn+1:end);
Y = Y(Tburn+1:end);
YA = YA(Tburn+1:end);
Ytilde = Ytilde(Tburn+1:end);
B = B(Tburn+1:end);
D = D(Tburn+1:end);
C = C(Tburn+1:end);
Q = Q(Tburn+1:end);
F = F(Tburn+1:end);
TAU = TAU(Tburn+1:end);
PM = PM(Tburn+1:end);
R = R(Tburn+1:end);


Ddistr = NaN(nd,1); %debt distribution
for i=1:nd
    Ddistr(i,1) = mean(D==d(i))*100;
end

eval(['save simu_eg_' outputloss])
```

```
% statistics.m
% this file computes the statistics and the debt distribution

% compute trade balance as fraction of output
TB_Y = (Ytilde - C)./Ytilde;

% bad financial standing (BFS): either default in current period F==1 or bad
% standing after signal B==1
x_bad = find(F==1|B==1);

% good financial standing (GFS): those who are in good financial standing (B==0)
% and do not default (F==0)
x_good = find(F==0&B==0);

% average per-period loss of BFS as share of total output, conditional on BFS
loss = mean((Y(x_bad)-Ytilde(x_bad))./Y(x_bad));

% default frequency
def_freq = mean(F)*4*100;

% other variables needed
dy = D(x_good)./Ytilde(x_good);        % debt/output
pm = PM(x_good);                       % country premium
tb = TB_Y(x_good);                     % trade balance over output
temp1 = corrcoef(pm,Ytilde(x_good));
corr_pm_y = temp1(2,1);                % correlation risk premium and output
temp2 = corrcoef(pm,tb);
corr_pm_tb = temp2(2,1);               % correlation risk premium and trade balance
    over output

Statistics = table;
Statistics.Av_loss      = loss*100;
Statistics.Def_prob     = def_freq;
Statistics.Mean_dy      = mean(dy)*100;
Statistics.Mean_prem    = mean(pm);
Statistics.SD_prem      = std(pm);
Statistics.Rho_prem_y   = corr_pm_y;
Statistics.Rho_prem_tb  = corr_pm_tb;
Statistics
```