Macroeconomics II
Problem Set 4
Suggested Solutions

Jose E. Gallegos

May 5, 2020

Science: In the first lectures of this second part of the course we have gone through international businesscycle facts and the open endowment economy, where the main difference has been the introduction of the trade balance and current account. In this problem set, given three distinct settings of the open endowment economy model, we will study the effects of permanent changes in producticity, anticipated interest rate changes and conclude by obtaining the predicted second moments of the model.

Exercise 1: An Economy with Endogenous Labor Supply

Consider a small open economy with preferences described by the utility function

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U\left(c_t, h_t\right)$$

where U is a period utility function given by

$$U(c, h) = -\frac{1}{2}\left[(\bar{c} - c)^2 + h^2\right]$$

where $\bar{c}$ is a satiation point. The household's budget constraint is given by

$$d_t = (1 + r)d_{t-1} + c_t - y_t$$

where d_t denotes real debt acquired in period t and due in period $t + 1$, and $r > 0$ denotes the world interest rate. Output, y_t, is produced according to

$$y_t = Ah_t$$

Households are also subject to the no-Ponzi-game constraint

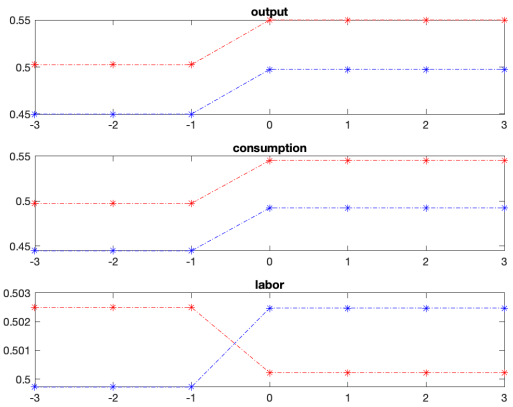
$$\lim_{j \to \infty} \mathbb{E}_t \frac{d_{t+j}}{(1 + r)^j} \leq 0$$

(a) Assuming that $\beta = (1 + r)^{-1}$, derive the equilibrium laws of motion of consumption, labor, debt, the trade balance and the current account.

(b) Assume that in period 0, unexpectedly, the productivity parameter A increases permanently to $A' > A$. Plot the effect of this shock on output, consumption, labor, the trade balance, the current account, and the stock of debt considering two cases:

$$\begin{aligned} \text{Case 1}: &\quad A = 0.9 \text{ and } A' = 1.1A \\ \text{Case 2}: &\quad A = 1 \text{ and } A' = 1.1A \end{aligned}$$

For both cases assume that $\bar{c} = 1, r = 0.05$ and that the initial real debt is 0.1. Comment your findings.



```
% Code adapted from the work by Mattias Almgren (IIES)
clear all;
clc;


%% Parameters
cbar     = 1;          % satiation point
r        = 0.05;       % exogeneous interest rate
d_init   = 0.1;        % initial level of real debt
Np       = 7;          % number periods in simulation

A1 = 0.9;              % initial productivitiy, Case 1
A2 = 1;                % initial productivitiy, Case 2

% Construct vector of shocks: shock affecting in period 4
Avec1    = [ones(floor(Np/2),1)*A1; ones(round(Np/2),1)*1.1*A1 ]; %productivity
    vector, Case 1
Avec2    = [ones(floor(Np/2),1)*A2; ones(round(Np/2),1)*1.1*A2 ]; %productivity
    vector, Case 2
Amat     = [Avec1 Avec2]; %Combine matrices

%% Simulation
% Preallocate vectors for storing results
y        = NaN(Np,2);
c        = NaN(Np,2);
h        = NaN(Np,2);
tb       = NaN(Np,2);
ca       = NaN(Np,2);
d        = NaN(Np,2);
t        = NaN(Np,2);    % time

for jj = 1:2
    %period 1
    c(1,jj)    = (1/(1+Amat(1,jj)^2))  * (Amat(1,jj)^2 * cbar - r * d_init);
    h(1,jj)    = (Amat(1,jj)/(1+Amat(1,jj)^2)) * (cbar+r * d_init);
    y(1,jj)    = Amat(1,jj)*h(1,jj);
    tb(1,jj)   = y(1,jj)-c(1,jj);
    ca(1,jj)   = 0;
    d(1,jj)    = d_init;
    t(1,jj)    = 1-round(Np/2);

    %periods 2 through Np
    for tt = 2:Np
        t(tt,jj)   = tt-round(Np/2);
        c(tt,jj)   = (1/(1+Amat(tt,jj)^2)) * (Amat(tt,jj)^2*cbar-r*d(tt-1,jj));
        h(tt,jj)   = (Amat(tt,jj)/(1+Amat(tt,jj)^2)) * (cbar + r*d(tt-1,jj));
        y(tt,jj)   = Amat(tt,jj)*h(tt,jj);
        tb(tt,jj)  = y(tt,jj)-c(tt,jj);
```

```
47          ca(tt,jj)  = tb(tt,jj) - r*d(tt-1,jj);
48          d(tt,jj)   = d(tt-1,jj);
49      end
50
51  end
52
53  %plot results
54  subplot(3,1,1)
55  plot(t,y(:,1),'-.b*')
56  hold on
57  plot(t,y(:,2),'-.r*')
58  title('output')
59  subplot(3,1,2)
60  plot(t,c(:,1),'-.b*')
61  hold on
62  plot(t,c(:,2),'-.r*')
63  title('consumption')
64  subplot(3,1,3)
65  plot(t,h(:,1),'-.b*')
66  hold on
67  plot(t,h(:,2),'-.r*')
68  title('labor')
69  saveas(gcf,'shock.png')
```

Exercise 2: Anticipated Interest Rate Decline

Consider a small open endowment economy enjoying free capital mobility. Preferences are described by the utility function

$$\sum_{t=0}^{\infty} \beta^t \ln c_t$$

with $\beta \in (0,1)$. Agents have access to an internationally traded bond paying the interest rate r_t when held from period t to period $t+1$. The representative household starts period zero with an asset position b_{-1}. Each period $t \geq 0$, the household receives an endowment y_t. Households know the time paths of $\{r_t\}$ and $\{y_t\}$ with certainty. The sequential budget constraint of the household and borrowing limit are given by, respectively

$$c_t + \frac{b_t}{1+r_t} = y_t + b_{t-1}$$
$$\lim_{j\to\infty} \frac{b_{t+j}}{\prod_{s=0}^{j}(1+r_{t+s})} \geq 0$$

(a) Derive the household's present value budget constraint.

(b) Derive the equilibrium paths of consumption and assets.

Assume now that in period 0 it is learned that in period $t^* \geq 0$ the interest rate will decline temporarily. Specifically, the new path of the interest rate is

$$r_t = \begin{cases} r_t & \text{for all } t \geq 0 \text{ and } t \neq t^* \\ r'_{t^*} < r_{t^*} & \text{for } t = t^* \end{cases}$$

(c) Consider a storage economy with $y_t = 0$ for all t and $b_{-1} > 0$. Find the impact of this anticipated interest rate cut on consumption at the date of announcement; that is, find $\ln c'_0/c_0$ where c'_t denotes the equilibrium path of consumption under the new interest rate path, and c_t denotes the equilibrium path of consumption under the old interest rate path. Discuss whether this anticipated future rate cut stimulates demand at the time it is announced. Provide intuition.

(d) Consider now an endowment economy with $b_{-1} = 0$ and $y_t = y > 0$ for all t. Find the impact on consumption of this anticipated interest rate cut and analyze whether the impact in period 0 is equal in size to the anticipated interest rate cut and whether it depends on the anticipation horizon t^*. That is, do anticipated interest rate cuts have a smaller effect on current consumption the further in the future they will take place? Provide intuition for your findings.

Exercise 3: Predicted Second Moments

Consider a small open endowment economy with preferences described by the utility function

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[-\frac{1}{2} (c_t - \bar{c})^2 \right]$$

The sequential budget and the sequence of borrowing constraints are given, respectively, by

$$c_t + (1+r) d_{t-1} = y_t + d_t$$
$$\lim_{j \to \infty} \mathbb{E}_t \frac{d_{t+j}}{(1+r)^j} \leq 0$$

Output, y_t, follows an $AR(1)$ process

$$y_t - \bar{y} = \rho (y_{t-1} - \bar{y}) + \varepsilon_t$$

where $\varepsilon_t \overset{\text{iid}}{\sim} N\left(0, \sigma_\varepsilon^2\right)$. Assuming that $\rho = 0.9, \sigma_\varepsilon = 0.03, \bar{y} = 1, r = 1/\beta - 1 = 0.1, d_{-1} = \bar{y}/2$, and $y_{-1} = \bar{y}$, discuss the results after completing the following procedure

(i) Simulate the economy for 100 years and discard the first 50 years.

(ii) Compute growth rates of output and consumption and the trade-balance-to-output ratio.

(iii) Compute the respective standard deviations of the growth rates and the correlation between output growth and the trade-balance-to-output ratio (i.e., σ_{gy}, σ_{gc} and $\rho_{gy,tby}$).

(iv) Replicate steps 1-3 1,000 times while, for each replication, keeping record of σ_{gy}, σ_{gc} and $\rho_{gy,tby}$.

(v) Report the average of σ_{gy}, σ_{gc} and $\rho_{gy,tby}$ over the 1,000 replications.

```
% Code adapted from the work by Mattias Almgren (IIES)
clear all;
clc;

%% set parameters and starting values
par.rho                 = 0.9;      % AR(1) parameter for income process
par.sigma_varepsilon    = 0.03;     % variance of output shock
ybar                    = 1;        % mean output
par.r                   = 0.1;      % exogeneous interest rate
d_minus1                = ybar/2;   % starting value savings
y_minus1                = ybar;     % starting value income

%% Simulation
% simulate output process once
par.T           = 100;      % number periods to simulate.
par.Tdiscard    = 50;       % number periods to discard.
par.Niter       = 1000;     % number of replications

moments         = NaN(par.Niter,3); % pre-allocate matrix for storage of moments
rng(1234)                           % seed for random number generator. For
    reproducibility.
for iter = 1:1000
    % preallocate vectors to store realized outcome
    ytemp       = NaN(par.T,1);
    ctemp       = NaN(par.T,1);
    dtemp       = NaN(par.T,1);
    tbtemp      = NaN(par.T,1);
    ytemp(1)    = y_minus1;         % manually insert starting value for output
    dtemp(1)    = d_minus1;         % manually insert starting value for savings

    for tt = 2:par.T
        ytemp(tt)   = par.rho * ( ytemp(tt-1) - ybar ) + ybar + random('Normal',0,
            par.sigma_varepsilon,1,1);
        ctemp(tt)   = ybar + (par.r / (1 + par.r - par.rho) ) * ( ytemp(tt) - ybar
            ) - par.r*dtemp(tt-1);
        dtemp(tt)   = dtemp(tt-1) - ( (1-par.rho) / ( 1 + par.r - par.rho ) ) * (
            ytemp(tt) - ybar );
        tbtemp(tt)  = ( (1-par.rho) / (1+par.r-par.rho) ) * (ytemp(tt) - ybar) +
            par.r * dtemp(tt-1);
    end

    % compute trade-balance-to-output
    tby     = tbtemp./ytemp;

    % calculate growth rates, and discard par.Tdiscard first periods
    gy = log(ytemp) - lagmatrix(log(ytemp),1);
    gy = gy(par.Tdiscard+1:end);

```

```
44      gc = log(ctemp) - lagmatrix(log(ctemp),1);
45      gc = gc(par.Tdiscard+1:end);
46
47      tby = tby(par.Tdiscard+1:end);
48
49      % compute standard deviations of respective growth series
50      sigma_gy     = std(gy);
51      sigma_gc     = std(gc);
52      rho_gytby    = corr(gy,tby);
53
54      % store moments in result matrix
55      moments(iter,1) = sigma_gy;
56      moments(iter,2) = sigma_gc;
57      moments(iter,3) = rho_gytby;
58
59  end
60
61  average_moments = mean(moments);
62
63  disp(['sigma_gy = ',num2str(average_moments(1)),', sigma_gc = ',num2str(
        average_moments(2)),', rho_gytby = ',num2str(average_moments(3))])
```