Tariffs as Taxes on Capital

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Abstract

This paper explores the macroeconomic consequences of levying tariffs on imported investment goods, which directly affect the household's investment Euler equation. First, we construct a new multi-country and multi-sector investment input-output matrix to trace investment goods through international production chains. Second, we embed this in an open-economy New Keynesian model with production networks. In a uniform US tariff experiment, this channel more than doubles the impact contraction in GDP, with investment-goods exposure as the best predictor of aggregate output losses. Holding the average tariff fixed, redesigning its composition to avoid the investment network cuts cumulative domestic output losses by two-thirds.

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1. Introduction

Amid rising geopolitical tensions, tariffs have returned to the center of macroeconomic policy debate. The sectoral composition of this policy is central to the discussion: tariffs are levied good by good, and the literature has shown that accounting for this composition matters for assessing their macroeconomic effects (Caliendo and Parro 2015; Kalemli-Özcan *et al.* 2025). The sectoral margin matters well beyond final consumption: its quantitative weight greater for intermediate inputs, which account for the bulk of international trade and propagate tariff shocks across production networks (Hummels *et al.* 2001; Acemoglu *et al.* 2012). Yet an additional margin remains comparatively unexplored: a growing share of traded goods is used to build industries' capital stock. The import intensity of these investment goods is high, comparable to that of intermediate inputs and well above that of final consumption.[1] This paper argues that the investment margin is distinct and first order for the aggregate impact of tariffs.

Tariffs on capital goods propagate differently because they combine the standard contemporaneous effects of tariffs with an intertemporal investment margin. Like tariffs on intermediate inputs, they raise the domestic price of imported goods and affect costs, demand, and production. Accounting for the investment network adds a new direct effect: tariffs raise the price of the investment goods that firms use to build capital, so the cost of accumulating it rises. Investment then falls not only because its return is lower but because it has become more expensive to undertake, and this decline operates through the forward-looking investment Euler equation, before the capital stock has had time to adjust. The macroeconomic response to tariffs thus depends not only on exposure to traded goods, but also whether those goods are accumulated as capital.

Measuring this investment channel of tariffs requires an object that is absent from standard network models' calibrations: a multi-country investment input-output matrix. Input-output tables with bilateral sector-country detail are abundant for intermediate goods. Equivalent data on which sectors and countries supply the goods used in capital formation by each sector is missing in standard international databases. Existing sector-by-sector investment matrices have been built almost entirely for the United States, from the BEA Capital Flow Table (Vom Lehn and Winberry 2022; Foerster *et al.* 2022). We construct the international counterpart by combining national accounts with input-output tables, linking flows of

[1] Combining our newly constructed investment input–output matrix (described further below) with the corresponding consumption and intermediate-input matrices from the 2022 OECD ICIO tables, the trade intensity of investment goods for the US is 9.1 percent, nearly equal to the 8.7 percent for intermediates and well above the 5.7 percent for consumption — where trade intensity is defined as the sum of exports and imports divided by the sum of absorption and production.

investment goods across origin and destination sector-country pairs.[2]

We use the investment matrix to compare tariff propagation across consumption, intermediates, and capital uses in a multi-country, multi-sector New Keynesian model. The model's key feature is that investment, consumption, and intermediate demand are all treated as distinct uses of traded goods: we calibrate a consumption matrix and an intermediate-input matrix, and embed the constructed investment-goods input-output matrix. The world effectively contains five economies: the United States, China, Spain, the Rest of the Euro Area, and the Rest of the World; each have 44 production sectors. Households consume goods from different countries and sectors; firms use domestic and foreign intermediate inputs; and sectoral capital stocks are accumulated using investment bundles assembled from domestic and foreign goods. Tariffs are destination-origin-sector wedges, with effects on real activity given the sticky prices and wedges; monetary policy then responds endogenously to inflation and real activity. Distinctly from other papers in the literature, the sectoral composition of tariff packages additionally affects aggregate outcomes through the cost of accumulating capital, bringing the investment-network logic of Vom Lehn and Winberry (2022) and Peri *et al.* (2026) to trade policy.

The construction of the investment input-output matrix is the paper's main data contribution. With this matrix embedded in the model, the paper delivers three substantive findings. First, capital accumulation is a quantitatively large amplification channel for tariffs. In a uniform 10 percentage-point US import tariff experiment, US real GDP falls by 2.54 percent on impact in the baseline model, compared with 1.20 percent in an otherwise comparable economy without investment.[3] Employment falls by 3.02 percent rather than 1.07 percent, and investment falls by 14.52 percent. These magnitudes are larger than reduced-form panel estimates but the comparison is not one-for-one: our experiment is a large uniform shock and reports the impact response of a single open economy in general equilibrium, not a medium-run cross-country average.[4] The disciplining point is that the sign, persistence, and order of magnitude are consistent with the historical evidence on tariffs (den Besten and Känzig 2025). The model's large real effects come from a standard macroeconomic propa-

[2]Contemporaneous work constructs related investment matrices for other purposes. Ding (2026) builds a global matrix of capital-service coefficients for a period ending in 2007, used to study long-run gains from trade, and Casal and Caunedo (2024) infer investment linkages from occupations rather than national accounts. Relative to these, our matrix is designed to trace bilateral flows of investment goods and can be used to study both supply and demand shocks propagating through the investment network, as in the analysis of carbon pricing and border adjustment in Delgado-Téllez *et al.* (2025) and of public investment under NGEU in Fernández-Cerezo *et al.* (2024).

[3]The No Investment counterfactual removes physical capital and investment dynamics, but reallocates the baseline capital cost share to intermediate inputs. It therefore keeps exposure to traded goods while shutting down the intertemporal investment margin. Appendix C.1 gives the full construction.

[4]Furceri *et al.* (2022) find an output decline of roughly 0.4 percent five years after a one-standard-deviation (3.6 percentage-point) tariff increase.

gation margin—tariffs raise the price of investment goods, investment contracts, and the decline in investment demand propagates to output and labor demand—rather than from implausibly high pass-through.

Second, the aggregate effects of sectoral tariffs depend on which sectors the tariff targets and how those sectors are used. A tariff schedule should therefore be summarized by three exposure measures: final-demand exposure, intermediate-input exposure, and investment-goods exposure. These measures rank sectors differently and explain different outcomes. In the US sectoral experiments, investment-goods import exposure explains 77 percent of the cross-sector variation in aggregate real GDP response and 92 percent of the variation in aggregate investment response, while final-demand exposure explains 87 percent of the variation in CPI inflation. The result separates two questions that are often conflated. Which tariffs raise consumer prices? Those on goods that enter household expenditure. Which tariffs reduce real activity? Those on goods that enter production and capital formation.

Third, this composition channel substantially impacts the aggregate effect of tariff policy. First, we test this in an empirically relevant tariff episode, the realized US-China tariff changes in 2025–26, where realized tariffs were not randomly assigned across sectors. The baseline predicts a large short-run contractions in the United States and China, but the realized sectoral allocation is less contractionary than a uniform-equivalent tariff package with the same import-weighted aggregate bilateral tariff path. Then, we explore what this implies for the optimal composition of tariff packages for a given tariff level. We first rank individual sectoral tariffs by their macroeconomic incidence at home and abroad; the most effective targets combine a small impact on domestic output with a large impact on foreign output. Then, holding the import-weighted average tariff fixed, we ask which sectoral mix minimizes the domestic output loss, and which maximizes the loss imposed abroad at no additional domestic cost. Shifting tariffs away from capital goods and upstream inputs cuts the cumulative domestic loss by two-thirds, because it avoids the slow erosion of the capital stock that makes tariff damage persistent. These optimal compositions are characterized by the same exposure measures that organize the sectoral experiments, with investment-goods exposure doing most of the work.

Related Literature. This paper first contributes to the literature on tariffs and trade policy. Classic quantitative trade models emphasize the welfare consequences of trade barriers and the role of input-output linkages in shaping gains from trade (Hummels *et al.* 2001; Caliendo and Parro 2015). Empirical work on recent US tariffs studies pass-through, effects on prices and economic activity, and distributional incidence (Amiti *et al.* 2020; Fajgelbaum *et al.* 2020; Cavallo *et al.* 2021; den Besten and Känzig 2025). On the theory side, a growing macroeconomic literature studies the dynamic general-equilibrium effects of tariffs in economies without input-output linkages (Auclert *et al.* 2025; Auray *et al.* 2026), the design of optimal tariffs (Itskhoki and Mukhin 2025; Dávila *et al.* 2025), and the optimal monetary-policy response to trade policy (Bergin and Corsetti 2023; Monacelli 2025; Bianchi and Coulibaly 2025; Guerrieri *et al.* 2025). Closest to this paper, Kalemli-Özcan *et al.* (2025) study the aggregate effects of tariffs in a multi-country model with input-output linkages and trade in intermediate goods. Relative to this work, the paper identifies traded capital goods as a distinct macroeconomic propagation channel and how the sectoral composition of tariffs can change aggregate outcomes, holding fixed aggregate tariff intensity, once investment goods are properly accounted for. The optimal-tariff literature characterizes the welfare-optimal *level* of tariffs, with import tariffs uniform across goods in canonical settings (Costinot *et al.* 2015; Ignatenko *et al.* 2025), and a sanctions literature traces frontiers of maximal damage in static trade models (de Souza *et al.* 2024; Sturm 2024). We instead fix the average tariff and characterize its optimal short-run composition, a margin that operates through nominal rigidities and the investment network, and has no counterpart in either literature.

The paper also builds on the macroeconomic production-network literature. Long and Plosser (1983) and Acemoglu *et al.* (2012) show how input-output linkages transmit sectoral shocks. Baqaee and Farhi (2019), Baqaee and Rubbo (2023), and Rubbo (2023) develop quantitative frameworks in which nonlinearities, markups, and nominal rigidities shape aggregation. Atalay (2017) and Boehm *et al.* (2023) discipline key substitution margins. Our model uses these insights in an open-economy tariff environment, but separates exposure by use: the same sector can be unimportant for household prices and important for capital formation.

Finally, the paper relates to the macroeconomic literature on investment dynamics and capital accumulation. Since Kydland and Prescott (1982), capital has been central to business-cycle propagation. Christiano *et al.* (2005) and Justiniano *et al.* (2011) show that investment dynamics and investment-specific disturbances are important for aggregate fluctuations. Vom Lehn and Winberry (2022) and Peri *et al.* (2026) show that the sectoral composition of investment demand matters for comovement and multipliers. We bring this investment-network logic to tariffs, showing that the macroeconomic effects of trade policy depend on whether tariffs hit the goods that build the capital stock.

Roadmap. Section 2 describes the data. Section 3 presents the model. Section 4 derives a simple analytical benchmark showing how sectoral tariffs pass through differently depending on whether imported goods enter consumption, intermediate use, or capital formation. Section 5 calibrates the economy and studies uniform and sectoral tariff experiments. Section 6 applies the model to the realized US-China tariff changes in 2025–26 and derives cost-minimizing and damage-maximizing tariff compositions at a fixed average tariff. Section 7 concludes.

2. Investment Input-Output Data

Quantifying the investment channel of tariffs requires knowing where each sector's capital goods are produced. Standard international input-output tables do not contain this information. They record intermediate-input flows in full bilateral sector-to-sector detail, and they report gross fixed capital formation by origin sector-country and destination country; the investing sector, however, is not identified. The data show that machinery produced in Japan is installed in Germany, but not whether it equips German automakers, hospitals, or farms. In a model that treats investment as a single final-demand aggregate, this omission is immaterial. For trade policy it is not: a tariff on capital goods is a shock to the sectors that buy them, and exposure to that shock varies widely across investing sectors.

Sector-level investment data of this kind have so far existed only for the United States. Vom Lehn and Winberry (2022) construct a US investment network from the BEA Capital Flow Table and show that the sectoral composition of capital-goods demand shapes co-movement over the business cycle; Foerster *et al.* (2022) use related capital-flow data to study long-run sectoral trends. We construct the corresponding international object: a matrix of capital-goods flows recording origin country, origin sector, destination country, and destination sector. The two dimensions absent from the US constructions—the origin and destination countries—are precisely the dimensions on which trade policy operates. The remainder of this section describes the construction and documents the features of the matrix that drive the quantitative results.

2.1. The investment input-output matrix

For each destination country k and investing sector i, let $s^I_{kl\,ij}$ denote the share of investment expenditure spent on goods produced by sector j in country l, with $\sum_l \sum_j s^I_{kl\,ij} = 1$. The conditional matrix s^I is the capital-formation analogue of a sourcing matrix: it describes the composition of the investment bundle before any normalization by sector size. For the quantitative analysis, the same flows are also scaled by the investment intensity of the destination sector, $\omega^I_{kl\,ij} \equiv \chi_{ki} s^I_{kl\,ij}$, where χ_{ki} is investment expenditure by sector (k, i) relative to its gross sales. Thus $\sum_l \sum_j \omega^I_{kl\,ij} = \chi_{ki}$. The first object describes the composition of the investment bundle; the second scales that composition by the investment intensity of the destination sector and is directly comparable to the standard intermediate-input matrix.

No single data source contains these four dimensions. National accounts identify the assets purchased by each sector, but not the country and sector that produced those assets. OECD ICIO tables identify bilateral gross fixed capital formation flows by origin sector-country and destination country, but aggregate over the investing sectors within the destination. Our matrix combines the two sources: national accounts identify which destination sectors purchase which asset types, and international input-output tables identify where the corresponding capital goods are produced.

The bridge is an asset-to-producer-sector mapping. Starting from the national-accounts asset taxonomy, we form thirteen operational asset classes after the aggregations and refinements documented in Appendix D. These classes include construction structures, machinery, transport equipment, digital equipment, software and databases, research and development, and other intellectual-property assets. For each class we identify the corresponding NACE Rev. 2 producer sector. Most mappings are direct: structures map to construction, motor vehicles to C29, digital equipment to the C26–C27 producer-sector pair reported in Table D.1, software and databases to J62_63, and research and development to M and P. A few assets require destination-use information. Transport equipment combines aircraft, ships, and motor vehicles, which are produced by different sub-sectors and used by different industries; we split these flows by destination of use. Pharmaceutical capital is similarly assigned to its dominant user sector.

The matrix is then assembled in three steps. First, national accounts provide the share of each sector-country's investment in each asset class and the share of national investment in each asset class undertaken by each destination sector. Second, the bridge translates OECD ICIO capital-goods flows from origin sector-country to destination country into asset-level flows from origin asset-country to destination country. Third, the asset-level flows are allocated across destination sectors within each receiving country using the sector-investment shares from the first step.[5] The procedure preserves aggregate capital-goods flows from OECD ICIO while adding the missing destination-sector dimension. Asset-specific depreciation rates are computed as median implied depreciation rates over the pre-pandemic period, separately by asset class, and then mapped to producing sectors through the same bridge.

2.2. Empirical patterns of the investment network

The resulting matrix has three empirical features that are not visible in standard input-output data and that shape the quantitative results.

Investment-goods supply is concentrated. Capital goods are supplied by a narrow set of

[5]Where the sector-by-asset composition of investment is not directly observed for a country, we do not import it from other countries' sectoral distributions. We combine three country-specific pieces of information: the weight of each sector in the country's value added, which provides a size-based prior for how much of each asset a sector would purchase; the observed total purchases of each asset class from the OECD ICIO flows; and a projection of the sector-asset intensities that adjusts the prior so that, aggregated across sectors, it reproduces the observed asset totals. The sectoral composition of investment is thus anchored in the country's own sectoral structure and disciplined by its own observed asset purchases; only the finer sector-by-asset intensity is projected.

sectors. Construction, machinery, motor vehicles, other transport equipment, and electronics account for most tangible investment goods, while software, research and development, and selected professional services account for the capitalized-services margin. Many sectors that are important suppliers of intermediate inputs supply essentially no capital goods. The investment network is therefore not a relabeling of the intermediate-input network. It is a distinct map of how production capacity is built.

International exposure is concentrated within tradable capital goods. Structures are produced locally, so construction is large in investment but weakly exposed to direct cross-border sourcing. By contrast, machinery, electronics, motor vehicles, and other transport equipment are both central to capital formation and heavily traded. These sectors are where the international dimension of the investment matrix matters most. A tariff on the average imported good and a tariff on an imported capital good are therefore different shocks even before any model is specified: they hit different locations in the use structure of the economy.

Use is widespread. Almost every sector invests, but sectors buy different capital bundles. Manufacturing and transportation sectors are intensive users of machinery, transport equipment, and electronics; service sectors rely more heavily on structures, software, databases, and R&D. This asymmetry between concentrated supply and dispersed use is the key empirical reason to measure investment flows at the destination-sector level. A small set of supplying sectors can be connected to a broad cross-section of investing sectors, and those connections are not recoverable from aggregate investment data.

Taken together, these features explain why the matrix must be measured rather than proxied. It is international, because the origin of capital goods determines exposure to tariffs and other border wedges. It is sectoral on both sides, because the sectors that supply capital goods are not the sectors that use them. And it is distinct from the intermediate-input matrix, because capital formation follows its own pattern of supply and use. The remaining calibrated objects of the quantitative model are described in Section 5.1.

3. Theoretical Framework

The world economy contains K countries, indexed by k, and I sectors in each country, indexed by i. Within each country-sector pair there is a unit mass of monopolistically competitive firms, indexed by $f \in (0, 1)$. Firms produce differentiated varieties using labor, a bundle of intermediate inputs, and capital services. Both intermediate-input bundles and investment bundles are sourced from domestic and foreign sectors. Tariffs are destination-origin-sector wedges, so the same foreign sector can face different wedges depending on the importing country.

Two features make the model useful for the question in this paper. First, capital accumulation is sector specific: households own the capital stocks used by each sector and choose the optimal level of investment sector by sector. Second, investment is itself a produced bundle with a sectoral and international sourcing matrix. Thus, a tariff on imported machinery, electronics, or vehicles is not only a tax on current absorption. It also raises the price of accumulating in a way that is specific to the investing sector given its investment bundle; this is the capital used in future production. Because prices and wages are sticky, these relative-price movements are not undone by nominal adjustment and have real effects in the short run.

3.1. Households

Each country is populated by a representative household. The household consumes the final composite good, supplies labor, owns the domestic capital stocks, receives firm profits, and trades nominal bonds. Period utility is

$$U_t = U(C_{k,t}) - N_{k,t}^{1+\varphi}/(1+\varphi), \tag{1}$$

where $N_{k,t}$ is aggregate labor, $U(C_{k,t}) = C_{k,t}^{1-\sigma}/(1-\sigma)$, and φ is the inverse Frisch elasticity. Wage setting follows the standard sticky-wage environment of Erceg *et al.* (2000); wage rigidities can vary across countries but are common across sectors within a country.

Aggregate consumption is a constant-returns-to-scale composite of sectoral consumption. Sectoral consumption is, in turn, a composite of goods produced by the same sector in different countries:

$$C_{k,t} = \mathcal{C}_k\left(\{C_{ki,t}\}_{i=1}^{I}\right) \quad \text{and} \quad C_{ki,t} = \mathcal{C}_{ki}\left(\{C_{kl\,i,t}\}_{l=1}^{K}\right), \tag{2}$$

where $C_{ki,t}$ is country k's consumption of sector i, and $C_{kl\,i,t}$ is country k's consumption of the good produced by sector i in country l. Varieties within an origin-sector are aggregated as in Dixit and Stiglitz (1977):

$$C_{kl\,i,t} = \left(\int_0^1 C_{kl\,if,t}^{(\epsilon_{ki}^{p}-1)/\epsilon_{ki}^{p}} df\right)^{\epsilon_{ki}^{p}/(\epsilon_{ki}^{p}-1)} \tag{3}$$

where ϵ_{ki}^{p} is the elasticity of substitution across varieties.

The household's nominal budget constraint is

$$P_{k,t}^{C} C_{k,t} + \sum_{i=1}^{I} P_{ki,t}^{I} I_{ki,t} + \frac{B_{k,t}}{1+i_{k,t}} + \mathcal{E}_{k,t}^{K} \sum_{h\in\mathcal{H}} Q_t^h B_{k,t}^h \leq \sum_{i=1}^{I} R_{ki,t} K_{ki,t} \tag{4}$$

$$+W_{k,t}N_{k,t} + \mathcal{E}^K_{k,t} \sum_{h \in \mathcal{H}} (Q^h_t + D^h_t) B^h_{k,t-1} + B_{k,t-1} + \Pi_{k,t} - T_{k,t}$$

where $W_{k,t}$ is the nominal wage, $\Pi_{k,t}$ are profits of domestic firms, and $T_{k,t}$ is a lump-sum tax or transfer. The price $P^C_{k,t}$ is the consumption price index implied by (2). The price $P^I_{ki,t}$ is the price of the investment bundle used to accumulate the capital stock $K_{ki,t}$ in sector i. Sectoral capital evolves according to

$$K_{ki,t} = (1 - \delta_{ki}) K_{ki,t-1} + I_{ki,t-1} \tag{5}$$

We follow Vom Lehn and Winberry (2022) and let the household own the sectoral capital stocks and rent them to firms at rates $R_{ki,t}$.[6]

There are two financial assets. The first, $B_{k,t}$, is a one-period nominal bond traded domestically and paying the country-specific gross nominal return $1 + i_{k,t}$. The second is a set $\mathcal{H}$ of internationally traded bonds with prices Q^h_t and dividends D^h_t.[7] International bonds are denominated in the currency of country K, so $\mathcal{E}^K_{k,t}$ denotes the bilateral nominal exchange rate between country k and country K.

Cost minimization inside the consumption nest gives the allocation across sectors, origins, and varieties:

$$\frac{\partial \mathcal{C}_k}{\partial C_{ki,t}} = \frac{P^C_{ki,t}}{P^C_{k,t}}, \quad \frac{\partial \mathcal{C}_{ki}}{\partial C_{kli,t}} = \frac{(1 + \tau_{kli,t}) P^k_{li,t}}{P^C_{ki,t}}, \quad \text{and} \quad C_{klif,t} = \left(\frac{P_{lif,t}}{P_{li,t}} \right)^{-\epsilon^p_{ki}} C_{kli,t}, \tag{6}$$

where $P^C_{ki,t}$ is the sectoral consumption price index. The producer-currency price index for sector i goods produced in country l is

$$P_{li,t} \equiv \left(\int_0^1 P^{1-\epsilon^p_{ki}}_{lif,t} df \right)^{\frac{1}{1-\epsilon^p_{ki}}}, \qquad P^k_{li,t} = \mathcal{E}^l_{k,t} P_{li,t}, \tag{7}$$

where $P^k_{li,t}$ denotes that origin-sector price in country k's currency.[8] The wedge $\tau_{kli,t}$ is a tariff paid by agents in country k on purchases of sector i goods produced in country l, so the buyer price is $(1 + \tau_{kli,t}) P^k_{li,t}$.

[6] If firms owned their capital stocks instead, the same household would own the firms. Investment decisions would still be valued with the household's stochastic discount factor.

[7] This specification follows Egorov and Mukhin (2023). If $\mathcal{S}$ is the set of states, financial markets are complete when $\mathcal{H} = \mathcal{S}$. Setting $\mathcal{H} = 1$ gives the usual incomplete-markets economy with one internationally traded bond.

[8] Under constant returns, $P^C_{k,t} C_{k,t} = \sum_{i=1}^{I} P^C_{ki,t} C_{ki,t}$ and $P^C_{ki,t} C_{ki,t} = \sum_{l=1}^{K} P^k_{li,t} C_{kli,t}$.

The first-order condition for domestic bonds is the usual consumption Euler equation:

$$U'(C_{k,t}) = \beta \mathbb{E}_t \left[U'(C_{k,t+1})(1 + i_{k,t})/(1 + \pi^C_{k,t+1}) \right], \tag{8}$$

where $1 + \pi^C_{k,t+1} \equiv P^C_{k,t+1}/P^C_{k,t}$ is gross CPI inflation.

The key dynamic margin in the paper is the sectoral investment Euler equation:

$$P^I_{ki,t} U'(C_{k,t}) = \beta \mathbb{E}_t \left\{ U'(C_{k,t+1}) \left[R_{ki,t+1} + P^I_{ki,t+1}(1 - \delta_{ki}) \right] \right\} \tag{9}$$

It equates the marginal utility cost of buying one more unit of the sector-i investment bundle to the discounted value of next period's rental income and undepreciated capital. Tariffs affect this condition whenever they change $P^I_{ki,t}$.

3.2. Firms

Each country has I industries, and each industry has a unit mass of differentiated-goods producers.

Production. Firm f in country-sector (k, i) produces output $Y_{kif,t}$ with labor, intermediate inputs, and capital services:

$$Y_{kif,t} = F_{ki} \left(K_{kif,t}, N_{kif,t}, X_{kif,t} \right). \tag{10}$$

The intermediate-input bundle is nested. A firm first combines sectoral intermediate goods, then sources each sectoral input across countries:

$$X_{kif,t} = \mathcal{X}_{ki} \left(\{X_{kijf,t}\}_{j=1}^{I} \right) \quad \text{and} \quad X_{kijf,t} = \mathcal{X}_{kij} \left(\{X_{kl\,ijf,t}\}_{l=1}^{K} \right), \tag{11}$$

where $X_{kijf,t}$ is firm f's demand for sector j inputs, and $X_{kl\,ijf,t}$ is the part of that demand sourced from country l. Varieties inside origin-sector (l, j) are aggregated as

$$X_{kl\,ijf,t} = \left(\int_0^1 X_{kl\,ijff',t}^{(\epsilon^p_{kj}-1)/\epsilon^p_{kj}} df' \right)^{\epsilon^p_{kj}/(\epsilon^p_{kj}-1)} \tag{12}$$

Investment goods are organized in the same spirit, but their economic role is different. They determine the price at which sectoral capital can be accumulated. The bundle $I_{ki,t}$ is a

CRS composite of sectoral capital goods, each sourced across countries:

$$I_{ki,t} = \mathcal{I}_{ki}\left(\{I_{kij,t}\}_{j=1}^{I}\right) \quad \text{and} \quad I_{kij,t} = \mathcal{I}_{kij}\left(\{I_{kl\,ij,t}\}_{l=1}^{K}\right), \tag{13}$$

where $I_{kij,t}$ is investment demand by destination sector i for goods produced by sector j, and $I_{kl\,ij,t}$ is the component sourced from country l. The origin-sector investment good is itself a Dixit and Stiglitz (1977) aggregate over varieties:

$$I_{kl\,ij,t} = \left(\int_0^1 I_{kl\,ijf',t}^{(\epsilon_{kj}^p-1)/\epsilon_{kj}^p}\, df'\right)^{\epsilon_{kj}^p/(\epsilon_{kj}^p-1)} \tag{14}$$

Cost minimization delivers the static demand conditions for labor, intermediate inputs, and capital services:

$$W_{k,t} = \mathrm{MC}_{ki,t}\frac{\partial F_{ki}}{\partial N_{kif,t}}, \quad P_{kij,t}^X = \mathrm{MC}_{ki,t}\frac{\partial F_{ki}}{\partial X_{kijf,t}}, \quad R_{ki,t} = \mathrm{MC}_{ki,t}\frac{\partial F_{ki}}{\partial K_{kif,t}} \tag{15}$$

and the allocation of intermediate-input demand across sectors and countries:

$$\frac{\partial \mathcal{X}_{ki}}{\partial X_{kijf,t}} = \frac{P_{kij,t}^X}{P_{ki,t}^X}, \quad \frac{\partial \mathcal{X}_{kij}}{\partial X_{kl\,ijf,t}} = \frac{(1+\tau_{klj,t})P_{lj,t}^k}{P_{kij,t}^X}, \quad X_{kl\,ijff',t} = \left(\frac{P_{ljf',t}}{P_{lj,t}}\right)^{-\epsilon_{kj}^p} X_{kl\,ijf,t} \tag{16}$$

Above, $P_{ki,t}^X$ denotes the price index of the intermediate bundle used by sector i in country k, and $P_{kij,t}^X$ is the price index of the sector-j input used by sector i. Tariffs apply to imported intermediate goods in the same way as to consumption goods.[9]

Investment demand across origin sectors and countries follows the analogous conditions:

$$\frac{\partial \mathcal{I}_{ki}}{\partial I_{kij,t}} = \frac{P_{kij,t}^I}{P_{ki,t}^I}, \quad \frac{\partial \mathcal{I}_{kij}}{\partial I_{kl\,ij,t}} = \frac{(1+\tau_{klj,t})P_{lj,t}^k}{P_{kij,t}^I}, \quad \text{and} \quad I_{kl\,ijf',t} = \left(\frac{P_{ljf',t}}{P_{lj,t}}\right)^{-\epsilon_{kj}^p} I_{kl\,ij,t}, \tag{17}$$

Under constant returns, all firms in a country-sector choose the same input mix. Nominal marginal cost is therefore common within (k, i) and equals the unit cost:

$$\begin{gathered}\mathrm{MC}_{ki,t} = \min_{N_{ki,t}, X_{kl\,ij,t}, K_{ki,t}} W_{ki,t}N_{ki,t} + R_{ki,t}K_{ki,t} + \sum_{j=1}^{I}\sum_{l=1}^{K}(1+\tau_{klj,t})P_{lj,t}^k X_{kl\,ij,t} \\ \text{s.t. } F_{ki}(K_{ki,t}, N_{ki,t}, X_{ki,t}) = 1,\end{gathered} \tag{18}$$

[9] The intermediate-input aggregators imply $P_{ki,t}^X X_{kif,t} = \sum_{j=1}^{I} P_{kij,t}^X X_{kijf,t}$ and $P_{kij,t}^X X_{kijf,t} = \sum_{l=1}^{K}(1 + \tau_{klj,t})P_{lj,t}^k X_{kl\,ijf,t}$.

Functional-form Assumptions on Aggregators. For the quantitative analysis, production is CES:

$$Y_{fki,t} = \left[\widetilde{\varsigma}_{ki}^{\frac{1}{\psi}} K_{fki,t}^{\frac{\psi-1}{\psi}} + \widetilde{\alpha}_{ki}^{\frac{1}{\psi}} N_{fki,t}^{\frac{\psi-1}{\psi}} + \widetilde{\vartheta}_{ki}^{\frac{1}{\psi}} X_{fki,t}^{\frac{\psi-1}{\psi}} \right]^{\frac{\psi}{\psi-1}}. \tag{19}$$

Consumption, intermediate inputs, and investment goods are CES aggregates. We introduce an energy/non-energy layer because energy is a particularly important imported input and has a short-run elasticity that need not equal the elasticity among other goods:

(20)

$$C_{k,t} = \left[\widetilde{\beta}_{kC}^{\frac{1}{\gamma}} (C_{k,t}^{E})^{\frac{\gamma-1}{\gamma}} + \left(1 - \widetilde{\beta}_{kC}\right)^{\frac{1}{\gamma}} (C_{k,t}^{M})^{\frac{\gamma-1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}}, \quad I_{ki,t} = \left[\widetilde{\beta}_{kiI}^{\frac{1}{\varrho}} (I_{ki,t}^{E})^{\frac{\varrho-1}{\varrho}} + (1 - \widetilde{\beta}_{kiI})^{\frac{1}{\varrho}} (I_{ki,t}^{M})^{\frac{\varrho-1}{\varrho}} \right]^{\frac{\varrho}{\varrho-1}},$$

$$X_{ki,t} = \left[\widetilde{\beta}_{kiX}^{\frac{1}{\phi}} (X_{ki,t}^{E})^{\frac{\phi-1}{\phi}} + (1 - \widetilde{\beta}_{kiX})^{\frac{1}{\phi}} (X_{ki,t}^{M})^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}},$$

where $C_{k,t}^E$ and $C_{k,t}^M$ denote energy and non-energy consumption, $X_{ki,t}^E$ and $X_{ki,t}^M$ denote energy and non-energy intermediate use by sector i, and $I_{ki,t}^E$ and $I_{ki,t}^M$ denote energy and non-energy investment goods for sector i.[10] Capital is the sum of the newly purchased investment bundle and undepreciated capital, as in (5).[11] The energy and non-energy baskets are

$$C_{k,t}^{E} = \left[\sum_{i \in I_E} (\widetilde{\nu}_{ki}^{C})^{\frac{1}{\eta}} C_{ki,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \qquad C_{k,t}^{M} = \left[\sum_{i \in I_M} (\widetilde{\upsilon}_{ki}^{C})^{\frac{1}{\iota}} C_{ki,t}^{\frac{\iota-1}{\iota}} \right]^{\frac{\iota}{\iota-1}}, \tag{21}$$

$$I_{ki,t}^{E} = \left[\sum_{j \in I_E} (\widetilde{\nu}_{kij}^{I})^{\frac{1}{\varkappa}} I_{kij,t}^{\frac{\varkappa-1}{\varkappa}} \right]^{\frac{\varkappa}{\varkappa-1}}, \qquad I_{ki,t}^{M} = \left[\sum_{j \in I_M} (\widetilde{\upsilon}_{kij}^{I})^{\frac{1}{\varpi}} I_{kij,t}^{\frac{\varpi-1}{\varpi}} \right]^{\frac{\varpi}{\varpi-1}}, \tag{22}$$

$$X_{ki,t}^{E} = \left[\sum_{j \in I_E} (\widetilde{\nu}_{kij}^{X})^{\frac{1}{\chi}} X_{kij,t}^{\frac{\chi-1}{\chi}} \right]^{\frac{\chi}{\chi-1}}, \qquad X_{ki,t}^{M} = \left[\sum_{j \in I_M} (\widetilde{\upsilon}_{kij}^{X})^{\frac{1}{\xi}} X_{kij,t}^{\frac{\xi-1}{\xi}} \right]^{\frac{\xi}{\xi-1}}, \tag{23}$$

where I_E and I_M are the sets of energy and non-energy sectors. The last layer sources each

[10] The household makes the investment decision for each sector, using the investment aggregator associated with that sector.

[11] As in Vom Lehn and Winberry (2022), we do not track the detailed vintage composition of past capital bundles, but we allow for sector-specific depreciation rates.

sectoral good across countries:

(24)

$$C_{ki,t} = \left[\sum_{l=1}^{K} (\widetilde{\zeta}^{C}_{kl\,i})^{\frac{1}{\delta}} C_{kl\,i,t}^{\frac{\delta-1}{\delta}} \right]^{\frac{\delta}{\delta-1}}, \quad I_{kij,t} = \left[\sum_{l=1}^{K} (\widetilde{\zeta}^{I}_{kl\,ij})^{\frac{1}{\lambda}} I_{kl\,ij,t}^{\frac{\lambda-1}{\lambda}} \right]^{\frac{\lambda}{\lambda-1}} \quad \text{and} \quad X_{kij,t} = \left[\sum_{l=1}^{K} (\widetilde{\zeta}^{X}_{kl\,ij})^{\frac{1}{\mu}} X_{kl\,ij,t}^{\frac{\mu-1}{\mu}} \right]^{\frac{\mu}{\mu-1}}.$$

Price Setting. Firms set prices in a staggered manner (Calvo 1983). A firm in country-sector (k, i) can reset its price with probability $1 - \theta_{ki}$ each period, allowing price flexibility to vary by country and sector.

We assume producer currency pricing (PCP) throughout. A resetting firm chooses its price in its own currency by solving

$$\max_{\overline{P}_{ki,t}} \mathbb{E}_t \sum_{s=0}^{\infty} \text{SDF}_{t,t+s} (\theta^{p}_{ki})^s \left(\overline{P}_{ki,t} - (1 - \tau_{ki}) \text{MC}_{ki,t+s} \right) \mathcal{D}_{ki,t+s}, \tag{25}$$

where $\text{SDF}_{t,t+s} = \beta^s U'(C_{k,t+s})/U'(C_{k,t})$ is the household's stochastic discount factor and $\mathcal{D}_{ki,t+s}$ is aggregate demand for the firm's variety. The term τ_{ki} is a country-sector production subsidy that can offset steady-state markup distortions.[12]

3.3. Government

The government has a fiscal authority and a central bank. The fiscal authority issues domestic debt, finances production subsidies, rebates or collects lump-sum taxes, and receives tariff revenue. Its budget constraint is

$$\begin{aligned} &\frac{B_{k,t}}{1 + i_{k,t}} + T_{k,t} + \sum_{l \neq k} \sum_{i=1}^{I} \left(\tau_{kl\,i,t} P^{k}_{l\,i,t} C_{kl\,i,t} + \sum_{j=1}^{I} \tau_{kl\,i,t} P^{k}_{l\,i,t} X_{kl\,ji,t} + \sum_{j=1}^{I} \tau_{kl\,i,t} P^{k}_{l\,i,t} I_{kl\,ji,t} \right) \\ &\qquad = B_{k,t-1} + \sum_{i=1}^{I} \tau_{k,i} MC_{ki,t} Y_{ki,t}, \end{aligned} \tag{26}$$

where $Y_{ki,t}$ is output of sector i in country k.

The central bank sets the nominal interest rate. The policy rule used in the quantitative model is stated below with the market-clearing conditions because it closes the aggregate equilibrium.

[12] This is the standard subsidy used to remove steady-state distortions from monopolistic competition; see Galí (2015).

3.4. Monetary Authority

Each country has a monetary authority. For countries outside a monetary union, the policy rate follows a Taylor rule. For countries inside a monetary union, the common policy rate responds to union-wide variables and member exchange rates are fixed.

Non-members of Currency Unions. Each central bank follows a Taylor rule:

$$\widehat{i}_{k,t} = \rho_{kr}\widehat{i}_{k,t-1} + (1-\rho_{kr})\left(\phi_{k\pi}\pi_{k\phi,t} + \phi_{ky}\widehat{y}_{k,t}\right) + \varepsilon^r_{k,t} \tag{27}$$

where $\widehat{i}_{k,t}$ is the deviation of the net nominal interest rate from steady state, ρ_{kr} is interest-rate smoothing, $\{\phi_{k\pi}, \phi_{ky}\}$ are the responses to the targeted inflation index $\pi_{k\phi,t}$ and real GDP, and $\widehat{y}_{k,t}$ is the log-deviation of real GDP from steady state. The monetary policy shock is $\varepsilon^r_{k,t} \sim \mathcal{N}\left(0, (\sigma_{kr})^2\right)$.

The targeted inflation index can be CPI, producer-price inflation, the GDP deflator, or any weighted average $\pi_{k\phi,t} = \mathbf{\Phi}^\intercal \boldsymbol{\pi}_{k,t} = \sum_{l=1}^{K}\sum_{i=1}^{I}\phi_{kli}\pi_{kli,t}$, where $\sum_{l=1}^{K}\sum_{i=1}^{I}\phi_{kli} = 1$ and $\pi_{kki,t} = \pi_{ki,t}$. When ϕ_{kli} equals the consumption expenditure share, the central bank targets headline CPI inflation.

Members of Currency Unions. Suppose that a subset $K^{MU} \subset K$ of countries belongs to a monetary union. A union-wide central bank sets the common nominal interest rate to stabilize union-wide inflation and output, π^{MU}_t and $\widehat{y}^{MU}_t$.[13]

$$\widehat{i}_{k^{MU},t} = \rho^{\mathrm{MU}r}_{k\mathrm{MU}}\widehat{i}_{k^{MU},t-1} + \left(1-\rho^{\mathrm{MU}r}_{k\mathrm{MU}}\right)\left(\phi_{\mathrm{MU}\pi}\pi^{\mathrm{MU}}_t + \phi_{\mathrm{MU}y}\widehat{y}^{\mathrm{MU}}_t\right) + \varepsilon^r_{\mathrm{MU},t} \tag{28}$$

The union-wide variables are GDP-weighted averages of member outcomes:

$$\pi^{\mathrm{MU}}_t = \sum_{k\in K^{MU}}\phi^{\mathrm{MU}}_k\pi_{k\phi,t} \quad \text{and} \quad \widehat{y}^{\mathrm{MU}}_t = \sum_{k\in K^{MU}}\phi^{\mathrm{MU}}_k\widehat{y}_{k,t}, \tag{29}$$

where $\phi^{\mathrm{MU}}_k = \mathcal{Y}_k/\sum_{l\in K^{MU}}\mathcal{Y}_l$ is member country k's steady-state nominal-GDP weight in the union.

Member countries peg their bilateral nominal exchange rates to the union anchor:

$$\mathcal{E}_{k,k^{MU},t} = \mathcal{E}_{k,k^{MU}} \quad \forall k \in K^{MU} \tag{30}$$

where $\mathcal{E}_{k,k^{MU}}$ is the bilateral nominal exchange rate in steady state.

[13] The location of the union-wide central bank is innocuous as long as it targets union-wide variables.

3.5. Market Clearing and GDP

Goods market clearing requires that each country-sector's output equals demand from all destinations and all uses: final consumption, intermediate inputs, and investment goods. Thus,

$$Y_{ki,t} = \sum_{l=1}^{K} C_{l\,ki,t} + \sum_{l=1}^{K}\sum_{j=1}^{I} X_{l\,kji,t} + \sum_{l=1}^{K}\sum_{j=1}^{I} I_{l\,kji,t}. \tag{31}$$

Labor market clearing requires aggregate labor supply to equal sectoral labor demand:

$$N_{k,t} = \sum_{i=1}^{I} N_{ki,t}. \tag{32}$$

The aggregate resource constraint links the change in country k's net foreign asset position to its trade balance:

$$\sum_{h\in\mathcal{H}} Q_t^h B_{k,t}^h - \sum_{h\in\mathcal{H}} (Q_t^h + D_t^h) B_{k,t-1}^h = P_{k\mathrm{EXP},t}\mathrm{EXP}_{k,t} - P_{k\mathrm{IMP},t}\mathrm{IMP}_{k,t} \tag{33}$$

where the first term on the right-hand side is nominal exports,

$$P_{k\mathrm{EXP},t}\mathrm{EXP}_{k,t} = \sum_{l\neq k}^{K}\sum_{i=1}^{I} P_{ki,t}\left[C_{l\,ki,t} + \sum_{j=1}^{I}\left(X_{l\,kji,t} + I_{l\,kji,t}\right)\right], \tag{34}$$

and the second term is nominal imports,

$$P_{k\mathrm{IMP},t}\mathrm{IMP}_{k,t} = \sum_{l\neq k}^{K}\sum_{i\in I} P_{l\,i,t}^{k}\left[C_{kl\,i,t} + \sum_{j=1}^{I}\left(X_{kl\,ji,t} + I_{kl\,ji,t}\right)\right]. \tag{35}$$

Nominal GDP is absorption plus net exports:

$$\mathcal{Y}_{k,t} = P_{k,t}^{C} C_{k,t} + \sum_{i=1}^{I} P_{ki,t}^{I} I_{ki,t} + P_{k\mathrm{EXP},t}\mathrm{EXP}_{k,t} - P_{k\mathrm{IMP},t}\mathrm{IMP}_{k,t} \tag{36}$$

The GDP deflator is the ratio of nominal GDP at current prices to the same quantities valued at steady-state prices:

$$(37)\qquad P_{kY,t} = \frac{P^C_{k,t}C_{k,t} + \sum_{i=1}^{I} P^I_{ki,t}I_{ki,t} + P_{k\mathrm{EXP},t}\mathrm{EXP}_{k,t} - P_{k\mathrm{IMP},t}\mathrm{IMP}_{k,t}}{P^C_{k}C_{k,t} + \sum_{i=1}^{I} P^I_{ki}I_{ki,t} + P_{k\mathrm{EXP}}\mathrm{EXP}_{k,t} - P_{k\mathrm{IMP}}\mathrm{IMP}_{k,t}},$$

Real GDP is therefore

$$(38)\qquad Y_{k,t} = \mathcal{Y}_{k,t}/P_{kY,t}.$$

3.6. Tariffs

The destination-origin-sector tariff wedge introduced in (6) follows

$$(39)\qquad \tau_{klj,t} = \rho^{\tau}_{klj}\tau_{klj,t-1} + \varepsilon^{\tau}_{klj,t}, \qquad l \neq k,$$

where

$$\varepsilon^{\tau}_{klj,t} \sim \mathcal{N}\left(0, (\sigma^{\tau}_{klj})^2\right).$$

For domestic purchases, $\tau_{kkj,t} = 0$.

4. Analytical Mechanism

A tariff is levied on an origin-sector good, but its macroeconomic incidence depends on where that good is used. The same foreign sector can enter household consumption, firms' intermediate-input bundles, and the investment bundles used to build sectoral capital. What differs across uses is the buyer-price index that absorbs the tariff wedge and the equilibrium condition in which that price index appears. This section derives three use-specific exposure measures that map a sectoral tariff into the model's three price wedges, and shows why the investment wedge is distinct from the others. The formal derivations are collected in Appendix B.

Consider a local tariff change around the zero-tariff steady state. Let $\tau_{ks,t}$ denote the wedge from a common tariff that country k applies to all foreign origins $l \neq k$ in sector s; bilateral experiments restrict the sums over l to the affected origins. Producer prices and exchange rates are held fixed in the pass-through step, while the rental rate of capital moves because it is pinned down by the investment Euler equation.

Use-specific exposure. Let $p^C_{k,t}$, $p^X_{ki,t}$, and $p^I_{ki,t}$ denote the log consumption price index, the log price of sector i's intermediate-input bundle, and the log price of sector i's investment bundle in country k. A hat denotes a price expressed relative to the domestic consumption price,

$\widehat{p}^X_{ki,t} \equiv p^X_{ki,t} - p^C_{k,t}$ and $\widehat{p}^I_{ki,t} \equiv p^I_{ki,t} - p^C_{k,t}$. Using $\beta^C_{kl\,i}$ for country k's household expenditure on origin (l, i) as a share of GDP, $\omega^X_{kl\,is}$ and $\omega^I_{kl\,is}$ for sector i's imported intermediate-input and investment purchases of (l, s) relative to its gross sales, ϑ_{ki} and χ_{ki} for sector i's intermediates and investment cost shares, and λ_{ki} for its Domar weight, the three exposure measures are

$$FD_{ks} \equiv \frac{\sum_{l\neq k} \beta^C_{kl\,s}}{\sum_{l=1}^K \sum_{i=1}^I \beta^C_{kl\,i}}, \tag{40}$$

$$e^X_{kis} \equiv \frac{\sum_{l\neq k} \omega^X_{kl\,is}}{\vartheta_{ki}}, \qquad IO_{ks} \equiv \sum_{l\neq k} \sum_{i=1}^I \lambda_{ki}\omega^X_{kl\,is}, \tag{41}$$

$$e^I_{kis} \equiv \frac{\sum_{l\neq k} \omega^I_{kl\,is}}{\chi_{ki}}, \qquad KI_{ks} \equiv \sum_{l\neq k} \sum_{i=1}^I \lambda_{ki}\omega^I_{kl\,is}. \tag{42}$$

FD_{ks} is the imported sector-s share of household expenditure; e^X_{kis} and e^I_{kis} are the corresponding shares in sector i's input and investment bundles; and IO_{ks} and KI_{ks} are their Domar-weighted, economy-wide counterparts. The investment matrix is gross-output normalized: if $s^I_{kl\,is}$ denotes the conditional sourcing share inside the investment bundle, then $\omega^I_{kl\,is} = \chi_{ki}s^I_{kl\,is}$ and dividing by χ_{ki} in e^I_{kis} recovers the conditional imported share.

To first order, the tariff passes through to the three price indices in proportion to these exposures:

$$p^C_{k,t} = FD_{ks}\,\tau_{ks,t}, \qquad \widehat{p}^X_{ki,t} = \left(e^X_{kis} - FD_{ks}\right)\tau_{ks,t}, \qquad \widehat{p}^I_{ki,t} = \left(e^I_{kis} - FD_{ks}\right)\tau_{ks,t}. \tag{43}$$

Final-demand exposure is the direct CPI pass-through. Intermediate-input exposure determines whether the tariff raises firms' input costs relative to the consumption numeraire; investment-goods exposure determines whether it raises the relative price of accumulating capital. The capital channel is active for sector i when $e^I_{kis} > FD_{ks}$.

Why the investment wedge is distinct. A tariff on intermediate inputs and a tariff on investment goods both raise a relative price that, through the rental rate in the case of investment, feeds into firms' marginal costs. The investment wedge differs in that it also enters the household's intertemporal investment Euler equation, and not only the static cost-minimization problem. Combining the log-linear investment Euler equation with the stochastic discount factor gives the user-cost condition

$$\left[1 - \beta(1-\delta_{ki})\right]\mathbb{E}_t\widehat{r}_{ki,t+1} = \widehat{\mathfrak{r}}_{k,t} + \widehat{p}^I_{ki,t} - \beta(1-\delta_{ki})\mathbb{E}_t\,\widehat{p}^I_{ki,t+1}, \tag{44}$$

where $\widehat{r}_{ki,t} \equiv r_{ki,t} - p^C_{k,t}$ and $\widehat{\mathfrak{r}}_{k,t} \equiv -\mathbb{E}_t \widehat{m}_{k,t+1}$ is the real risk-free return implied by the household stochastic discount factor. If the tariff decays at rate ρ^{τ}_{ks}, combining (43) and (44) gives the direct investment-price component of the expected real rental-rate response, holding $\widehat{\mathfrak{r}}_{k,t}$ fixed:

$$\mathbb{E}_t \widehat{r}_{ki,t+1} = \mathcal{A}_{ki}(\rho^{\tau}_{ks})\left(e^I_{kis} - FD_{ks}\right)\tau_{ks,t}, \qquad \mathcal{A}_{ki}(\rho) \equiv \frac{1-\beta(1-\delta_{ki})\rho}{1-\beta(1-\delta_{ki})}. \tag{45}$$

The amplification factor satisfies $\mathcal{A}_{ki}(1) = 1$ for a permanent tariff and $\mathcal{A}_{ki}(\rho) > 1$ for $\rho < 1$: a transitory tariff makes investing today expensive relative to investing tomorrow and therefore moves the direct user-cost component more than one for one with the relative investment price. An intermediate-input tariff, which enters only the contemporaneous resource constraint, has no such intertemporal amplification. Holding the real risk-free return, expected output, and marginal costs fixed, the direct impact response of sectoral investment is $\widehat{i}_{ki,t} = -\frac{\psi}{\delta_{ki}}\mathcal{A}_{ki}(\rho^{\tau}_{ks})(e^I_{kis} - FD_{ks})\tau_{ks,t}$, where ψ is the elasticity of substitution between capital and other inputs. The quantitative model then adds the endogenous responses of output, marginal costs, the exchange rate, and monetary policy (Appendix B).

Organizing predictions. The analytical mapping provides the organizing logic for the quantitative exercises: FD_{ks} captures the direct CPI loading of a sectoral tariff, IO_{ks} captures its aggregate loading on current intermediate-input costs, and KI_{ks} captures its aggregate loading on the price of capital formation. Because the investment demand directed to origin sector (l, s) is proportional to $\sum_{i=1}^{I} \lambda_{ki}\omega^I_{kl\,is}\,\widehat{i}_{ki,t}$, a tariff that raises the relative price of investment contracts demand for the foreign sectors that supply capital goods on impact, before the physical capital stock has adjusted. Section 5 shows that these exposures rank the cross-sector responses of CPI inflation, real activity, and investment in the calibrated model, with investment exposure being at the core of the aggregate impact in both real output and investment.

5. Results

This section uses the calibrated model to ask how tariffs propagate when capital is both a factor of production and an object of international trade. We proceed in four steps. Section 5.1 describes the calibration. Section 5.2 defines the tariff experiments and the counterfactual economies used to isolate the mechanisms. Section 5.3 studies the aggregate response to a uniform US import tariff, comparing the baseline economy with economies that remove capital or remove the investment-goods network. Section 5.4 then turns to sector-by-sector tariff experiments, showing which sectors account for the aggregate effects and why the answer differs across output, investment, and prices.

5.1. Model Calibration

We calibrate the model at the quarterly frequency to five economies: the United States, China, Spain, the Rest of the Euro Area, and the Rest of the World. Each economy contains 44 production sectors, listed in Appendix Table E.1. The calibration keeps the sectoral and international structure of the model in Section 3, while allowing factor shares, input-output linkages, investment-goods linkages, markups, price rigidities, and trade exposures to vary by country and sector. Table 1 summarizes the main calibrated objects.

TABLE 1. Calibration

Block	Parameter	Value	Target or source
Economies	Number of economies, K	5	United States, China, Spain, Rest of the Euro Area, Rest of the World
Sectors	Number of sectors, I	44	OECD ICIO sectoral classification
Preferences	Discount factor, β	0.99	Quarterly frequency
Preferences	Intertemporal elasticity, σ^{-1}	1	Log utility benchmark
Preferences	Inverse Frisch elasticity, φ	1	Frisch elasticity guided by Chetty *et al.* (2011)
Stationarity	Borrowing-premium slope, γ_*	0.001	Small stationarity device as in Schmitt-Grohe and Uribe (2003)
Consumption	Consumption quasi-shares, $\widetilde{\beta}^C_k$, $\widetilde{\nu}^C_{ki}$, $\widetilde{\upsilon}^C_{ki}$, $\widetilde{\varsigma}^C_{kl\,i}$	Data	Chosen to reproduce 2022 OECD ICIO consumption expenditure shares by country, origin, and sector
Consumption	Energy/non-energy elasticity, γ_C	0.4	Böhringer and Rivers (2021)
Consumption	Energy sectoral elasticity, η_C	0.9	Short-run substitution evidence in Atalay (2017) and Boehm *et al.* (2023)
Consumption	Non-energy sectoral elasticity, ι_C	0.9	Short-run substitution evidence in Atalay (2017) and Boehm *et al.* (2023)
Consumption	Origin elasticity, δ_C	1	Cobb-Douglas benchmark across origins within sector

Notes: The table reports the baseline quarterly calibration. Country-sector shares and bilateral-sectoral matrices are read from the data set used by the MATLAB calibration routine. Euro Area aggregates reported below combine Spain and the Rest of the Euro Area using model-implied weights.

Table 1 continued

Block	**Parameter**	**Value**	**Target or source**
Investment	Energy/non-energy elasticity, γ_I	0.4	Same energy substitution benchmark as consumption
Investment	Energy sectoral elasticity, η_I	0.2	Low short-run substitutability across capital-goods sectors
Investment	Non-energy sectoral elasticity, ι_I	0.2	Low short-run substitutability across capital-goods sectors
Investment	Origin elasticity, δ_I	1	Cobb-Douglas benchmark across origins within sector
Production	Capital-labor-intermediate elasticity, ψ	0.5	Moderate short-run substitution in production
Production	Energy/non-energy intermediate elasticity, γ_X	0.4	Böhringer and Rivers (2021)
Production	Energy intermediate elasticity, η_X	0.2	Low short-run substitution in intermediate use
Production	Non-energy intermediate elasticity, ι_X	0.2	Low short-run substitution in intermediate use
Production	Origin elasticity for intermediates, δ_X	1	Cobb-Douglas benchmark across origins within sector
Production	Production quasi-shares, $\widetilde{\varsigma}_{ki}$, $\widetilde{\alpha}_{ki}$, $\widetilde{\vartheta}_{ki}$	Data	Chosen to reproduce sector-country capital, labor, and intermediate-input cost shares, including wage-bill shares from Eurostat FIGARO
Intermediate inputs	Intermediate-input quasi-shares, $\widetilde{\beta}^X_{ki}$, $\widetilde{\nu}^X_{kij}$, $\widetilde{\upsilon}^X_{kij}$, $\widetilde{\zeta}^X_{kl\,ij}$	Data	Chosen to reproduce 2022 OECD ICIO input-output tables

Notes: The table reports the baseline quarterly calibration. Country-sector shares and bilateral-sectoral matrices are read from the data set used by the MATLAB calibration routine. Euro Area aggregates reported below combine Spain and the Rest of the Euro Area using model-implied weights.

Table 1 continued

Block	**Parameter**	**Value**	**Target or source**
Investment network	Investment quasi-shares, $\widetilde{\beta}^I_{ki}$, $\widetilde{\nu}^I_{kij}$, $\widetilde{\upsilon}^I_{kij}$, $\widetilde{\zeta}^I_{kl\,ij}$	Data	Chosen to reproduce the investment input-output matrix constructed in Section 2.1
Capital	Depreciation rate, δ_{ki}	Data	Asset-specific rates from EU KLEMS: median implied depreciation over the pre-pandemic period by asset class, mapped to sectors via the asset-to-sector bridge; see Section 2.1
Prices	Price Calvo parameter, θ^p_{ki}	Data	PRISMA micro-price evidence in Gautier *et al.* (2024); energy prices from Dhyne *et al.* (2006)
Monetary policy	Interest-rate smoothing, ρ_r	0.7	Standard Taylor-rule calibration
Monetary policy	Inflation coefficient, ϕ_π	1.5	Standard Taylor-rule calibration (Galí 2015)
Monetary policy	Real GDP coefficient, ϕ_y	0.125	Standard Taylor-rule calibration (Galí 2015)
Tariff process	Tariff persistence, ρ^τ_{klj}	0.99	Baseline AR(1) process for tariff wedges, common across bilateral sectoral wedges

Notes: The table reports the baseline quarterly calibration. Country-sector shares and bilateral-sectoral matrices are read from the data set used by the MATLAB calibration routine. Euro Area aggregates reported below combine Spain and the Rest of the Euro Area using model-implied weights.

Households. The household side is calibrated conservatively. We set $\beta = 0.99$, $\sigma = 1$, and $\varphi = 1$, with the Frisch elasticity guided by Chetty *et al.* (2011). The borrowing premium γ_* is set to 0.001, so that net foreign assets are stationary without making short-run exchange-rate movements depend on a large financial friction, following the logic in Schmitt-Grohe and Uribe (2003). Consumption is a nested aggregate over energy and non-energy goods, sectors, and origins. The quasi-share parameters are chosen so that, after log-linearization, the model reproduces observed country-sector consumption expenditure shares from the 2022 OECD ICIO tables. We set the energy versus non-energy elasticity to 0.4, in line with Böhringer and Rivers (2021), and the within-basket elasticities to values close to the short-run estimates in Atalay (2017). Direct household purchases of foreign energy goods are set to zero in the calibration, so that foreign energy exposure enters through firms and investment rather than through an implausibly large direct household-import margin.

Production and investment. Firms combine labor, capital, and intermediate inputs. Labor shares α_{ki} are calibrated from sector-country wage-bill shares, while intermediate-input shares ϑ_{ki} and the input-output matrix $\omega^X_{kl\,ij}$ are read from the 2022 OECD ICIO tables. The elasticities in the intermediate-input nest follow the evidence in Atalay (2017), and the energy substitution elasticity follows Böhringer and Rivers (2021). Investment is treated symmetrically as a network object: $\omega^I_{kl\,ij}$ describes how investment by destination sector (k, i) is sourced from origin country l and origin sector j, and is constructed from the investment input-output matrix described in Section 2.1. This is important because capital is not only a stock in production; it is also a demand category with its own international sourcing pattern. The macroeconomic relevance of this margin is familiar from models with time to build and capital adjustment, such as Kydland and Prescott (1982) and Christiano *et al.* (2005), and from the literature on investment-specific shocks, including Justiniano *et al.* (2011). Recent network evidence in Vom Lehn and Winberry (2022) and Peri *et al.* (2026) makes the same point for the US at the sectoral level: the composition of investment demand matters for aggregate propagation.

Given the investment expenditure share χ_{ki} and the depreciation rate δ_{ki}, the capital cost share is pinned down by the steady-state Euler equation,

$$\varsigma_{ki} = \frac{1 - \beta(1 - \delta_{ki})}{\beta\delta_{ki}}\chi_{ki}. \tag{46}$$

This distinction matters. The investment matrix measures spending on capital goods, while ς_{ki} is the production-side cost share that enters marginal costs. The calibration rescales the relevant shares when needed so that labor, capital, and intermediate-input shares are internally consistent sector by sector.

Nominal rigidities and policy. Sectoral price rigidities are taken from the PRISMA micro-price evidence in Gautier *et al.* (2024). Because that evidence is thinner for energy goods, we complement it with Dhyne *et al.* (2006), which implies much more frequent price adjustment in energy-related sectors. Monetary policy follows a standard Taylor rule. We set the interest-rate smoothing coefficient to 0.7 and the inflation and real GDP coefficients to 1.5 and 0.125, respectively, as in Galí (2015). The tariff experiments are short-run exercises, so the trade elasticities in consumption, investment, and intermediates are set to one, consistent with the evidence that trade responses are smaller at cyclical horizons than in the long run (Boehm *et al.* 2023).

5.2. Tariff Experiments and Counterfactual Economies

Tariff experiments. The model tariff wedge follows $\tau_{klj,t} = \rho^{\tau}_{klj}\tau_{klj,t-1} + \varepsilon^{\tau}_{klj,t}$, with $\rho^{\tau}_{klj} = 0.99$ in the baseline calibration. The main aggregate experiment is a 10 percentage-point innovation to the uniform US import tariff on all non-US origins and all 44 sectors. The sectoral experiment applies the same 10 percentage-point innovation one sector at a time and records the contribution of each tariffed sector to aggregate outcomes. In both cases, the first quarters are the relevant horizon for the mechanism: capital prices and investment respond immediately, while the stock of capital moves only gradually.

Counterfactual economies. To discern the role of investment and of the investment-goods network, we compare the baseline with two counterfactual economies. In the No Investment economy, capital is removed from production and from dynamics. To keep the comparison disciplined, the disappearing capital cost share is reassigned to intermediate inputs, leaving labor shares and markups unchanged. Since tariffs act on goods, this makes the no-investment comparison demanding: the counterfactual keeps a strong direct goods channel even though it removes capital as a dynamic margin. In the No Investment Network economy, capital remains in production but the detailed sectoral investment-goods network is replaced by one aggregate investment good and one aggregate capital stock per country. Sectoral capital services are then allocated through a static reallocation block with elasticity ν_K, set to one in the baseline counterfactual following Peri *et al.* (2026). Appendix C.1 and Appendix C.2 give the full construction.

5.3. Macroeconomic Dynamics after Uniform Tariffs

We begin with a uniform tariff because it gives the cleanest view of the model's aggregate propagation. The United States raises import tariffs by 10 percentage points on every foreign source and every sector. Recent work reaches a broad consensus that tariffs are contractionary in the short run, whether identified empirically from US history (den Besten and Känzig 2025) or studied in open-economy New Keynesian models with trade in intermediates but abstracting from capital (Auclert *et al.* 2025; Auray *et al.* 2026; Kalemli-Özcan *et al.* 2025). We ask how much of this contraction comes from treating capital as a produced good whose price and sourcing pattern respond to tariffs.

Figure 1 reports the US impulse responses to the uniform tariff. Each panel plots one macroeconomic variable over the quarters following the shock, and the three lines compare the baseline economy (solid), the No Investment economy (dashed), and the No Investment Network economy (dash-dotted). Investment is a central feature shaping the aggregate effects of tariffs. On impact, US real GDP falls by 2.54 percent in the baseline, compared with 1.20

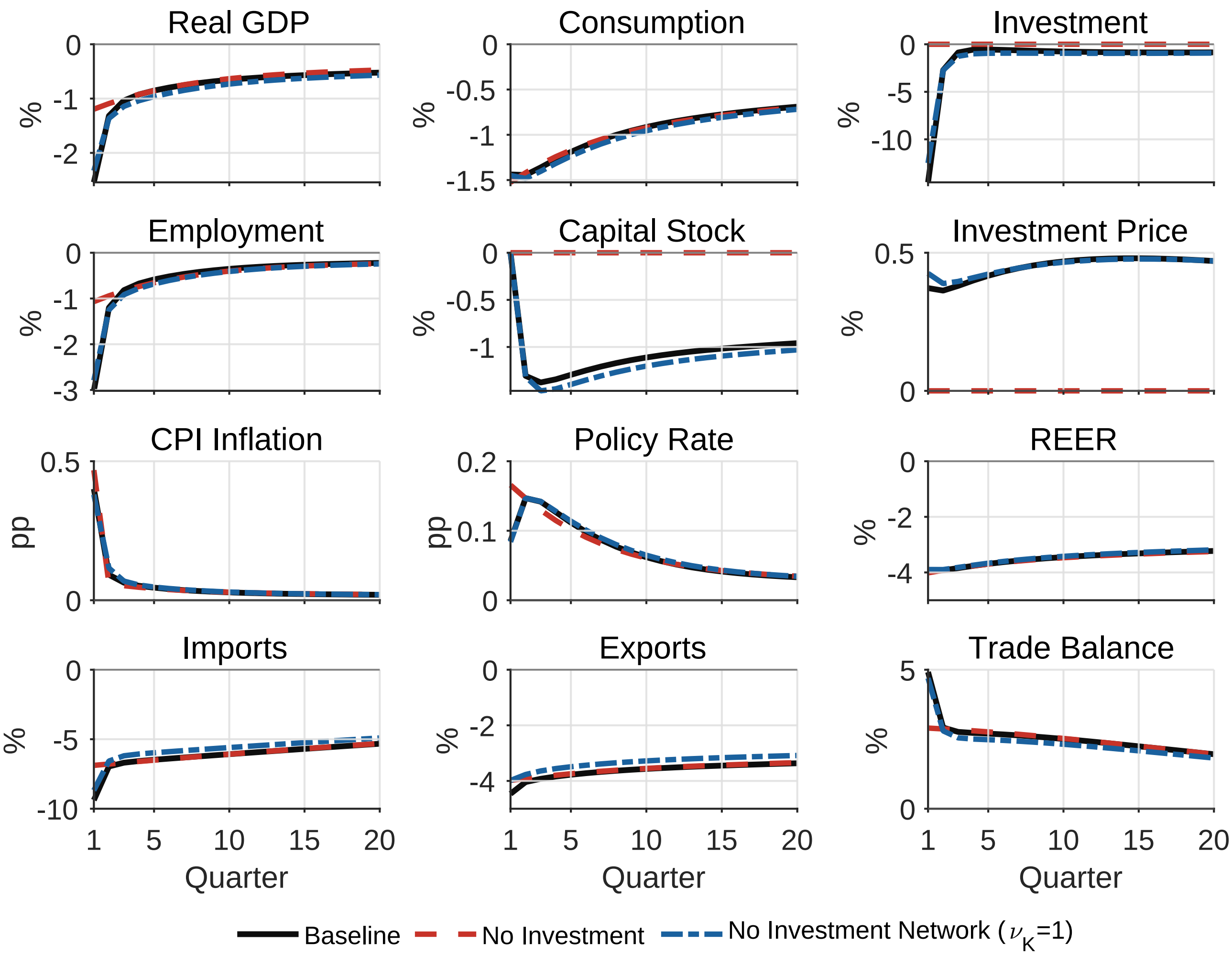

FIGURE 1. Uniform US tariff experiment: model comparison. The figure reports US impulse responses to a 10 percentage-point uniform US import tariff, in percent deviations from steady state (annualized percentage points for inflation and the policy rate). The solid line is the baseline economy, the dashed line the No Investment economy, and the dash-dotted line the No Investment Network economy with $\nu_K = 1$. In the panels for investment, capital, and the investment price, which are absent from the No Investment economy, the dashed line is plotted at zero as a reference. The real effective exchange rate (REER) is defined so that an increase is an appreciation of the US dollar.

percent without capital. Employment falls by 3.02 percent instead of 1.07 percent. Investment, absent by construction in the No Investment economy, collapses by 14.52 percent in the baseline; in the panels for investment, capital, and the investment price, the dashed line is accordingly plotted at zero as a reference rather than as a model-implied response. Imports contract by 9.38 percent, and the trade balance improves by 4.92 percent as the fall in imports dominates the fall in exports.

The result is not an artifact of giving the baseline a larger static exposure to tariffs. Recall that the No Investment calibration reallocates the full capital cost share into intermediates, the margin that tariffs hit most directly. If capital were merely a relabeling of intermediate demand, this counterfactual would deliver a response as large as the baseline. It does not. The reason is that capital adds a dynamic demand margin. A tariff raises the price of imported investment goods, and the Euler equation brings forward a sharp fall in investment, which lowers sectoral output demand, labor demand, and hence aggregate output.

The same mechanism matters for spillovers. The Euro Area and China responses, reported in Appendix Figures E1 and E2, are smaller in absolute value than the US response but are amplified by a similar factor. Chinese exports fall by 2.11 percent in the baseline and by 1.00 percent without capital. Roughly half of the Chinese export spillover is therefore tied to the US investment margin. This is the trade-composition effect: a tariff that depresses investment demand also depresses the trade in goods used to make capital.

The No Investment Network comparison in Figure 1 keeps capital in the model but removes the sectoral investment-goods network. The real GDP difference is more modest than in the No Investment comparison. Still, the investment network leaves a clear signature. In the baseline, the US investment price rises by 0.37 percent on impact; in the No Investment Network economy with $\nu_K = 1$, it rises by 0.43 percent. The detailed investment network dampens the aggregate investment-price response because sectoral investment bundles reweight heterogeneous price movements. At the same time, trade flows are more responsive when the network is active: US imports fall by 9.38 percent in the baseline, compared with 8.68 percent without the network, and exports fall by 4.46 percent compared with 3.98 percent.[14]

The two comparisons give a clean decomposition. Removing capital eliminates the dynamic demand margin and cuts the fall in US real GDP by more than half. Removing only the detailed investment network leaves that dynamic margin in place, but changes which goods are expensive, which sectors reduce purchases, and which foreign suppliers lose demand.

5.4. Macroeconomic Dynamics after Sectoral Tariffs

The uniform tariff is informative about aggregate propagation, but it hides where the propagation comes from. We therefore run 44 sectoral tariff experiments. In each experiment, the United States raises the import tariff by 10 percentage points on one sector, applied to all non-US origins, and we compute the contribution of that sectoral tariff to aggregate real GDP, CPI inflation, investment, and employment in each region. We report 12-quarter cumulative contributions for the baseline incidence results, and use impact and cumulative responses when comparing the baseline model with the No Investment and No Investment Network economies.

[14] Appendix Figure E3 reports the full US set of outcomes and compares $\nu_K = 1$ with a high-elasticity robustness case, $\nu_K = 1000$. The corresponding Euro Area and China figures are reported in Appendix Figures E4 and E5.

The sectoral exercise is organized around the use-specific exposure measures introduced in Section 4. We specialize those objects to the United States. Let u denote the United States, let s denote the foreign sector subject to the tariff, and let $l \neq u$ denote a foreign origin. All objects are steady-state expenditure shares, so the absence of a time subscript denotes the calibration point. We write

$$\mathsf{FD}_s \equiv FD_{us}, \qquad \mathsf{IO}_s \equiv IO_{us}, \qquad \mathsf{KI}_s \equiv KI_{us}.$$

The three measures summarize the direct use of imported sector-s goods in household absorption, production, and capital formation. The first, FD_s, is final-demand exposure. The second, IO_s, is the intermediate-input exposure. The third, KI_s, is investment-goods exposure. All three are defined in (40). The Domar weights convert sector-level imported-input and investment-goods purchases, measured relative to each using sector's sales, into aggregate exposure measures expressed relative to US GDP. Thus, IO_s measures aggregate exposure to imported intermediate inputs from sector s, while KI_s measures the corresponding exposure through goods used to build capital.

These objects are not shocks and are not sufficient statistics for the full general-equilibrium response. Rather, they are steady-state exposure measures that the analytical benchmark predicts should organize different margins of tariff incidence: FD_s for CPI incidence, IO_s for the production-cost channel, and KI_s for investment and real activity.

Figure 2 reports the contributions of each tariffed sector to US real GDP, CPI inflation, investment, and employment. The largest real GDP contractions come from electronics, motor vehicles, retail, machinery and equipment, and energy mining. These are not simply the sectors with the largest final consumption exposure. They are sectors that either supply investment goods directly, sit upstream in production, or act as distribution margins. This distinction echoes the network logic in Acemoglu *et al.* (2012) and Baqaee and Farhi (2019): size alone is not enough; the use of the good in the network determines propagation.

The main mechanism is clearest in investment. Tariffs on electronics, motor vehicles, and machinery and equipment dominate the US investment response. These are precisely the sectors whose imports enter capital formation. Given the calibration in Table 1, the investment nest has low short-run substitution across sectors, with $\eta_I = \iota_I = 0.2$. A tariff on a capital-good sector, therefore, cannot be easily undone by switching the composition of investment within the quarter. The investment price rises, the Euler equation lowers desired investment, and the contraction feeds into output and employment.

The price response follows a different ranking. CPI incidence is much more closely tied to final-demand exposure. This separation between real activity and prices is useful. It shows that the model is not merely assigning large effects to sectors because they are

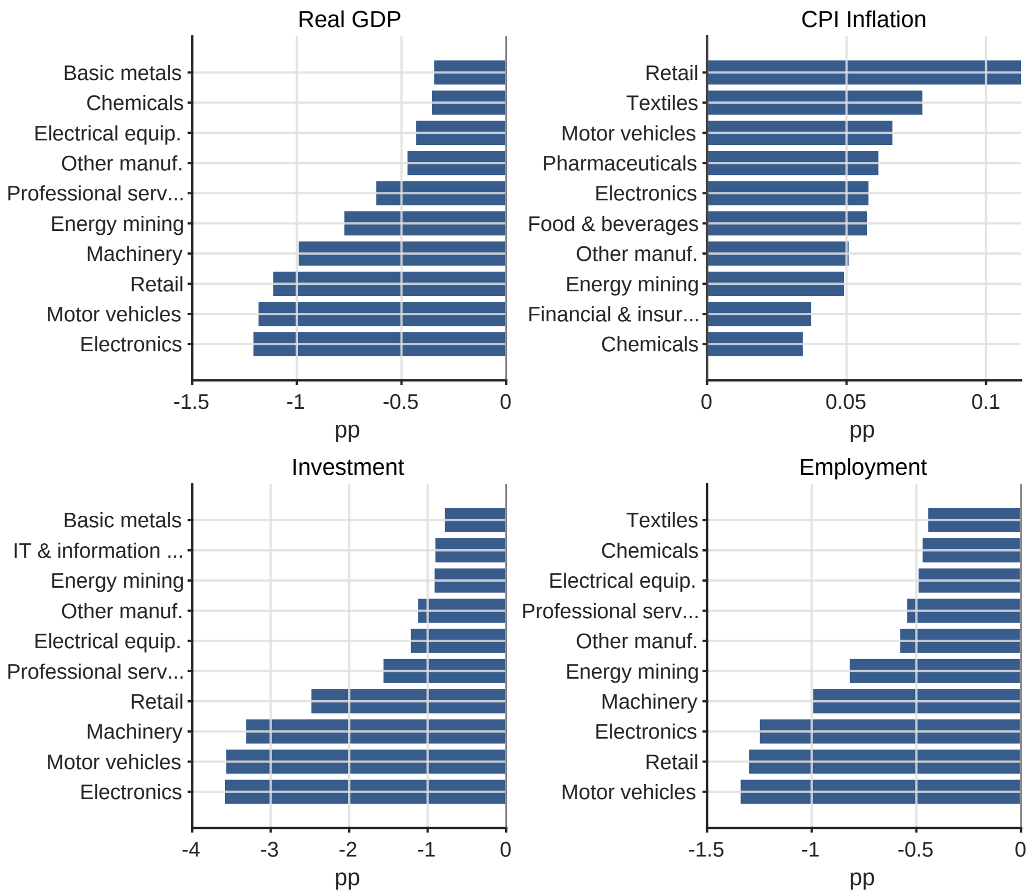

FIGURE 2. Sectoral tariff incidence in the United States. Bars report 12-quarter cumulative contributions from sector-specific US import tariff shocks for real GDP, CPI inflation, investment, and employment. The panels focus on the sectors with the largest US contributions; Appendix Figures E6–E14 report the full 44-sector country panels and additional outcomes.

visible in household expenditure. Real GDP, employment, and investment respond through production and capital formation; consumer prices respond through final demand. The empirical literature on recent tariffs has shown that pass-through to consumer prices is substantial but heterogeneous (Amiti *et al.* 2020; Cavallo *et al.* 2021). The model adds the insight that the sectors with the largest pass-through to activity need not be those with the largest pass-through to CPI.

5.4.1. Sectoral Incidence and Sectoral Characteristics

Figure 3 relates the US sectoral responses to the exposure measures in equations (40)–(42). Investment-goods exposure explains 77 percent of the cross-sector variation in US real GDP incidence and 92 percent of the variation in US investment incidence. Final-demand exposure explains only 25 percent of the real GDP variation and 12 percent of the investment variation. For CPI inflation, the ranking reverses: final-demand exposure explains 87 percent of the

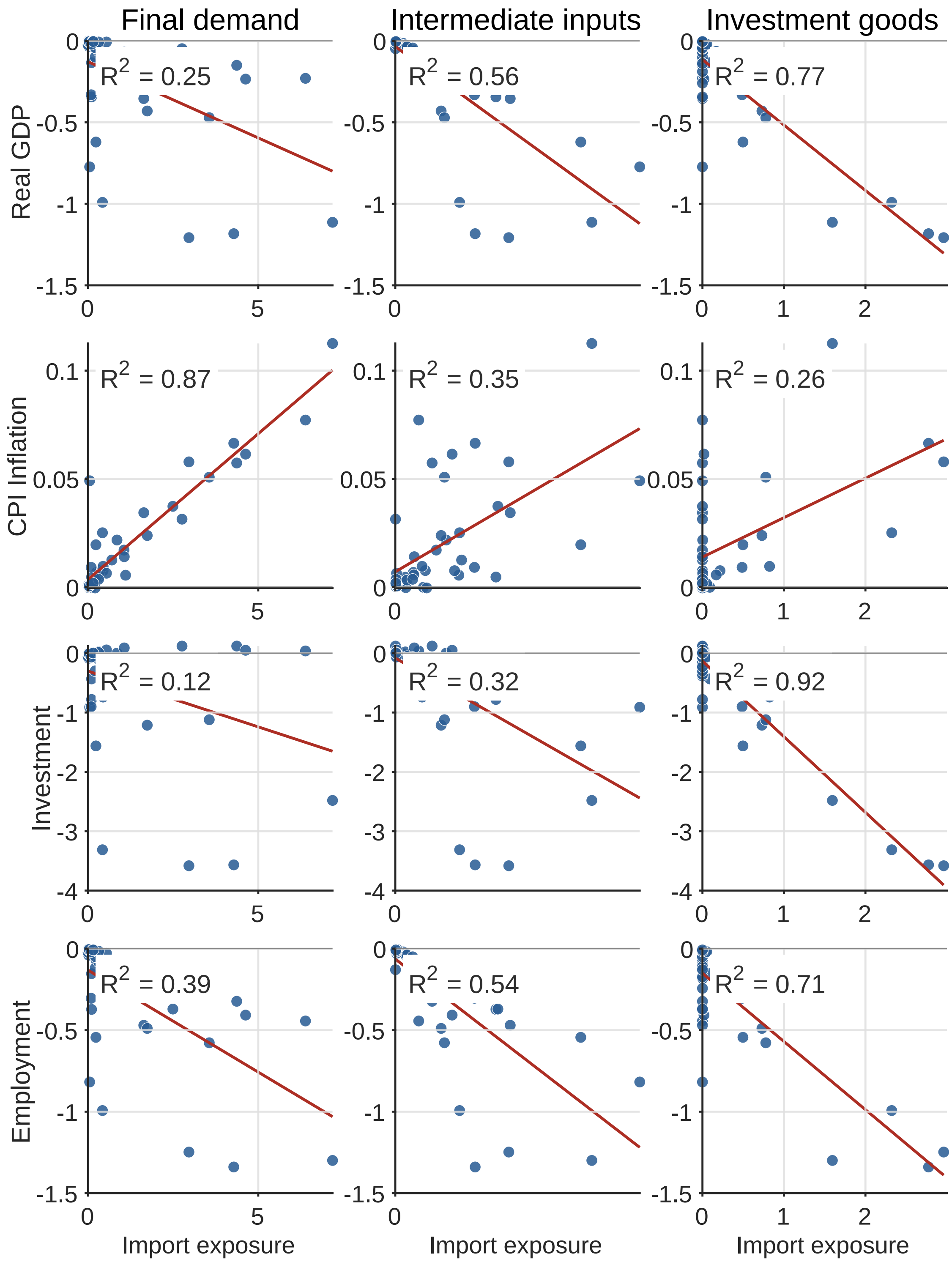

FIGURE 3. US sectoral incidence and structural exposure. Columns report sectoral responses of real GDP, CPI inflation, investment, and employment. Rows plot each response against final-demand, intermediate-input, and investment-goods import exposure. Appendix Figures E15–E17 report the corresponding panels for all regions.

variation, while investment-goods exposure explains 26 percent. Appendix Table E.2 reports the corresponding R^2 statistics.

The scatter plots provide a compact test of the mechanism. If tariffs worked mainly as household consumption taxes, final-demand exposure would organize all outcomes. It does not. If tariffs worked mainly as static production-cost shocks, intermediate-input exposure would dominate. It is informative for real GDP, but it explains less of the variation than investment-goods exposure, which is the strongest predictor of real activity. This is why electronics, motor vehicles, and machinery matter more for real activity than their consumption shares alone would imply.

5.4.2. Comparison with Counterfactual Models

The two counterfactual economies sharpen the interpretation of the sectoral results.

For each sector and each outcome present in both models, we define a counterfactual gap as the baseline contribution minus the contribution in the counterfactual economy; negative entries mean that the baseline model produces a more contractionary response.[15] Table 2 summarizes the leading US impact gaps.

The No Investment comparison shows that the missing capital margin is not a neutral rescaling of the sectoral bars. It is front-loaded and concentrated in real quantities. On impact, the largest US real GDP gaps stem from tariffs on retail, motor vehicles, and electronics; the employment gaps from the same sectors are over twice as large as the output gaps. CPI gaps are much smaller. The capital channel is therefore a real-activity and trade-composition channel, not primarily a sectoral price-incidence channel.

The No Investment Network comparison asks: conditional on having capital, does the sectoral sourcing of investment goods matter? The answer is yes, but not as a uniform multiplier. At impact, removing the investment-goods network mainly changes the incidence of tariffs on textiles, food and beverages, and pharmaceuticals. Over the 12-quarter horizon, reported in Appendix Table E.3, the sectors tha result in the largest gaps are instead electronics, motor vehicles, and other transport equipment. The sign also reverses for the cumulative investment gaps from these sectors: the economy without the detailed investment network eventually produces the larger cumulative investment contraction. This sign change is informative: the investment network reallocates propagation across sectors and over time.

The same incidence logic extends to the other regions.[16] China is most exposed through electronics, textiles, other manufacturing, electrical equipment, and motor vehicles, while the

[15]When the outcome is investment and the counterfactual is the No Investment economy, the table reports the baseline investment response because investment is absent by construction in that counterfactual.

[16]Appendix Figures E1–E5 report the corresponding macro counterfactuals for the Euro Area and China, and Appendix Figures E6–E14 report the country-sector panels behind Figure 2.

Counterfactual gap	Outcome	Horizon	Largest US sectoral gaps
Baseline minus No Investment	Real GDP	Impact	Retail (–0.182), motor vehicles (–0.169), electronics (–0.155)
Baseline minus No Investment	Employment	Impact	Retail (–0.415), motor vehicles (–0.364), electronics (–0.326)
Baseline response (No Investment absent)	Investment	Impact	Retail (–1.802), motor vehicles (–1.707), electronics (–1.522)
Baseline minus No Investment Network	Real GDP	Impact	Textiles (–0.061), food and beverages (–0.044), pharmaceuticals (–0.042)
Baseline minus No Investment Network	Employment	Impact	Textiles (–0.096), food and beverages (–0.071), electronics (+0.068)
Baseline minus No Investment Network	Investment	Impact	Textiles (–0.410), pharmaceuticals (–0.298), food and beverages (–0.290)

Notes: Entries are impact percentage-point contributions. A negative gap means that the baseline model is more contractionary than the counterfactual for that sectoral tariff. A positive gap means that the counterfactual is more contractionary. The investment row for the No Investment economy is a baseline response, not a subtraction, because investment is absent in that counterfactual. The No Investment Network economy keeps capital accumulation but removes the sectoral input-output network in capital goods. Appendix Table E.3 reports the corresponding 12-quarter cumulative gaps.

TABLE 2. Sectoral counterfactual comparisons in the United States

Euro Area and the Rest of the World show broader exposure through pharmaceuticals, retail, motor vehicles, chemicals, professional services, and energy-related sectors. The uniform-tariff response in Section 5.3 is therefore an aggregate of uneven sectoral shocks: some sectors matter because they are consumption goods, others because they are intermediates, and the sectors that matter most for the capital channel are those that enter investment-goods production. This is the sense in which capital changes not only the magnitude of tariff transmission, but also its incidence.

6. The 2025–26 US–China Trade War: From Realized Impacts to Optimal Tariff Composition

The experiments in Section 5 use clean tariff shocks to identify mechanisms. This section puts these mechanisms to work in two exercises. The first feeds the realized 2025–26 US–China tariff schedule into the model and asks what it implies for output, inflation, investment, and employment, and how much its composition—rather than its average size—drives the result (Sections 6.1–6.2). Because these tariffs were not randomly assigned across sectors and other shocks were also at work, we treat the observed schedule as a model input, not an identified natural experiment. The second holds the average tariff fixed and asks how it should be distributed across sectors, ranking them by their macroeconomic incidence at home and abroad and deriving the compositions that minimize the domestic output loss or maximize the loss imposed abroad at unchanged domestic cost (Section 6.3).

The tariff object fed into the model is the sectoral change in the day-weighted effective tariff rate, measured in percentage points. We first construct quarterly tariff levels from 2025Q1 through 2026Q1 and treat 2025Q1 as the pre-episode base. The model shocks begin in 2025Q2. Since the model wedge follows $\tau_{klj,t} = \rho^{\tau}_{klj}\tau_{klj,t-1} + \varepsilon^{\tau}_{klj,t}$, we choose the innovation $\varepsilon^{\tau}_{klj,t}$ each quarter so that the simulated wedge $\tau_{klj,t}$ reproduces the observed quarter-to-quarter effective tariff change for that sector and bilateral direction. The two directions are US tariffs on Chinese goods and Chinese tariffs on US goods. The affected goods are concentrated in manufacturing and other traded-goods sectors, while many service sectors have zero effective tariff changes. The realized package is therefore a natural stress test for the paper's mechanism: the macroeconomic effect depends not only on how much tariffs rise, but also on whether the goods hit by tariffs are used in consumption, production, or investment.

Figure 4 summarizes the resulting tariff schedule that we feed into the model: the top row plots the realized effective sectoral tariff changes for each bilateral direction, and the bottom row the uniform-equivalent changes that preserve the import-weighted aggregate tariff change while removing sectoral dispersion. We construct the quarterly tariff measures from product-level information in the IMF Tariff Tracker. The raw data record tariff rates at the HS2022 product level and the date on which each rate becomes applicable. For each product, we convert the sequence of tariff announcements into daily validity spells: a tariff is assumed to remain in force until the day before the next recorded change, and the last observed spell is extended to the end of the sample window. We then intersect these spells with calendar quarters, from 2025Q1 through 2026Q1, and count the number of days for which each tariff is active within each quarter. The 2025Q1 level is used only to initialize the change series; it is not a model shock quarter.

The product-level quarterly tariff is an import-weighted average over these daily spells. The weight of a tariff spell is the value of imports of the product multiplied by the number of active days in the quarter. This procedure gives larger weight to economically relevant products and lets within-quarter tariff changes enter in proportion to the number of days for which they are in force. We then map the resulting HS2022 tariff measures into the 44 sectors used in the theoretical model. HS2022 products are first concorded to HS2017 using the UNSTATS HS2022–HS2017 conversion table. HS2017 products are then mapped into OECD TiVA/NACE sectors using the OECD HS–ISIC/NACE conversion key. Finally, tariffs are aggregated from products to the 44 model sectors with the same import-exposure weights.

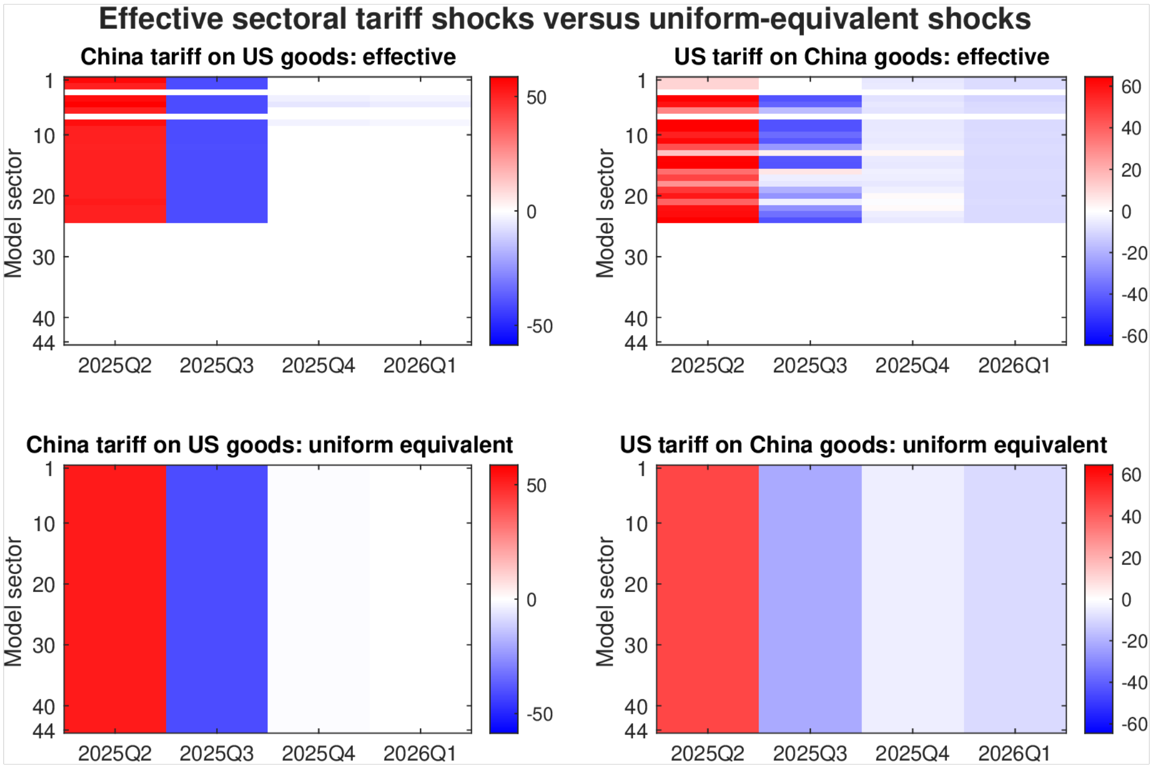

Figure 1: Effective sectoral tariff shocks versus uniform-equivalent shocks. The uniform-equivalent shocks preserve the import-weighted aggregate tariff change in each direction and quarter, but remove cross-sector heterogeneity.

FIGURE 4. Realized sectoral tariff shocks and uniform-equivalent shocks. The top row plots the effective sectoral tariff changes for Chinese tariffs on US goods and US tariffs on Chinese goods; the first change is 2025Q2 relative to the 2025Q1 base. The bottom row plots the uniform-equivalent tariff changes that preserve the import-weighted aggregate tariff change by bilateral direction and quarter while removing sectoral dispersion.

Service and non-goods sectors with no direct tariff exposure are assigned zero tariffs, yielding a balanced quarterly sector-level data set for the realized US–China tariff experiment.

6.1. Implied Macro Dynamics across Models

Figure 5 reports the model-implied dynamics across regions. The baseline predicts a large real contraction in the two economies directly involved in the tariff episode. On impact, US real GDP falls by 2.35 percent and Chinese real GDP falls by 1.31 percent. Investment is much more elastic: it falls by 13.08 percent in the United States and by 5.27 percent in China. Employment falls by 3.85 percent in the United States and by 4.14 percent in China. CPI inflation rises on impact by 0.32 percentage points in the United States and by 0.08 percentage points in China, before reversing as activity contracts.

The comparison across models is the same as in the cleaner experiments, but the realized

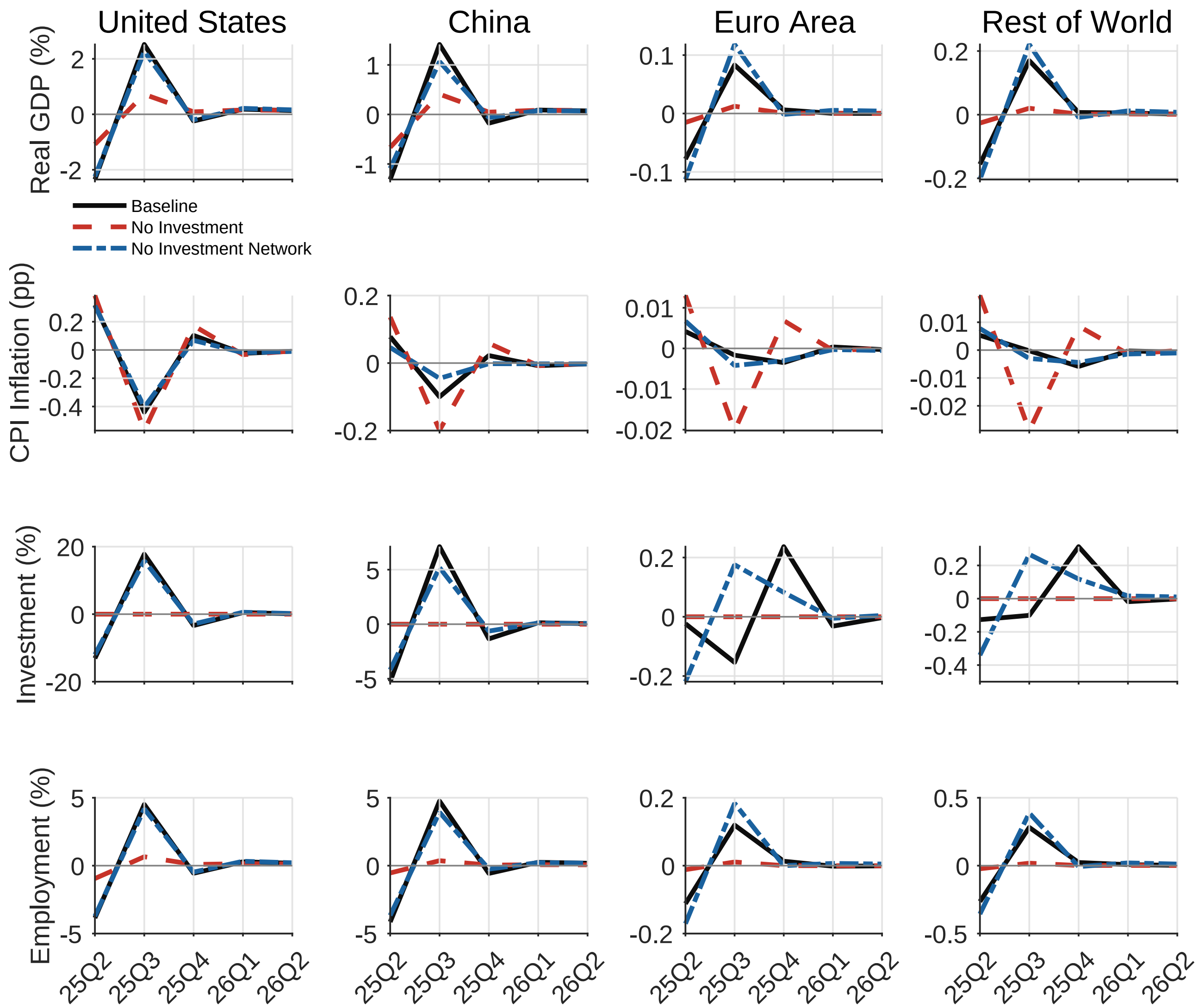

FIGURE 5. Realized 2025–26 tariff experiment across regions and models. Rows report real GDP, CPI inflation, investment, and employment; columns report the United States, China, the Euro Area, and the Rest of the World. The solid line is the baseline, the dashed line the No Investment economy, and the dash-dotted line the No Investment Network economy. Real variables are in percent deviations from steady state; CPI inflation is in percentage points.

tariff package makes the economic interpretation sharper. In the No Investment economy, the impact fall in US real GDP is 1.08 percent instead of 2.35 percent, and the fall in Chinese real GDP is 0.66 percent instead of 1.31 percent. Recall that this is not because the No Investment economy is weakly exposed to goods trade: the missing capital cost share is reassigned to intermediates. Instead, the key is the forward-looking nature of investment goods.

Removing only the investment-goods network has a smaller aggregate effect. The No Investment Network economy predicts a US real GDP decline of 2.25 percent, close to the 2.35 percent baseline decline, and a Chinese decline of 1.09 percent, close to the 1.31 percent baseline decline. Yet this closeness should not be read as irrelevance. It means that the existence of capital is the first-order aggregate force in this realized bilateral episode, while the detailed investment network mainly pins down which sectors, when tariffed, are responsible for the largest aggregate losses. The next subsection shows that this allocation effect is substantial.

A bilateral decomposition clarifies the mechanism. We split the total response into the part driven by US tariffs on Chinese goods and the part driven by Chinese tariffs on US goods, by re-running the realized experiment with each bilateral direction's tariffs in isolation. In the baseline, US tariffs on Chinese goods account for most of the US impact response: they reduce US real GDP by 1.73 percent and US investment by 9.86 percent. Chinese tariffs on US goods add another 0.62 percent decline in US real GDP and a 3.22 percent decline in US investment. For China, the two directions are more balanced. US tariffs on Chinese goods reduce Chinese real GDP by 0.75 percent and investment by 2.80 percent; Chinese tariffs on US goods reduce Chinese real GDP by 0.57 percent and investment by 2.48 percent. Retaliation therefore contributes meaningfully, but the bulk of the US contraction is self-inflicted: it is driven by the tariffs that the United States levies on the goods it imports from China.

6.2. Sectoral Distribution of Tariffs and Its Relevance

An aggregate effective tariff rate is not enough to characterize the realized 2025–26 package. To see why, compare the realized sectoral tariff changes with a uniform-equivalent tariff path. For each direction d and quarter t, define

$$\Delta\tau^{\text{uni}}_{d,t} = \sum_i \omega_{d,i,t-1}\Delta\tau^{\text{eff}}_{d,i,t}, \tag{47}$$

where $\omega_{d,i,t-1}$ is sector i's lagged share in the affected bilateral import bundle for direction d. In this uniform-equivalent counterfactual, every sector receives the same tariff change $\Delta\tau^{\text{uni}}_{d,t}$, so the import-weighted aggregate tariff change is identical to the realized package while the cross-sector distribution is removed. For Chinese tariffs on US goods, the uniform-equivalent path is 52.17, −41.00, −0.58, and −0.45 percentage points. For US tariffs on Chinese goods, it is 46.63, −21.22, −4.11, and −9.25 percentage points. Figure 4 plots the realized sectoral shocks and their uniform-equivalent counterparts.

Figure 6 reports the difference between the realized effective package and its uniform-equivalent counterpart. Positive values mean that the realized sectoral allocation is less contractionary than the uniform allocation with the same import-weighted aggregate tariff path. The result is economically large. In the baseline, the realized package raises US real GDP by 0.95 percentage points relative to the uniform-equivalent package, employment by 1.59 percentage points, and investment by 4.93 percentage points. For China, the corresponding

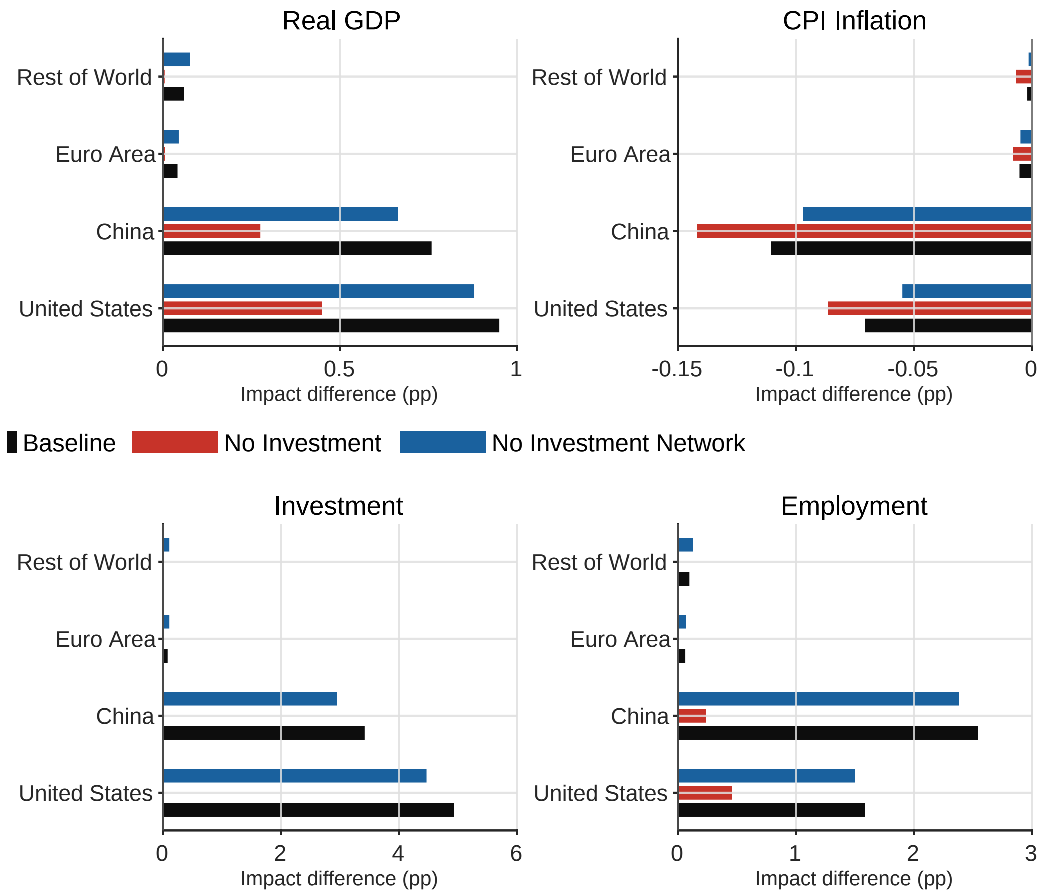

FIGURE 6. Effective minus uniform-equivalent responses. Bars report impact differences in percentage points for real GDP, CPI inflation, investment, and employment. Positive values mean that the realized sectoral tariff allocation is less contractionary than the uniform-equivalent allocation.

differences are 0.76, 2.54, and 3.42 percentage points. The CPI inflation comparison, reported in the same figure, shows the opposite pattern, with the No Investment model exhibiting the largest impact difference even as this is small across the board: the composition that softens real activity losses need not imply the same ranking for consumer-price pressure.

This comparison answers a simple identification question. If only the aggregate bilateral tariff intensity mattered, the effective and uniform-equivalent packages would have the same macroeconomic effects. They do not. The realized sectoral allocation is less damaging because the tariff changes are not spread uniformly over all traded goods. They are concentrated in sectors whose exposure to US and Chinese investment demand is smaller than in the uniform-equivalent benchmark. In this sense, sectoral composition is not a detail to be averaged away: it is part of the shock itself, and the investment network determines how it maps into capital formation.

The last comparison asks whether the capital-goods input-output network changes sectoral

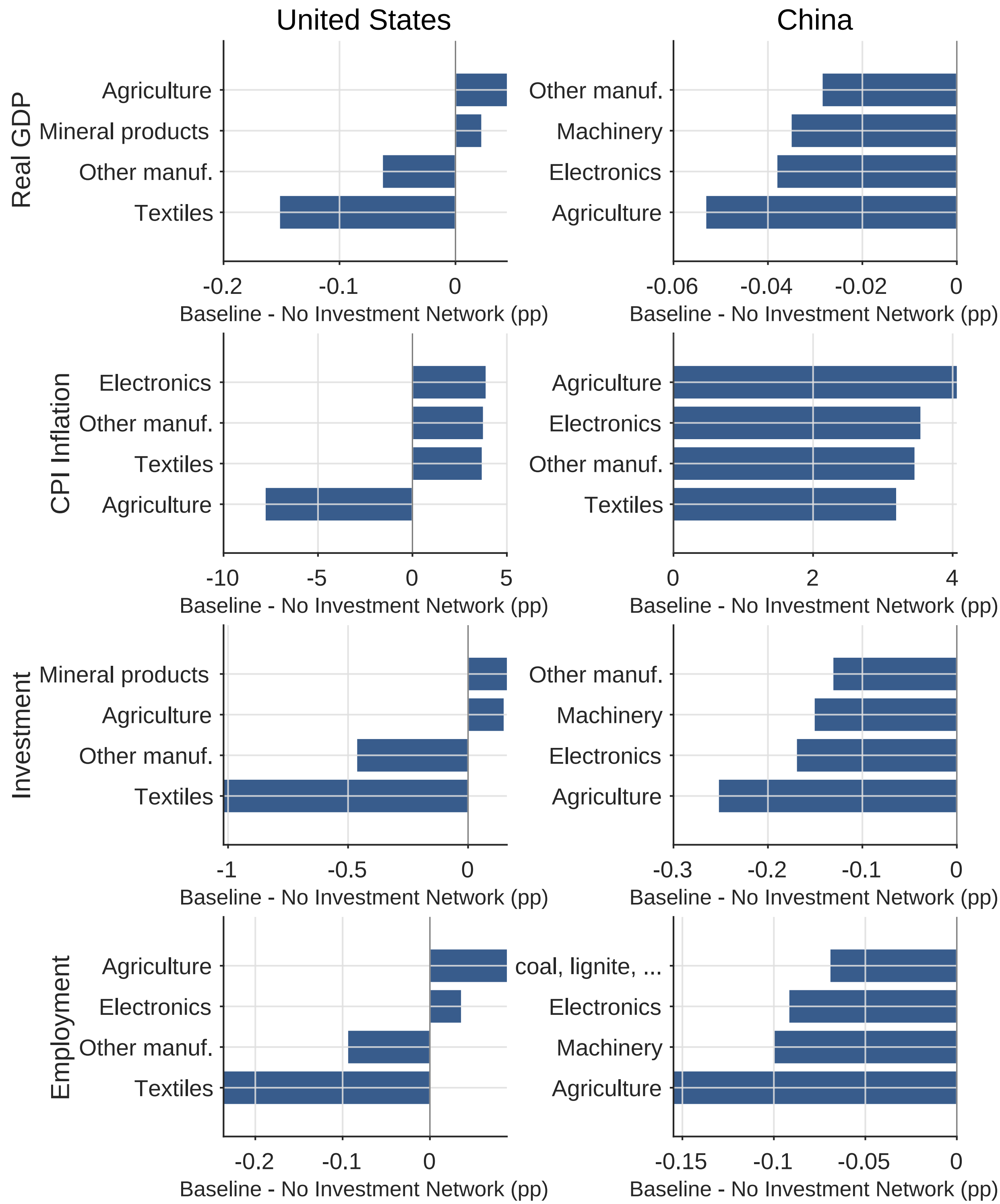

FIGURE 7. Largest sectoral impact gaps under realized effective tariffs. The gap is Baseline minus No Investment Network for real GDP, CPI inflation, investment, and employment. Negative values mean that the baseline response is more contractionary for that sector.

incidence even when the aggregate baseline and No Investment Network paths are close. For each sector and outcome, define the impact gap as the baseline contribution to aggregate values minus the No Investment Network contribution under the realized effective tariff package. Negative values mean that the baseline response is more contractionary.

Figure 7 reports the largest gaps for real GDP, CPI inflation, investment, and employment. In the United States, textiles and other manufacturing account for the largest differences in real GDP, employment, and investment. In China, agriculture, electronics, machinery, and other manufacturing are the most impactful sectors.

The message is not that the investment network always makes tariffs more contractionary; it is more precise. The investment network changes which sectoral price increases matter for investment demand, which sectors reduce output, and which foreign suppliers lose demand. Two tariff packages with similar aggregate effects can therefore imply different sectoral incidence once capital and investment networks are active. This is the same logic as in Section 5.4, now applied to the realized 2025–26 tariff path rather than to clean one-sector-at-a-time experiments.

6.3. Strategic Incidence and the Optimal Composition of Tariffs

The sectoral heterogeneity documented above raises a natural question: which tariffs should a government choose? We answer it in two steps. Section 6.3.1 ranks individual sectoral tariffs by their macroeconomic incidence at home and abroad. Section 6.3.2 turns to entire packages: holding the import-weighted average tariff fixed, it characterizes the compositions that are optimal with respect to transparent macroeconomic objectives—minimizing the domestic output loss, or maximizing the loss imposed abroad at unchanged domestic cost.

6.3.1. Sector-by-Sector Incidence

If a government wanted to target tariff-equivalent import wedges across sectors, which sectors would deliver the largest macroeconomic effect abroad for a given macroeconomic cost at home?[17]

For each bilateral direction and sector, we compute the response to a 10 percentage-point tariff-equivalent wedge and summarize the cumulative twelve-quarter GDP effects on the imposing country and on the targeted country. Let k be the country imposing the

[17] We do not interpret this exercise as an optimal-tariff calculation in the welfare-theoretic sense. In canonical trade models, optimal tariffs reflect a trade-off between terms-of-trade gains, tariff revenue, and domestic distortions, possibly augmented by political-economy or macroeconomic objectives (Bagwell and Staiger 1999; Broda *et al.* 2008; Ossa 2014; Itskhoki and Mukhin 2025). Our log-linear quantitative model is not solved to second order and therefore does not deliver a consumption-equivalent welfare ranking of sectoral tariffs. Instead, we use it to construct a transparent measure of strategic macroeconomic incidence.

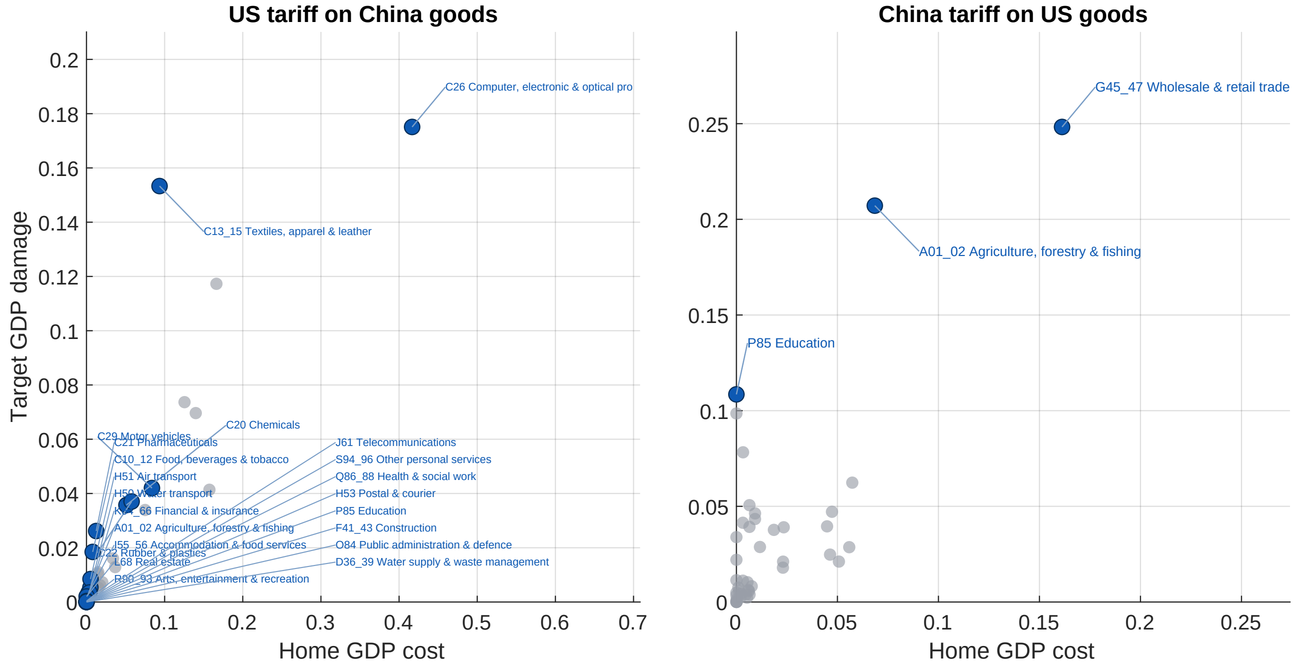

FIGURE 8. Strategic incidence of bilateral sectoral import wedges. Each point is a sector. The horizontal axis is the cumulative twelve-quarter GDP cost in the country imposing the wedge, and the vertical axis is the cumulative twelve-quarter GDP damage in the targeted country, both after a 10 percentage-point bilateral tariff-equivalent wedge. Blue points denote sectors on the damage-cost frontier: no other sector produces weakly larger target-country damage at weakly lower domestic cost.

wedge, l the targeted origin, and i the targeted sector, so the shock is to the bilateral wedge $\tau_{kli,t}$ of the model. The 44-sector ICIO classification includes services; for service sectors the wedge should be read as a tariff-equivalent import cost rather than a literal customs tariff on merchandise. Define $\Delta y_i^k(k,l) \equiv \sum_{t=0}^{11} \widehat{y}_{k,t}(\tau_{kli})$ as the cumulative response of real GDP, equation (A.109), in the imposing country, and $\Delta y_i^l(k,l)$ analogously in the targeted origin. We define domestic macroeconomic cost as $C_i^k(k,l) = \max\{-\Delta y_i^k(k,l), 0\}$ and target-country damage as $D_i^l(k,l) = \max\{-\Delta y_i^l(k,l), 0\}$. Positive GDP responses are therefore not mechanically penalized. The baseline strategic score is $D_i^l(k,l) - C_i^k(k,l)$, and Appendix Table E.5 reports how the top sector changes when the domestic-cost weight is varied. This score is a descriptive frontier of macroeconomic incidence, not a social-welfare objective.

Figure 8 plots the resulting damage-cost frontier. For US tariff-equivalent wedges on Chinese sectors, the highest GDP-based strategic score is obtained in textiles, apparel, and leather, followed by pharmaceuticals and food, beverages, and tobacco. Electronics generates larger damage to China in absolute value, but it also entails a substantially larger US GDP cost, so it falls in the ranking once domestic cost receives unit weight. Conversely, for Chinese tariff-equivalent wedges on US sectors, agriculture has the highest score, followed by education, accommodation and food services, and wholesale and retail trade. These rankings reflect the interaction of bilateral absorption shares with the propagation of tariff shocks through production and investment-input networks. They also reinforce the caution emphasized by recent macro-tariff work: once input linkages, expenditure switching, and dynamic demand responses are taken seriously, sectoral import wedges can be contractionary at home even when they depress activity abroad (Auclert *et al.* 2025). Appendix Table E.4 reports the top-ranked sectors, and Appendix Figures E18–E19 plot the corresponding ranking bars.

6.3.2. The Optimal Composition of a Tariff Package

The rankings above consider one wedge at a time. This subsection asks which entire tariff-equivalent schedule a government should choose.[18] The thought experiment holds the overall size of the package fixed and varies only where the wedges fall. A package assigns a wedge τ_{kli} to each imported sectoral good or service, where, as in the model, k is the importing country (here the United States), $l \neq k$ the origin country, and i the sector. We measure the size of a package by the average wedge paid on one dollar of US imports,

$$\bar{\tau} = \sum_{l \neq k} \sum_{i} \omega_{li}^{\mathrm{IMP}} \tau_{kli}. \tag{48}$$

All packages considered have the same size, $\bar{\tau} = 10$ percentage points. The benchmark is the uniform package—the 10 percentage-point tariff on every import of Section 5.3—and the alternative packages redistribute the same average protection across goods, with each individual tariff between 0 and 50 percentage points. Within this set, the *cost-minimizing* composition minimizes the cumulative twelve-quarter US real GDP loss, and the *damage-maximizing* composition maximizes the cumulative Chinese GDP loss subject to the US loss being no larger than under the uniform package.[19]

The composition margin is large. Moving from the uniform to the cost-minimizing package cuts the cumulative US output loss from 11.4 to 3.5 percentage points—by more than two-thirds—at the same average tariff, while the damage imposed on China barely changes (0.8 versus 1.0 percentage points; Table 3). The optimal composition is the one the paper's

[18] As in the previous subsection, "optimal" refers to these transparent macroeconomic-incidence objectives, not to welfare-theoretic optimality. The exercise formalizes a selection problem that governments already solve informally when drafting retaliation lists screened for domestic exposure.

[19] At the twelve-quarter cumulative horizon every sectoral contribution is weakly negative for both the imposing and the targeted country, so these signed objectives coincide with the truncated measures C_i^k and D_i^l of Section 6.3.1; the damage-maximizing program is the package-level counterpart of the score $D_i^l - \eta\, C_i^k$, with η equal to the shadow price of the domestic-cost constraint. The damage-maximizing problem is also the package-level analogue of the sanctions-design problem studied in static trade models (de Souza *et al.* 2024; Sturm 2024).

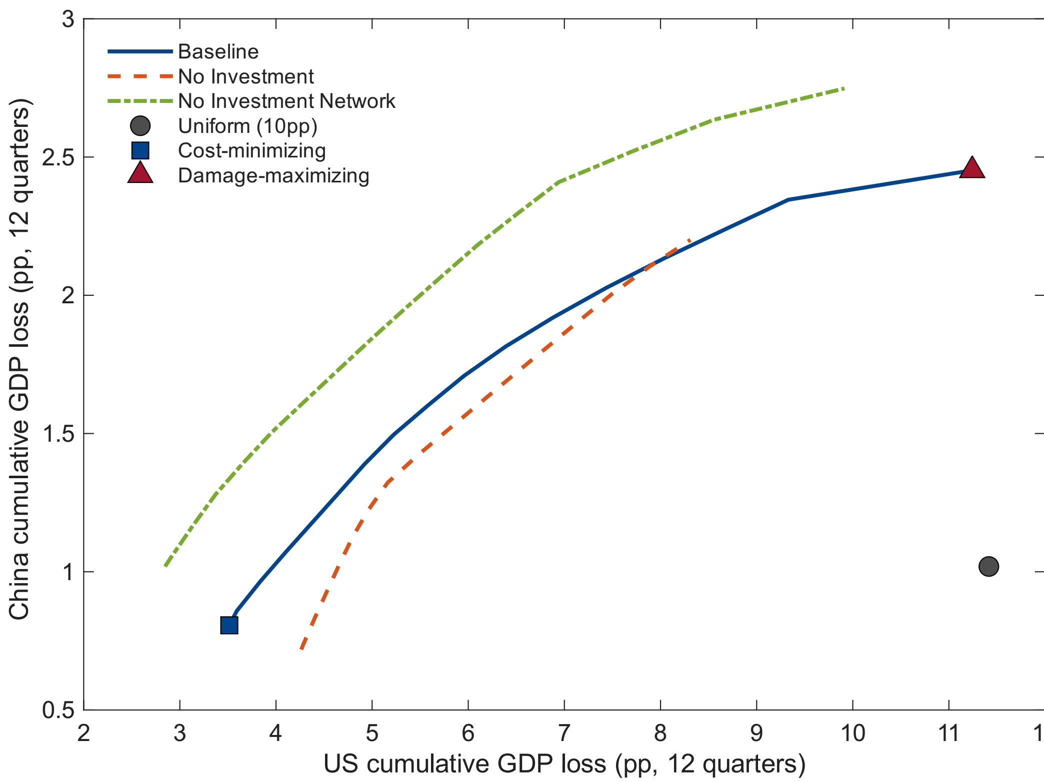

FIGURE 9. The composition frontier. Each line traces, for a given economy, the smallest cumulative twelve-quarter US real GDP loss consistent with a given cumulative twelve-quarter GDP loss imposed on China, across all sectoral tariff compositions with the same 10 percentage-point average tariff, equation (48) (44 sectoral instruments applied to all origins, individual tariffs capped at 50 percentage points). The solid line is the baseline economy, the dashed line the No Investment economy, and the dash-dotted line the No Investment Network economy. Markers report the uniform, cost-minimizing, and damage-maximizing packages in the baseline economy. Origin-specific (bilateral) instruments, reported in the text, expand the frontier further.

mechanism predicts: zero tariffs on capital goods and their inputs (electronics, machinery, transport equipment, metals), high tariffs on consumer goods and services (food, textiles, pharmaceuticals, accommodation). Sparing investment goods avoids the persistent loss from a shrinking capital stock, which is where most of the cumulative damage lies. The gain has a price: consumer-price inflation is moderately higher under the cost-minimizing package (a cumulative 1.01 versus 0.87 percentage points), the final-demand versus investment-goods trade-off of Section 5.4.

The same margin can be turned outward. At no additional cost to the United States, reallocating tariffs toward the goods China supplies—electronics, fabricated metals, textiles—raises China's cumulative loss from 1.0 to 2.5 percentage points; letting tariffs differ by origin raises it to 5.6, more than five times the uniform package; and adding a ceiling on euro-area spillovers preserves that damage while reducing the euro-area loss from 2.6 to 0.7 percentage points (Table 3). Figure 9 traces the full set of attainable pairs of US and Chinese losses: the uniform package sits deep in the interior, dominated in both dimensions at once.

Package	US GDP	China GDP	Euro Area GDP	US CPI
Uniform	-11.42	-1.02	-2.62	0.87
Cost-minimizing	-3.51	-0.81	-2.62	1.01
Damage-maximizing (sectoral)	-11.24	-2.45	-1.17	0.87
Damage-maximizing (bilateral)	-11.42	-5.63	-1.46	0.75
Spillover-constrained (bilateral)	-11.42	-5.63	-0.65	0.74

Notes: Cumulative twelve-quarter responses in percentage points. Every package satisfies equation (48): a 10 percentage-point import-weighted average US tariff with a 50 percentage-point per-instrument cap. Sectoral packages use 44 instruments applied to all origins; bilateral packages use 176 origin-sector instruments. The damage-maximizing packages maximize the cumulative Chinese GDP loss subject to the US loss not exceeding its uniform-package level; the spillover-constrained variant additionally requires the euro-area loss not to exceed one quarter of its uniform-package level.

TABLE 3. Cost-minimizing and damage-maximizing tariff compositions: cumulative twelve-quarter outcomes

Figure 9 also repeats the exercise in the No Investment economy: without capital, the uniform package would cost less (a cumulative 9.8 rather than 11.4 percentage points), but the best attainable package would cost more (4.3 rather than 3.5). Composition can thus more than undo the amplification that capital adds to a uniform tariff, because that amplification is concentrated in identifiable goods.

Moreover, the optimizer adds little beyond the paper's own statistics: ranking sectors by $[\bar{\mathcal{A}}(\rho)(\mathsf{KI}_s - \mathsf{FD}_s \sum_i \lambda_{ki}\chi_{ki}) + \mathsf{IO}_s]/\omega_s^{\text{IMP}}$—net investment-goods exposure and intermediate-input exposure per unit of import share $\omega_s^{\text{IMP}} \equiv \sum_{l \neq k} \omega_{ls}^{\text{IMP}}$, with $\bar{\mathcal{A}}(\rho)$ the average amplification factor of equation (45)—and filling the package by that ranking alone delivers 93 percent of the gains from the optimal package (Appendix F).

These results are not an artifact of the GDP metric. To confirm this, we re-solve the design problem under a dual-mandate stabilization loss that penalizes squared deviations of output, consumer-price inflation, and wage inflation. The loss is admittedly reduced-form—we do not derive it as a second-order approximation to household welfare—but a quadratic loss of this kind is a workhorse of the monetary-policy literature (Barnichon and Mesters 2023; Debortoli *et al.* 2019), and we let the weight on inflation span the welfare-based, dual-mandate, and welfare-optimized values that the latter surveys. Unlike the cumulative-GDP criterion, it prices inflation and is convex-quadratic rather than linear in the tariff vector, and its solution is an interior composition spread across fifteen sectors rather than a vertex. Even so, the size of the composition margin and its outward leverage are undiminished: the loss-minimizing package cuts the US stabilization loss by roughly three-quarters, and the damage-maximizing composition raises China's stabilization loss 5.5-fold at unchanged US loss, robustly across inflation weights—indeed, under every weighting of the output, consumer-price, and wage-inflation terms that we examine, the loss-designed package leaves the cumulative US GDP loss between 4.9 and 6.2 percentage points, against 11.4 under the uniform tariff (Appendix F.2, Figure F2).

Two caveats apply. First, we assume no retaliation: by linearity, a fixed retaliation path shifts all packages by the same constant and leaves every optimal composition unchanged, but an endogenous response would not. Second, we deliberately characterize the composition of a package of a given size rather than the welfare-optimal level of tariffs studied by the optimal-tariff literature (Itskhoki and Mukhin 2025; Dávila *et al.* 2025): the composition margin is where the investment network bites, and it has no counterpart in that literature.

7. Conclusion

Tariffs are taxes on imported goods, but which goods is the key macroeconomic question. This paper has shown that the answer matters because some imported goods are used to build capital. Capital is a bundle of investment goods from different countries and sectors; tariffs that tax these goods will raise the price of investing, lower investment demand, lower production and employment, and propagate across the network before the capital stock has meaningfully adjusted. This mechanism is quantitatively large. Removing capital from a uniform tariff increase experiment cuts the real GDP contraction by more than half and eliminates the investment collapse that drives much of the short-run response.

This affects how one should summarize tariff exposure. The final demand exposure of a tariffed sector is informative about CPI inflation. Intermediate-input exposure matters for production costs. Investment-goods exposure is the object that organizes much of the real GDP, employment, and investment response. This separation helps reconcile two facts that are both present in recent episodes: some tariffs are immediately visible in consumer prices, while others operate through firms' cost of expanding capacity. Analysis of the aggregate tariff rate fails to distinguish between these cases.

The realized US-China tariff changes in 2025–26 reinforce the point. The model predicts a large short-run contraction in the United States and China, especially in investment and employment, but the realized sectoral allocation is less contractionary than a uniform-equivalent package with the same import-weighted aggregate tariff path. Sectoral composition therefore matters for aggregate dynamics, not only for distribution. The lesson is not that every detail of a tariff schedule is equally important. It is that the margins connected to capital formation have first-order aggregate consequences. Holding the average tariff fixed, redesigning the sectoral mix around the investment network cuts the cumulative domestic output loss by two-thirds or multiplies the loss imposed abroad several-fold: which goods a tariff package targets is a policy choice in its own right.

This conclusion points to a broader research agenda. Trade policy is often evaluated using static exposure measures, while macroeconomic stabilization is often evaluated using aggregate shocks. Tariffs sit between these approaches. They are sectoral policies with aggregate consequences. Models that combine trade linkages, nominal rigidities, and capital accumulation can therefore discipline which tariff packages are inflationary, which are recessionary, and which reallocate activity across sectors without large aggregate effects. Our evidence suggests that the investment margin is essential for making those distinctions.

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Appendix

Appendix A. Model Derivation and Log-linearization

In this section, we derive model equilibrium conditions, outlining the final set of log-linearized equations. We further enlarge the theoretical model presented in the main text—which only contains exogenous perturbations to the foreign price wedges and the monetary policy shock—to accommodate the standard shocks in the literature: an internal demand shock, sectoral TFP, sectoral price cost-push shocks, and wage cost-push shocks.

A.1. Households

Each household is made up of a continuum of members, each specialized in a different labor service, indexed by $g \in [0,1]$. Income is pooled within each household, acting as a risk-sharing mechanism. The per-period utility function (1) is modified to accommodate nominal wage rigidities and a discount factor disturbance. Namely,

$$U_t = \left(\frac{C_{k,t}^{1-\sigma}}{1-\sigma} - \int_0^1 \frac{\mathcal{N}_{gk,t}^{1+\varphi}}{1+\varphi}\, dg \right) Z_{k,t}, \tag{A.1}$$

where $\mathcal{N}_{gk,t}$ denotes the labor supply of labor service g, $Z_{k,t}$ is an exogenous preference shifter, and σ denotes the inverse of the intertemporal elasticity of substitution.[20]

Consumption Demand Curves. The allocation optimal allocation between energy and non-energy goods is the result of a cost minimization programme min $P_{k,t}^{C,E} C_{k,t}^{E} + P_{k,t}^{C,M} C_{k,t}^{M}$ subject to (20). Similarly, the optimal allocation between energy (non-energy) consumption is the result of a cost minimization programme min $\sum_{i \in I_E} P_{ki,t}^{C} C_{ki,t}$ (min $\sum_{i \in I_M} P_{ki,t}^{C} C_{ki,t}$) subject to (21). Finally, the optimal allocation between the different consumption goods is the result of a cost minimization programme min $\sum_{l=1}^{K} (1+\tau_{kl\,i,t}) P_{l\,i,t}^{k} C_{kl\,i,t}$ subject to (24). The implied demand curves are given by

$$P_{k,t}^{C,E} = P_{k,t}^{C} \left(\frac{\widetilde{\varrho}_k^C C_{k,t}}{C_{k,t}^E} \right)^{\frac{1}{\gamma_C}} \quad \text{and} \quad P_{k,t}^{C,M} = P_{k,t}^{C} \left(\frac{(1-\widetilde{\varrho}_k^C) C_{k,t}}{C_{k,t}^M} \right)^{\frac{1}{\gamma_C}} \tag{A.2}$$

$$P_{ki,t}^{C} = P_{k,t}^{C,E} \left(\frac{\widetilde{\nu}_{ki}^C C_{k,t}^E}{C_{ki,t}} \right)^{\frac{1}{\eta_C}} \quad \forall \quad i \in I_E \quad \text{and} \quad P_{ki,t}^{C} = P_{k,t}^{C,M} \left(\frac{\widetilde{\upsilon}_{ki}^C C_{k,t}^M}{C_{ki,t}} \right)^{\frac{1}{\iota_C}} \quad \forall \quad i \in I_M \tag{A.3}$$

$$(1+\tau_{kl\,i,t}) P_{l\,i,t}^{k} = P_{ki,t}^{C} \left(\frac{\widetilde{\zeta}_{kl\,i}^C C_{ki,t}}{C_{kl\,i,t}} \right)^{\frac{1}{\delta_C}} \quad \forall \quad l \in K. \tag{A.4}$$

[20] Each household takes labor income as given since wages are set by labor unions and employment is decided by firms. The household still chooses consumption allocation, domestic and foreign bond positions, and sectoral investment/capital accumulation; these choices generate the bond Euler equations and the I sectoral investment Euler equations below.

The log-linearized versions of the consumption demand curves (A.2)-(A.4) are given by

$$\widehat{p}^{C,E}_{k,t} = \frac{1}{\gamma_C}(\widehat{c}_{k,t} - \widehat{c}^{E}_{k,t}) \quad \text{and} \quad \widehat{p}^{C,M}_{k,t} = \frac{1}{\gamma_C}(\widehat{c}_{k,t} - \widehat{c}^{M}_{k,t}) \tag{A.5}$$

$$\widehat{p}^{C}_{ki,t} - \widehat{p}^{C,E}_{k,t} = \frac{1}{\eta_C}(\widehat{c}^{E}_{k,t} - \widehat{c}_{ki,t}) \quad \text{and} \quad \widehat{p}^{C}_{ki,t} - \widehat{p}^{C,M}_{k,t} = \frac{1}{\iota_C}(\widehat{c}^{M}_{k,t} - \widehat{c}_{ki,t}) \tag{A.6}$$

$$\tau_{kli,t} + \widehat{p}^{k}_{li,t} - \widehat{p}^{C}_{ki,t} = \frac{1}{\delta_C}\left(\widehat{c}_{ki,t} - \widehat{c}_{kli,t}\right) \tag{A.7}$$

where $\widehat{p}^{C,E}_{k,t} = p^{C,E}_{k,t} - p^{C}_{k,t}$, $\widehat{p}^{C,M}_{k,t} = p^{C,M}_{k,t} - p^{C}_{k,t}$, $\widehat{p}^{C}_{ki,t} = p^{C}_{ki,t} - p^{C}_{k,t}$, and $\widehat{p}^{k}_{li,t} = p^{k}_{li,t} - p^{C}_{k,t}$ are well-defined as a ratio of prices.[21]

Currency Pricing. We assume producer currency pricing in all markets. Log-linearizing (7), we obtain

$$\widehat{p}^{k}_{li,t} = p^{k}_{li,t} - p^{C}_{k,t} = e^{l}_{k,t} + p_{li,t} - p^{C}_{k,t} = e^{l}_{k,t} + (p_{li,t} - p^{C}_{l,t}) + (p^{C}_{l,t} - p^{C}_{k,t}) = \widehat{p}_{li,t} + q^{l}_{k,t}, \tag{A.8}$$

$$\widehat{p}_{ki,t} = p_{ki,t} - p^{C}_{k,t} = \widehat{p}_{ki,t}, \tag{A.9}$$

where we have used the transformation $\widehat{p}_{li,t} = p_{li,t} - p^{C}_{l,t}$; and the definition of the bilateral real exchange rate between country k and country m: $\mathcal{Q}_{kl,t} = P_{lC,t}\mathcal{E}_{kl,t}/P^{C}_{k,t}$, which log-linearized takes the form

$$q^{l}_{k,t} = e^{l}_{k,t} + p^{C}_{l,t} - p^{C}_{k,t}. \tag{A.10}$$

Consumption Baskets. The log-linearized consumption aggregator (20) is given by

$$\widehat{c}_{k,t} = \varrho^{C}_{k}\widehat{c}^{E}_{k,t} + (1 - \varrho^{C}_{k})\widehat{c}^{M}_{k,t} \tag{A.11}$$

where $\varrho^{C}_{k} = \frac{P^{C,E}_{k}C^{E}_{k}}{P^{C}_{k}C_{k}} = (\widetilde{\varrho}^{C}_{k})^{\frac{1}{\gamma_C}}\left(\frac{C^{E}_{k}}{C_{k}}\right)^{\frac{\gamma_C-1}{\gamma_C}}$ and $(1 - \varrho_{k}) = \frac{P^{C,M}_{k}C^{M}_{k}}{P^{C}_{k}C_{k}} = (1 - \widetilde{\varrho}_{k})^{\frac{1}{\gamma_C}}\left(\frac{C^{M}_{k}}{C_{k}}\right)^{\frac{\gamma_C-1}{\gamma_C}}$ can be verified using the steady-state consumption aggregator (20) and the demand curves (A.2).

The log-linearized versions of the energy and non-energy consumption aggregators (21) are given by

$$\widehat{c}^{E}_{k,t} = \sum_{i\in I_E} \nu^{C}_{ki}\widehat{c}_{ki,t} \qquad \text{and} \qquad \widehat{c}^{M}_{k,t} = \sum_{i\in I_M} \upsilon^{C}_{ki}\widehat{c}_{ki,t} \tag{A.12}$$

[21]The individual price levels are not well-defined in steady state, but their ratio is.

where $\nu^C_{ki} = \frac{P^C_{ki} C_{ki}}{P^{C,E}_k C^E_k} = (\widetilde{\nu}^C_{ki})^{\frac{1}{\eta_C}} \left(\frac{C_{ki}}{C^E_k}\right)^{\frac{\eta_C - 1}{\eta_C}}$ and $\upsilon^C_{ki} = \frac{P^C_{ki} C_{ki}}{P^{C,M}_k C^M_k} = (\widetilde{\upsilon}^C_{ki})^{\frac{1}{\iota_C}} \left(\frac{C_{ki}}{C^M_k}\right)^{\frac{\iota_C - 1}{\iota_C}}$ can be verified using the steady-state energy and non-energy consumption aggregators (21) and the demand curves (A.3).

The log-linearized version of the final layer of the consumption aggregator, (24), is given by

$$\widehat{c}_{ki,t} = \sum_{l=1}^{K} \zeta^C_{kl\,i} \widehat{c}_{kl\,i,t} \tag{A.13}$$

where $\zeta^C_{kl\,i} = \frac{P_{l\,i} C_{kl\,i}}{P^C_{ki} C_{ki}} = (\widetilde{\zeta}^C_{kl\,i})^{\frac{1}{\delta_C}} \left(\frac{C_{kl\,i}}{C_{ki}}\right)^{\frac{\delta_C - 1}{\delta_C}}$ can be verified using the steady-state international consumption aggregator (24) and the consumption demand curves (A.4).

Price Indices. The different price indices can be derived by combining the consumption demand curves previously derived with the different consumption aggregators. The consumption price index, the energy and non-energy price index, and the consumption import price index are given by $P^C_{k,t} = \left[\widetilde{\varrho}^C_k (P^{C,E}_{k,t})^{1-\gamma_C} + (1 - \widetilde{\varrho}^C_k)(P^{C,M}_{k,t})^{1-\gamma_C}\right]^{\frac{1}{1-\gamma_C}}$, $P^{C,E}_{k,t} = \left[\sum_{i \in I_E} \widetilde{\nu}^C_{ki} (P^C_{ki,t})^{1-\eta_C}\right]^{\frac{1}{1-\eta_C}}$, $P^{C,M}_{k,t} = \left[\sum_{i \in I_M} \widetilde{\upsilon}^C_{ki} (P^C_{ki,t})^{1-\iota_C}\right]^{\frac{1}{1-\iota_C}}$, and $P^C_{ki,t} = \left\{\sum_{l=1}^{K} \widetilde{\zeta}^C_{kl\,i} [(1 + \tau_{kl\,i,t}) P^k_{l\,i,t}]^{1-\delta_C}\right\}^{\frac{1}{1-\delta_C}}$. Their log-linearized counterparts are given by

$$0 = \varrho^C_k \widehat{p}^{C,E}_{k,t} + (1 - \varrho^C_k) \widehat{p}^{C,M}_{k,t} \tag{A.14}$$

$$\widehat{p}^{C,E}_{k,t} = \sum_{i \in I_E} \nu^C_{ki} \widehat{p}^C_{ki,t} \quad \text{and} \quad \widehat{p}^{C,M}_{k,t} = \sum_{i \in I_M} \upsilon^C_{ki} \widehat{p}^C_{ki,t} \tag{A.15}$$

$$\widehat{p}^C_{ki,t} = \sum_{l=1}^{K} \zeta^C_{kl\,i} (\widehat{p}^k_{l\,i,t} + \tau_{kl\,i,t}) \tag{A.16}$$

Intertemporal Household Problem. International financial markets are incomplete, $\mathcal{H} = 1$ in (4), with households in each country only having access to two risk-free bonds. More precisely, the household in country k has access to a domestic bond, and an internationally traded bond, $B^K_{k,t}$, issued, without loss of generality, by country K and denominated in country K's currency. The agent maximizes the present discounted value of per-period utility flows (A.1), with discount factor β, subject to her budget constraint and a law of motion for capital, The household faces the following per-period budget constraint:

$$P^C_{k,t}C_{k,t} + \sum_{i=1}^{I} P^I_{ki,t}I_{ki,t} + \frac{B_{k,t}}{1+i_{k,t}} + B^K_{k,t}\left[1-\Gamma(\mathrm{NFA}^K_{k,t})\right]^{-1}\mathcal{E}^K_{k,t} + \Xi_{k,t} \leq$$

$$\sum_{i=1}^{I} R_{ki,t}K_{ki,t} + B_{k,t-1} + B^K_{k,t-1}\mathcal{E}^K_{k,t}(1+i_{K,t-1}) + \int_0^1 W_{gk,t}\mathcal{N}_{gk,t}dg + \Pi_{k,t} - T_{k,t} \tag{A.17}$$

where $\int_0^1 W_{gk,t}\mathcal{N}_{gk,t}dg$ is the nominal labor income received by the representative household, $P^I_{ki,t}I_{ki,t}$ denotes the investment spending by the representative household for the different sectors i, where investment is defined by the law of motion $I_{ki,t} = K_{ki,t+1} - (1-\delta_{ki})K_{ki,t}$, and $R_{ki,t}K_{ki,t}$ denotes the interest revenue from the capital holdings in each of those sectors, $\mathrm{NFA}^K_{k,t} = B^K_{k,t}\mathcal{E}^K_{k,t}$ is the net foreign asset position of households in the country k, and where $\Gamma(x) = \gamma_*\left(\exp\left\{x/\mathcal{Y}_{k,t}\right\}-1\right)$ is an external financial intermediary premium that depends on the economy-wide net holdings of internationally traded foreign bonds as a ratio to the national nominal GDP $\mathcal{Y}_{k,t}$, with $\gamma_* > 0$.[22] The sign convention is that positive $\Xi_{k,t}$ and positive $T_{k,t}$ are lump-sum payments by households to the fiscal authority; rebates or transfers to households are negative values of these fiscal instruments. Under this convention, the incurred intermediation premium is rebated through $\Xi_{k,t}$.

The above program delivers $2+I$ Euler equations for each country: one for each of the bonds plus one per sector i to account for investment in capital in that sector:

$$C^{-\sigma}_{k,t} = \mathbb{E}_t\beta C^{-\sigma}_{k,t+1}\frac{1+i_{k,t}}{1+\pi^C_{k,t+1}}\frac{Z_{k,t+1}}{Z_{k,t}} \tag{A.18}$$

$$C^{-\sigma}_{k,t} = \mathbb{E}_t\beta C^{-\sigma}_{k,t+1}\frac{1+i_{K,t}}{1+\pi^C_{k,t+1}}\left[1-\Gamma(\mathrm{NFA}^K_{k,t})\right]\frac{\mathcal{E}^K_{k,t+1}}{\mathcal{E}^K_{k,t}}\frac{Z_{k,t+1}}{Z_{k,t}} \quad \forall k \neq K \tag{A.19}$$

$$C^{-\sigma}_{k,t} = \mathbb{E}_t\beta C^{-\sigma}_{k,t+1}\frac{P^I_{ki,t+1}(1-\delta_{ki}) + R_{ki,t+1}}{P^I_{ki,t}}\frac{Z_{k,t+1}}{Z_{k,t}} \tag{A.20}$$

for all $i \in \{1,..,I\}$ and where we assume that the (log-)demand shock follows an AR(1) process:

$$z_{k,t} = \rho^z_k z_{k,t-1} + \varepsilon^z_{k,t} \tag{A.21}$$

where $z_{k,t} := \log Z_{k,t}$, and $\varepsilon^z_{k,t} \sim \mathcal{N}\left(0,(\sigma^z_k)^2\right)$.

[22] The role of this intermediation premium is to stabilize the net foreign asset position in response to transitory shocks, a common practice in open economies with incomplete financial markets (Schmitt-Grohe and Uribe 2003). Furthermore, this specification guarantees that, in the non-stochastic steady state, households have no incentive to hold foreign bonds and the economy's net foreign asset position is zero.

The log-linearized version of the household's first-order conditions (A.18)-(A.20) are given by:

(A.22)

$$\widehat{c}_{k,t} = -\frac{1}{\sigma}(i_{k,t} - \mathbb{E}_t\pi^C_{k,t+1}) + \mathbb{E}_t\widehat{c}_{k,t+1} + \frac{1}{\sigma}(1-\rho^z_k)z_{k,t}$$

(A.23)

$$\widehat{c}_{k,t} = -\frac{1}{\sigma}(i_{K,t} - \mathbb{E}_t\pi^C_{k,t+1}) + \mathbb{E}_t\widehat{c}_{k,t+1} + \frac{1}{\sigma}(1-\rho^z_k)z_{k,t} - \frac{1}{\sigma}\mathbb{E}_t\Delta e_{kK,t+1} + \frac{1}{\sigma}\gamma_*\text{nfa}^K_{k,t} \quad \forall k \neq K$$

$$\widehat{c}_{k,t} = -\frac{1}{\sigma}\left[\frac{(1-\delta_{ki})\chi_{ki}}{(1-\delta_{ki})\chi_{ki}+\delta_{ki}\varsigma_{ki}}\mathbb{E}_t\widehat{p}^I_{ki,t+1} + \frac{\delta_{ki}\varsigma_{ki}}{(1-\delta_{ki})\chi_{ki}+\delta_{ki}\varsigma_{ki}}\mathbb{E}_t\widehat{r}_{ki,t+1} - \widehat{p}^I_{ki,t} + \mathbb{E}_t\pi^C_{k,t+1}\right] + \mathbb{E}_t\widehat{c}_{k,t+1} + \frac{1}{\sigma}(1-\rho^z_k)z_{k,t}$$

(A.24)

$$= -\frac{1}{\sigma}\left\{\beta(1-\delta_{ki})\mathbb{E}_t\widehat{p}^I_{ki,t+1} + [1-\beta(1-\delta_{ki})]\mathbb{E}_t\widehat{r}_{ki,t+1} - \widehat{p}^I_{ki,t} + \mathbb{E}_t\pi^C_{k,t+1}\right\} + \mathbb{E}_t\widehat{c}_{k,t+1} + \frac{1}{\sigma}(1-\rho^z_k)z_{k,t}$$

for all $i \in \{1,..,I\}$ and where we define the different log-linear NFA positions as $\text{nfa}^K_{k,t} = B^K_{k,t}\mathcal{E}^K_{k,t}/\mathcal{Y}_k$ and $\text{nfa}^{\text{MU}}_{k,t} = B^{\text{MU}}_{k,t}\mathcal{E}_{k\text{MU},t}/\mathcal{Y}_k$ since $B^K_{k,t} = 0$ and $B^{\text{MU}}_{k,t} = 0$ in the steady state, and

$$\Delta e^l_{k,t} = e^l_{k,t} - e^l_{k,t-1}. \tag{A.25}$$

Additionally, since the price level is not well-defined in steady state but its ratio with respect to the consumption price level is, we define $\widehat{p}^I_{ki,t} = p^I_{ki,t} - p^C_{k,t}$ and $\widehat{r}_{ki,t} = r_{ki,t} - p^C_{k,t}$; $\varsigma_{ki} = \frac{R_{ki}K_{ki}}{P_{ki}Y_{ki}}$ denotes the (steady-state) capital expenditure share of total sales of firm i, and $\chi_{ki} = \frac{P^I_{ki}I_{ki}}{P_{ki}Y_{ki}} = \beta\delta_{ki}\varsigma_{ki}/[1-\beta(1-\delta_{ki})]$ denotes the (steady-state) investment expenditure share of total sales of firm i.[23]

Combining the log-linearized first-order conditions for the holdings of domestic and internationally traded bonds (A.22)-(A.23), yields a risk-adjusted Uncovered Interest Parity (UIP) condition,

$$i_{k,t} - i_{K,t} = \mathbb{E}_t\Delta e^K_{k,t+1} - \gamma_*\text{nfa}^K_{k,t}. \tag{A.26}$$

[23] We have used the law of motion of sectoral capital and (A.20), both in steady-state, to derive the relation between χ_{ki} and ς.

A.2. Firms

We augment the production function (19) to include a sectoral TFP shock $A_{ki,t}$, and allow for decreasing returns to scale $\psi_{ki} < 1$,

$$Y_{kif,t} = A_{ki,t}\left[\widetilde{\alpha}_{ki}^{\frac{1}{\psi}} N_{kif,t}^{\frac{\psi-1}{\psi}} + \widetilde{\varsigma}_{ki}^{\frac{1}{\psi}} K_{kif,t}^{\frac{\psi-1}{\psi}} + \widetilde{\vartheta}_{ki}^{\frac{1}{\psi}} X_{kif,t}^{\frac{\psi-1}{\psi}}\right]^{\frac{\psi}{\psi-1}\psi_{ki}}, \tag{A.27}$$

where the log-linearized law of motione of sectoral capital (5) is given by

$$k_{ki,t} = \delta_{ki} i_{ki,t-1} + (1-\delta_{ki}) k_{ki,t-1}, \tag{A.28}$$

and the (log-)TFP shock follows an AR(1) process:

$$a_{ki,t} \equiv \log A_{ki,t} = \rho_{ki}^{a} a_{ki,t-1} + \varepsilon_{ki,t}^{a}, \tag{A.29}$$

where $a_{ki,t} := \log A_{ki,t}$, and $\varepsilon_{ki,t}^{a} \sim \mathcal{N}\left(0, (\sigma_{ki}^{a})^2\right)$.

The optimal allocation between labor, capital, and intermediate inputs is the result of a cost minimization programme in each sector i $\min W_{k,t} N_{kif,t} + R_{ki,t} K_{f,ki,t} + P_{ki,t}^{X} X_{kif,t}$ subject to (A.27). The implied intermediate input demand curves are given by

$$W_{k,t} = \mathrm{MC}_{kif,t} A_{ki,t}^{\frac{\psi-1}{\psi\psi_{ki}}} \psi_{ki} Y_{kif,t}^{\frac{(1-\psi_{ki})(1-\psi)}{\psi\psi_{ki}}} \left(\frac{\widetilde{\alpha}_{ki} Y_{kif,t}}{N_{kif,t}}\right)^{\frac{1}{\psi}} \tag{A.30}$$

$$R_{ki,t} = \mathrm{MC}_{ki,t} A_{ki,t}^{\frac{\psi-1}{\psi\psi_{ki}}} \psi_{ki} Y_{kif,t}^{\frac{(1-\psi_{ki})(1-\psi)}{\psi\psi_{ki}}} \left(\frac{\widetilde{\varsigma}_{ki} Y_{kif,t}}{K_{kif,t}}\right)^{\frac{1}{\psi}} \tag{A.31}$$

$$P_{ki,t}^{X} = \mathrm{MC}_{ki,t} A_{ki,t}^{\frac{\psi-1}{\psi\psi_{ki}}} \psi_{ki} Y_{kif,t}^{\frac{(1-\psi_{ki})(1-\psi)}{\psi\psi_{ki}}} \left(\frac{\widetilde{\vartheta}_{ki} Y_{kif,t}}{X_{kif,t}}\right)^{\frac{1}{\psi}} \tag{A.32}$$

The log-linearized versions of the labor, intermediate inputs and capital demand curves (A.30)-(A.54) are given by

$$\widehat{w}_{k,t} - \widehat{\mathrm{mc}}_{ki,t} = \frac{\psi-1}{\psi\psi_{ki}} a_{ki,t} + \frac{(1-\psi_{ki})(1-\psi)}{\psi\psi_{ki}} \widehat{y}_{kif,t} + \frac{1}{\psi}\left(\widehat{y}_{kif,t} - \widehat{n}_{kif,t}\right) \tag{A.33}$$

$$\widehat{r}_{ki,t} - \widehat{\mathrm{mc}}_{ki,t} = \frac{\psi-1}{\psi\psi_{ki}} a_{ki,t} + \frac{(1-\psi_{ki})(1-\psi)}{\psi\psi_{ki}} \widehat{y}_{kif,t} + \frac{1}{\psi}\left(\widehat{y}_{kif,t} - \widehat{k}_{kif,t}\right) \tag{A.34}$$

$$\widehat{p}_{ki,t}^{X} - \widehat{\mathrm{mc}}_{ki,t} = \frac{\psi-1}{\psi\psi_{ki}} a_{ki,t} + \frac{(1-\psi_{ki})(1-\psi)}{\psi\psi_{ki}} \widehat{y}_{kif,t} + \frac{1}{\psi}\left(\widehat{y}_{kif,t} - \widehat{x}_{kif,t}\right) \tag{A.35}$$

where $\widehat{w}_{k,t} = w_{k,t} - p^C_{k,t}$, $\widehat{\mathrm{mc}}_{ki,t} = \mathrm{mc}_{ki,t} - p^C_{k,t}$ and $\widehat{p}^X_{ki,t} = p_{kiX,t} - p^C_{k,t}$.

Production Structure. The log-linearized version of the production function (A.27) is given by

$$\widehat{y}_{kif,t} = a_{ki,t} + \mathcal{M}^p_{ki}\alpha_{ki}\widehat{n}_{kif,t} + \mathcal{M}^p_{ki}\varsigma_{ki}\widehat{k}_{kif,t} + \mathcal{M}^p_{ki}\vartheta_{ki}\widehat{x}_{kif,t} \tag{A.36}$$

where the identities $\mathcal{M}^p_{ki}\alpha_{ki} = \mathcal{M}^p_{ki}\frac{W_k N_{ki}}{P_{ki}Y_{ki}} = \frac{N_{ki}}{Y_{ki}}\psi_{ki}Y_{ki}^{\frac{(1-\psi_{ki})(1-\psi)}{\psi\psi_{ki}}}\left(\frac{\widetilde{\alpha}_{ki}Y_{ki}}{N_{ki}}\right)^{\frac{1}{\psi}} = \psi_{ki}\left(\frac{W_k Y_{ki}^{\frac{1-\psi_{ki}}{\psi_{ki}}}}{\mathrm{MC}_{ki}\psi_{ki}}\right)^{1-\psi}\widetilde{\alpha}_{ki}$, $\mathcal{M}^p_{ki}\varsigma_{ki} = \mathcal{M}^p_{ki}\frac{R_{ki}K_{ki}}{P_{ki}Y_{ki}} = \frac{K_{ki}}{Y_{ki}}\psi_{ki}Y_{ki}^{\frac{(1-\psi_{ki})(1-\psi)}{\psi\psi_{ki}}}\left(\frac{\widetilde{\varsigma}_{ki}Y_{ki}}{K_{ki}}\right)^{\frac{1}{\psi}} = \psi_{ki}\left(\frac{R_{ki} Y_{ki}^{\frac{1-\psi_{ki}}{\psi_{ki}}}}{\mathrm{MC}_{ki}\psi_{ki}}\right)^{1-\psi}\widetilde{\varsigma}_{ki}$ and $\mathcal{M}^p_{ki}\vartheta_{ki} = \mathcal{M}^p_{ki}\frac{P^X_{ki}X_{ki}}{P_{ki}Y_{ki}} = \frac{X_{ki}}{Y_{ki}}\psi_{ki}Y_{ki}^{\frac{(1-\psi_{ki})(1-\psi)}{\psi\psi_{ki}}}\left(\frac{\widetilde{\vartheta}_{ki}Y_{ki}}{X_{ki}}\right)^{\frac{1}{\psi}} = \psi_{ki}\left(\frac{P^X_{ki} Y_{ki}^{\frac{1-\psi_{ki}}{\psi_{ki}}}}{\mathrm{MC}_{ki}\psi_{ki}}\right)^{1-\psi}\widetilde{\vartheta}_{ki}$ can be verified using the first-order conditions from the firms' problem (A.30)-(A.32) in steady-state and the standard monopolistic competition pricing condition in steady-state,

$$P_{ki} = \mathcal{M}^p_{ki}\mathrm{MC}_{ki}. \tag{A.37}$$

Using the previous identities, together with the production function (A.27) in steady-state, one can verify that

$$\mathcal{M}^p_{ki}\frac{W_k N_{ki}}{P_{ki}Y_{ki}} + \mathcal{M}^p_{ki}\frac{R_{ki}K_{ki}}{P_{ki}Y_{ki}} + \mathcal{M}^p_{ki}\frac{P^X_{ki}X_{ki}}{P_{ki}Y_{ki}} = \frac{W_k N_{ki}}{\mathrm{MC}^n_{ki}Y_{ki}} + \frac{R_{ki}K_{ki}}{\mathrm{MC}^n_{ki}Y_{ki}} + \frac{P^X_{ki}X_{ki}}{\mathrm{MC}^n_{ki}Y_{ki}} = \psi_{ki}. \tag{A.38}$$

The marginal cost of production is given by $\mathrm{MC}_{ki,t} = A_{ki,t}^{-1/\psi_{ki}}\psi_{ki}^{-1}Y_{ki,t}^{(1-\psi_{ki})/\psi_{ki}}\left[\widetilde{\alpha}_{ki}W_{k,t}^{1-\psi} + \widetilde{\varsigma}_{ki}R_{ki,t}^{1-\psi} + \widetilde{\vartheta}_{ki}(P^X_{ki,t})^{1-\psi}\right]^{\frac{1}{1-\psi}}$, and the log-linearized counterpart is given by

$$\widehat{\mathrm{mc}}_{ki,t} = \frac{1-\psi_{ki}}{\psi_{ki}}\widehat{y}_{ki,t} - \frac{1}{\psi_{ki}}a_{ki,t} + \frac{\mathcal{M}_{ki}}{\psi_{ki}}\alpha_{ki}\widehat{w}_{k,t} + \frac{\mathcal{M}_{ki}}{\psi_{ki}}\varsigma_{ki}\widehat{r}_{ki,t} + \frac{\mathcal{M}_{ki}}{\psi_{ki}}\vartheta_{ki}\widehat{p}^X_{ki,t} \tag{A.39}$$

Capital Inputs Demand Curves. Aggregate investment per sector is part of the household's intertemporal problem, but the composition of the investment bundle is solved intratemporarily. The optimal allocation of investment between energy and non-energy goods is the result of a cost minimization programme $\min P^{I,E}_{ki,t}I^E_{ki,t} + P^{I,M}_{ki,t}I^M_{ki,t}$ subject to (20). Similarly, the optimal allocation between energy (non-energy) capital inputs is the result of a cost minimization programme $\min \sum_{j\in I_E} P^I_{kij,t} I_{kij,t}$ ($\min \sum_{j\in I_M} P^I_{kij,t} I_{kij,t}$) subject to (22). Finally, the optimal allocation between the different capital goods is the result of a cost minimization programme $\min \sum_{l=1}^{K} (1+\tau_{kl\,j,t}) P^k_{l\,j,t} I_{kl\,ij,t}$ subject to (24). The implied investment input demand curves are given by

$$P^{I,E}_{ki,t} = P^I_{ki,t} \left(\frac{\widetilde{\varrho}^I_{ki} I_{ki,t}}{I^E_{ki,t}} \right)^{\frac{1}{\gamma_I}} \quad \text{and} \quad P^{I,M}_{ki,t} = P^I_{ki,t} \left(\frac{(1-\widetilde{\varrho}^I_{ki}) I_{ki,t}}{I^M_{ki,t}} \right)^{\frac{1}{\gamma_I}} \tag{A.40}$$

$$P^I_{kij,t} = P^{I,E}_{ki,t} \left(\frac{\widetilde{\nu}^I_{kij} I^E_{ki,t}}{I_{kij,t}} \right)^{\frac{1}{\eta_I}} \quad \forall \quad j \in I_E \quad \text{and} \quad P^I_{kij,t} = P^{I,M}_{ki,t} \left(\frac{\widetilde{\upsilon}^I_{kij} I^M_{ki,t}}{I_{kij,t}} \right)^{\frac{1}{\iota_I}} \quad \forall \quad j \in I_M \tag{A.41}$$

$$(1+\tau_{kl\,j,t}) P^k_{l\,j,t} = P^I_{kij,t} \left(\frac{\widetilde{\zeta}_{kl\,ijI} I_{kij,t}}{I_{kl\,ij,t}} \right)^{\frac{1}{\delta_I}} \quad \forall \quad l \in K \tag{A.42}$$

The log-linearized versions of the investment inputs demand curves (A.40)-(A.42) are given by

$$\widehat{p}^{I,E}_{ki,t} - \widehat{p}^I_{ki,t} = \frac{1}{\gamma_I} \left(\widehat{i}_{ki,t} - \widehat{i}^E_{ki,t} \right) \quad \text{and} \quad \widehat{p}^{I,M}_{ki,t} - \widehat{p}^I_{ki,t} = \frac{1}{\gamma_I} \left(\widehat{i}_{ki,t} - \widehat{i}^M_{ki,t} \right) \tag{A.43}$$

(A.44)

$$\widehat{p}^I_{kij,t} - \widehat{p}^{I,E}_{ki,t} = \frac{1}{\eta_I} \left(\widehat{i}^E_{ki,t} - \widehat{i}_{kij,t} \right) \quad \forall \quad j \in I_E \quad \text{and} \quad \widehat{p}^I_{kij,t} - \widehat{p}^{I,M}_{ki,t} = \frac{1}{\iota_I} \left(\widehat{i}^M_{ki,t} - \widehat{i}_{kij,t} \right) \quad \forall \quad j \in I_M$$

$$\tau_{kl\,j,t} + \widehat{p}^k_{l\,j,t} - \widehat{p}^I_{kij,t} = \frac{1}{\delta_I} \left(\widehat{i}_{kij,t} - \widehat{i}_{kl\,ij,t} \right) \quad \forall \quad l \in K \tag{A.45}$$

where $\widehat{p}^{I,E}_{ki,t} = p^{I,E}_{ki,t} - p^C_{k,t}$, $\widehat{p}^{I,M}_{ki,t} = p^{I,M}_{ki,t} - p^C_{k,t}$, and $\widehat{p}^I_{kij,t} = p^I_{kij,t} - p^C_{k,t}$ are well-defined as log price ratios relative to consumer prices.

Capital Baskets. The log-linearized investment input aggregator (20) is given by

$$\widehat{i}_{ki,t} = \varrho^I_{ki} \widehat{i}^E_{ki,t} + (1-\varrho^I_{ki}) \widehat{i}^M_{ki,t} \tag{A.46}$$

where $\varrho^I_{ki} = \frac{P^{I,E}_{ki} I^E_{ki}}{P^I_{ki} I_{ki}} = (\widetilde{\varrho}^I_{ki})^{\frac{1}{\gamma_I}} \left(\frac{I^E_{ki}}{I_{ki}} \right)^{\frac{\gamma_I - 1}{\gamma_I}}$ and $(1-\varrho^I_{ki}) = \frac{P^{I,M}_{ki} I^M_{ki}}{P^I_{ki} I_{ki}} = (1-\widetilde{\varrho}^I_{ki})^{\frac{1}{\gamma_I}} \left(\frac{I^M_{ki}}{I_{ki}} \right)^{\frac{\gamma_I - 1}{\gamma_I}}$ can be verified using the steady-state capital aggregator (20) and the capital demand curves (A.40). The log-linearized versions of the energy and non-energy investment aggregators (22) are given by

$$\widehat{i}^{E}_{ki,t} = \sum_{j\in I_E} \nu^{I}_{kij}\widehat{i}_{kij,t} \quad \text{and} \quad \widehat{i}^{M}_{ki,t} = \sum_{j\in I_M} \upsilon^{I}_{kij}\widehat{i}_{kij,t} \tag{A.47}$$

where $\nu^{I}_{kij} = \frac{P^{I}_{kij}I_{kij}}{P^{I,E}_{ki}I^{E}_{ki}} = (\widetilde{\nu}^{I}_{kij})^{\frac{1}{\eta_I}}\left(\frac{I_{kij}}{I^{E}_{ki}}\right)^{\frac{\eta_I-1}{\eta_I}}$ and $\upsilon^{I}_{kij} = \frac{P^{I}_{kij}I_{kij}}{P^{I,M}_{ki}I^{M}_{ki}} = (\widetilde{\upsilon}^{I}_{kij})^{\frac{1}{\iota_I}}\left(\frac{I_{kij}}{I^{M}_{ki}}\right)^{\frac{\iota_I-1}{\iota_I}}$ can be verified using the steady-state energy and non-energy capital aggregators (22) and the demand curves (A.41). The log-linearized version of the final layer of the investment sourcing aggregator, (24), is given by

$$\widehat{i}_{kij,t} = \sum_{l=1}^{K} \zeta^{I}_{kl\,ij}\widehat{i}_{kl\,ij,t} \tag{A.48}$$

where $\zeta^{I}_{kl\,ij} = \frac{P^{k}_{lj}I_{kl\,ij}}{P^{I}_{kij}I_{kij}} = (\widetilde{\zeta}^{I}_{kl\,ij})^{\frac{1}{\delta_I}}\left(\frac{I_{kl\,ij}}{I_{kij}}\right)^{\frac{\delta_I-1}{\delta_I}}$ can be verified using the steady-state international investment aggregator (24) and the demand curve (A.42).

Price Indices. The different price indices can be derived by combining the intermediate input demand curves previously derived with the other intermediate input aggregators. The capital price index, the energy and non-energy capital price index, and the capital import price index are given by $P^{I}_{ki,t} = \left[\widetilde{\varrho}^{I}_{ki}(P^{I,E}_{ki,t})^{(1-\gamma_I)} + (1-\widetilde{\varrho}^{I}_{ki})(P^{I,M}_{ki,t})^{1-\gamma_I}\right]^{\frac{1}{1-\gamma_I}}$, $P^{I,E}_{ki,t} = \left[\sum_{j\in I_E}\widetilde{\nu}^{I}_{kij}(P^{I}_{kij,t})^{1-\eta_I}\right]^{\frac{1}{1-\eta_I}}$, $P^{I,M}_{ki,t} = \left[\sum_{j\in I_M}\widetilde{\upsilon}^{I}_{kij}(P^{I}_{kij,t})^{1-\iota_I}\right]^{\frac{1}{1-\iota_I}}$, and $P^{I}_{kij,t} = \left\{\sum_{l=1}^{K}\widetilde{\zeta}^{I}_{kl\,ij}[(1+\tau_{kl\,j,t})P^{k}_{lj,t}]^{1-\delta_I}\right\}^{\frac{1}{1-\delta_I}}$. Their log-linearized counterparts are given by

$$\widehat{p}^{I}_{ki,t} = \varrho^{I}_{ki}\widehat{p}^{I,E}_{ki,t} + (1-\varrho^{I}_{ki})\widehat{p}^{I,M}_{ki,t}, \tag{A.49}$$

$$\widehat{p}^{I,E}_{ki,t} = \sum_{j\in I_E}\nu_{kij,I}\widehat{p}^{I}_{kij,t} \quad \text{and} \quad \widehat{p}^{I,M}_{ki,t} = \sum_{j\in I_M}\upsilon_{kij,I}\widehat{p}^{I}_{kij,t}, \tag{A.50}$$

$$\widehat{p}^{I}_{kij,t} = \sum_{l=1}^{K}\zeta^{I}_{kl\,ij}(\widehat{p}^{k}_{lj,t} + \tau_{kl\,j,t}). \tag{A.51}$$

Intermediate Input Demand Curves. The optimal allocation between energy and non-energy intermediate goods is the result of a cost minimization programme $\min P^{X,E}_{ki,t}X^{E}_{ki,t} + P^{X,M}_{ki,t}X^{M}_{ki,t}$ subject to (20). Similarly, the optimal allocation between energy (non-energy) intermediate inputs is the result of a cost minization programme $\min \sum_{j\in I_E} P^{X}_{kij,t}X_{kij,t}$ ($\min \sum_{j\in I_M} P^{X}_{kij,t}X_{kij,t}$) subject to (23). Finally, the optimal allocation between the different consumption goods is the result of a cost minimization programme $\min \sum_{l=1}^{K}(1+\tau_{kl\,j,t})P^{k}_{lj,t}X_{kl\,ij,t}$ subject to (24).

The implied intermediate input demand curves are given by

$$\text{(A.52)} \qquad P^{X,E}_{ki,t} = P^{X}_{ki,t}\left(\frac{\tilde{\varrho}^{X}_{ki}X_{ki,t}}{X^{E}_{ki,t}}\right)^{\frac{1}{\gamma_X}} \quad \text{and} \quad P^{X,M}_{ki,t} = P^{X}_{ki,t}\left(\frac{(1-\tilde{\varrho}^{X}_{ki})X_{ki,t}}{X^{M}_{ki,t}}\right)^{\frac{1}{\gamma_X}}$$

(A.53)

$$P^{X}_{kij,t} = P^{X,E}_{ki,t}\left(\frac{\tilde{\nu}^{X}_{kij}X^{E}_{ki,t}}{X_{kij,t}}\right)^{\frac{1}{\eta_X}} \quad \forall \quad j \in I_E \quad \text{and} \quad P^{X}_{kij,t} = P^{X,M}_{ki,t}\left(\frac{\tilde{\upsilon}^{X}_{kij}X^{M}_{ki,t}}{X_{kij,t}}\right)^{\frac{1}{\iota_X}} \quad \forall \quad j \in I_M$$

$$\text{(A.54)} \qquad (1+\tau_{klj,t})P^{k}_{lj,t} = P^{X}_{kij,t}\left(\frac{\zeta^{X}_{kl\,ij}X_{kij,t}}{X_{kl\,ij,t}}\right)^{\frac{1}{\delta_X}} \quad \forall \quad l \in K$$

The log-linearized versions of the labor and intermediate inputs demand curves (A.52)-(A.54) are given by

$$\text{(A.55)} \qquad \hat{p}^{X,E}_{ki,t} - \hat{p}^{X}_{ki,t} = \frac{1}{\gamma_X}\left(\hat{x}_{ki,t} - \hat{x}^{E}_{ki,t}\right) \quad \text{and} \quad \hat{p}^{X,M}_{ki,t} - \hat{p}^{X}_{ki,t} = \frac{1}{\gamma_X}\left(\hat{x}_{ki,t} - \hat{x}^{M}_{ki,t}\right)$$

(A.56)

$$\hat{p}^{X}_{kij,t} - \hat{p}^{X,E}_{ki,t} = \frac{1}{\eta_X}\left(\hat{x}^{E}_{ki,t} - \hat{x}_{kij,t}\right) \quad \forall \quad j \in I_E \quad \text{and} \quad \hat{p}^{X}_{kij,t} - \hat{p}^{X,M}_{ki,t} = \frac{1}{\iota_X}\left(\hat{x}^{M}_{ki,t} - \hat{x}_{kij,t}\right) \quad \forall \quad j \in I_M$$

$$\text{(A.57)} \qquad \tau_{klj,t} + \hat{p}^{k}_{lj,t} - \hat{p}^{X}_{kij,t} = \frac{1}{\delta_X}\left(\hat{x}_{kij,t} - \hat{x}_{kl\,ij,t}\right) \quad \forall \quad l \in K$$

where $\hat{p}^{X,E}_{ki,t} = p^{X,E}_{ki,t} - p^{C}_{k,t}$, $\hat{p}^{X,M}_{ki,t} = p^{X,M}_{ki,t} - p^{C}_{k,t}$, and $\hat{p}^{X}_{kij,t} = p^{X}_{kij,t} - p^{C}_{k,t}$ are well-defined as a ratio of prices.

Intermediate Inputs Baskets. The log-linearized intermediary input aggregator (20) is given by

$$\text{(A.58)} \qquad \hat{x}_{ki,t} = \varrho^{X}_{ki}\hat{x}^{E}_{ki,t} + (1-\varrho^{X}_{ki})\hat{x}^{M}_{ki,t}$$

where $\varrho^{X}_{ki} = \frac{P^{X,E}_{ki}X^{E}_{ki}}{P^{X}_{ki}X_{ki}} = (\tilde{\varrho}^{X}_{ki})^{\frac{1}{\gamma_X}}\left(\frac{X^{E}_{ki}}{X_{ki}}\right)^{\frac{\gamma_X-1}{\gamma_X}}$ and $(1-\varrho^{X}_{ki}) = \frac{P^{X,M}_{ki}X^{M}_{ki}}{P^{X}_{ki}X_{ki}} = (1-\tilde{\varrho}^{X}_{ki})^{\frac{1}{\gamma_X}}\left(\frac{X^{M}_{ki}}{X_{ki}}\right)^{\frac{\gamma_X-1}{\gamma_X}}$ can be verified using the steady-state intermediate input aggregator (20) and the input demand curves (A.52).

The log-linearized versions of the energy and non-energy intermediate input aggregators

(23) are given by

$$\widehat{x}^{E}_{ki,t} = \sum_{j\in I_E} \nu^{X}_{kij}\widehat{x}_{kij,t} \qquad \text{and} \qquad \widehat{x}^{M}_{ki,t} = \sum_{j\in I_M} \upsilon^{X}_{kij}\widehat{x}_{kij,t} \tag{A.59}$$

where $\nu^{X}_{kij} = \frac{P^{X}_{kij}X_{kij}}{P^{X,E}_{ki}X^{E}_{ki}} = (\widetilde{\nu}^{X}_{kij})^{\frac{1}{\eta_X}}\left(\frac{X_{kij}}{X^{E}_{ki}}\right)^{\frac{\eta_X-1}{\eta_X}}$ and $\upsilon^{X}_{kij} = \frac{P^{X}_{kij}X_{kij}}{P^{X,M}_{ki}X^{M}_{ki}} = (\widetilde{\upsilon}^{X}_{kij})^{\frac{1}{\iota_X}}\left(\frac{X_{kij}}{X^{M}_{ki}}\right)^{\frac{\iota_X-1}{\iota_X}}$ can be verified using the steady-state energy and non-energy intermediate input aggregators (23) and the demand curves (A.53).

The log-linearized version of the final layer of the intermediate input aggregator, (24), is given by

$$\widehat{x}_{kij,t} = \sum_{l=1}^{K} \zeta^{X}_{kl\,ij}\widehat{x}_{kl\,ij,t} \tag{A.60}$$

where $\zeta^{X}_{kl\,ij} = \frac{P_{kl\,j}X_{kl\,ij}}{P^{X}_{kij}X_{kij}} = (\widetilde{\zeta}^{X}_{kl\,ij})^{\frac{1}{\delta_X}}\left(\frac{X_{kl\,ij}}{X_{kij}}\right)^{\frac{\delta_X-1}{\delta_X}}$ can be verified using the steady-state international intermediate input aggregators (24) and the demand curve (A.54).

Price Indices. The different price indices can be derived by combining the intermediate input demand curves previously derived with the other intermediate input aggregators. The intermediate input price index, the energy and non-energy input price index, and the input import price index are given by $P^{X}_{ki,t} = \left[\widetilde{\varrho}^{X}_{ki}(P^{X,E}_{ki,t})^{1-\gamma_X} + (1-\widetilde{\varrho}^{X}_{ki})(P^{X,M}_{ki,t})^{1-\gamma_X}\right]^{\frac{1}{1-\gamma_X}}$, $P^{X,E}_{ki,t} = \left[\sum_{j\in I_E}\widetilde{\nu}^{X}_{kij}(P^{X}_{kij,t})^{1-\eta_X}\right]^{\frac{1}{1-\eta_X}}$, $P^{X,M}_{ki,t} = \left[\sum_{j\in I_M}\widetilde{\upsilon}^{X}_{kij}(P^{X}_{kij,t})^{1-\iota_X}\right]^{\frac{1}{1-\iota_X}}$, and $P^{X}_{kij,t} = \left\{\sum_{l=1}^{K}\widetilde{\zeta}^{X}_{kl\,ij}[(1+\tau_{kl\,j,t})P^{k}_{l\,j,t}]^{1-\delta_X}\right\}^{\frac{1}{1-\delta_X}}$. Their log-linearized counterparts are given by

$$\widehat{p}^{X}_{ki,t} = \varrho^{X}_{ki}\widehat{p}^{X,E}_{ki,t} + (1-\varrho^{X}_{ki})\,\widehat{p}^{X,M}_{ki,t} \tag{A.61}$$

$$\widehat{p}^{X,E}_{ki,t} = \sum_{j\in I_E}\nu^{X}_{kij}\,\widehat{p}^{X}_{kij,t} \quad \text{and} \quad \widehat{p}^{X,M}_{ki,t} = \sum_{j\in I_M}\upsilon^{X}_{kij}\,\widehat{p}^{X}_{kij,t} \tag{A.62}$$

$$\widehat{p}^{X}_{kij,t} = \sum_{l=1}^{K}\zeta^{X}_{kl\,ij}(\widehat{p}^{k}_{l\,j,t} + \tau_{kl\,j,t}). \tag{A.63}$$

A.3. Price–Setting

We extend our framework to allow for a time-varying elasticity of substitution between different good varieties, in order to micro-found price cost-push shocks. We extend (3) to

$$Y_{kl\,i,t} = \left(\int_0^1 (Y_{fkl\,i,t})^{(\epsilon^p_{ki,t}-1)/\epsilon^p_{ki,t}} df\right)^{\epsilon^p_{ki,t}/(\epsilon^p_{ki,t}-1)}, \tag{A.64}$$

where $Y_{kl\,i,t}$ denotes the demand in country l for sector i goods produced in country k. The implied sectoral price index is

$$P_{ki,t} = \left(\int_0^1 (P_{kif,t})^{1-\epsilon^p_{ki,t}} df\right)^{\frac{1}{1-\epsilon^p_{ki,t}}}. \tag{A.65}$$

Producers of each differentiated variety face the demand function

$$Y_{kl\,i,t+s|t} = \left(P_{kif,t}/P_{ki,t+s}\right)^{-\epsilon^p_{ki,t+s}} Y_{kl\,i,t+s} \tag{A.66}$$

where the elasticity is dated at the period in which sales occur; its steady-state value is used in the linearization below. Firms set prices à la Calvo, which implies that the aggregate price dynamics are described by the equation

$$(\Pi^p_{ki,t})^{1-\epsilon^p_{ki,t}} = \theta^p_{ki} + (1-\theta^p_{ki})\left(\overline{P}_{ki,t}/P_{ki,t-1}\right)^{1-\epsilon^p_{ki,t}}, \tag{A.67}$$

which can be written in log-linearized terms as

$$\pi_{ki,t} = (1-\theta^p_{ki})(\overline{p}_{ki,t} - p_{ki,t-1}). \tag{A.68}$$

The per-period nominal profits of the domestic firm producing good from sector i are then given by

$$\mathcal{P}_{ki,t} = \sum_{l=1}^{K} P_{ki,t} Y_{l\,ki,t} - \mathcal{C}_{ki,t}(Y_{ki,t}) \tag{A.69}$$

where $Y_{ki,t} = \sum_{l=1}^{K} Y_{l\,ki,t}$ denotes total demand across destination markets and $\mathcal{C}_{ki,t}(Y_{ki,t})$ denotes total costs of country k firms in their home currency.

Consider the pricing problem of a firm from country k and denote $\overline{P}_{ki,t}$ its optimallychosen reset price, resulting from the problem

$$\max_{\overline{P}_{ki,t}} \sum_{s=0}^{\infty} (\theta_{ki}^{p})^{s} \mathbb{E}_t \left\{ \frac{\Lambda_{t,t+s}}{P_{ki,t+s}} \left[\overline{P}_{ki,t} Y_{ki,t+s|t} - \mathcal{C}_{ki,t+s}\left(Y_{ki,t+s|t}\right)\right]\right\} \tag{A.70}$$

subject to the sequence of demand constraints (A.66). The optimality condition associated with the problem takes the form

$$\sum_{s=0}^{\infty} (\theta_{ki}^{p})^{s} \mathbb{E}_t \left\{ \frac{\Lambda_{t,t+s} Y_{ki,t+s|t}}{P_{ki,t+s}} [\overline{P}_{ki,t} - \mathcal{M}_{ki,t+s} \mathrm{MC}_{ki,t+s|t}] \right\} = 0. \tag{A.71}$$

A first-order Taylor expansion of (A.71) around the zero inflation steady state yields

$$\overline{p}_{ki,t} = (1-\beta\theta_{ki}^{p}) \sum_{s=0}^{\infty} (\beta\theta_{ki}^{p})^{s} \mathbb{E}_t \left(\mathrm{mc}_{ki,t+s|t} + \mu_{ki,t+s}\right). \tag{A.72}$$

The log marginal cost for an individual firm that last set its price in period t is given by (A.33),

$$\mathrm{mc}_{ki,t+s|t} = w_{k,t+s} - \frac{\psi-1}{\psi\psi_{ki}} a_{ki,t+s} - \log\psi_{ki} - \frac{(1-\psi_{ki})(1-\psi)}{\psi\psi_{ki}} y_{ki,t+s|t} - \frac{1}{\psi}\left[\log\widetilde{\alpha}_{ki} + y_{ki,t+s|t} - n_{ki,t+s|t}\right],$$

where $n_{ki,t+s|t}$ denotes the log employment in period $t+s$ for a firm that last reset its price in period t, and where we have made use of (A.33). Letting $\mathrm{mc}_{ki,t} = \int_0^1 \mathrm{mc}_{kif,t}\, df = (1-\theta_{ki}^{p}) \sum_{s=0}^{\infty} (\theta_{ki}^{p})^{s} \mathrm{mc}_{ki,t|t-s}$ represent the log average marginal cost, it follows that

$$\mathrm{mc}_{ki,t} = w_{k,t} - \frac{\psi-1}{\psi\psi_{ki}} a_{ki,t} - \log\psi_{ki} - \frac{(1-\psi_{ki})(1-\psi)}{\psi\psi_{ki}} y_{ki,t} - \frac{1}{\psi}\left[\log\widetilde{\alpha}_{ki} + y_{ki,t} - n_{ki,t}\right].$$

Thus, the following relation holds between firm-specific and economy-wide marginal costs

$$\mathrm{mc}_{ki,t+s|t} = \mathrm{mc}_{ki,t+s} - \frac{(1-\psi_{ki})(1-\psi)}{\psi\psi_{ki}} \left(y_{ki,t+s|t} - y_{ki,t+s}\right) - \frac{1}{\psi}\left[\left(y_{ki,t+s|t} - y_{ki,t+s}\right) - \left(n_{ki,t+s|t} - n_{ki,t+s}\right)\right].$$

Notice that, making use of both marginal cost expressions (A.30)-(A.32), the identity $x_{ki,t+s|t} - x_{ki,t+s} = n_{ki,t+s|t} - n_{ki,t+s}$ is satisfied. Hence, we can write

$$y_{ki,t+s|t} - y_{ki,t+s} = \left[\mathcal{M}_{ki}\frac{W_k N_{ki}}{P_{ki}Y_{ki}} + \mathcal{M}_{ki}\frac{R_{ki}K_{ki}}{P_{ki}Y_{ki}} + \mathcal{M}_{ki}\frac{P_{ki}^{X}X_{ki}}{P_{ki}Y_{ki}}\right](n_{ki,t+s|t} - n_{ki,t+s}) = \psi_{ki}(n_{ki,t+s|t} - n_{ki,t+s}),$$

where we have used the identity (A.38), and where we have used the linearized production function (A.36). We can finally write the relation between marginal costs as

$$\mathrm{mc}_{ki,t+s|t} = \mathrm{mc}_{ki,t+s} + \frac{1-\psi_{ki}}{\psi_{ki}}\left(y_{ki,t+s|t} - y_{ki,t+s}\right) = \mathrm{mc}_{ki,t+s} - \frac{(1-\psi_{ki})\epsilon^p_{ki}}{\psi_{ki}}\left(\overline{p}_{ki,t} - p_{ki,t+s}\right).$$

Introducing this last expression into the log-linearized firms' FOC (A.72), we can write

$$\overline{p}_{ki,t} = (1-\beta\theta^p_{ki})\sum_{s=0}^{\infty}(\beta\theta^p_{ki})^s\mathbb{E}_t\left[\mathrm{mc}_{ki,t+s} - \frac{(1-\psi_{ki})\epsilon^p_{ki}}{\psi_{ki}}\left(\overline{p}_{ki,t} - p_{ki,t+s}\right) + \mu_{ki,t+s}\right] \tag{A.73}$$

$$= (1-\beta\theta^p_{ki})\sum_{s=0}^{\infty}(\beta\theta^p_{ki})^s\mathbb{E}_t\left[\Theta_{ki}\mathrm{mc}_{ki,t+s} + (1-\Theta_{ki})\,p_{ki,t+s} + \Theta_{ki}\mu_{ki,t+s}\right] \tag{A.74}$$

where $\Theta_{ki} = \psi_{ki}/[\psi_{ki} + (1-\psi_{ki})\epsilon^p_{ki}]$.

Rewriting the (linearized) inflation dynamics (A.75) as

$$\overline{p}_{ki,t} - p_{ki,t} = \frac{\theta^p_{ki}}{1-\theta^p_{ki}}\pi_{ki,t}. \tag{A.75}$$

Subtracting $p_{ki,t}$ on both sides of (A.72), and inserting (A.75) on the LHS,

$$\frac{\theta^p_{ki}}{1-\theta^p_{ki}}\pi_{ki,t} = (1-\beta\theta^p_{ki})\sum_{s=0}^{\infty}(\beta\theta^p_{ki})^s\mathbb{E}_t\left[\Theta_{ki}(\mathrm{mc}_{ki,t+s} - p_{ki,t+s}) + p_{ki,t+s} - p_{ki,t} + \Theta_{ki}\mu_{ki,t+s}\right]$$

$$= \Theta_{ki}(1-\beta\theta^p_{ki})\sum_{s=0}^{\infty}(\beta\theta^p_{ki})^s\mathbb{E}_t\left(\mathrm{mc}_{ki,t+s} - p_{ki,t+s} + \mu_{ki,t+s}\right)$$

$$+ (1-\beta\theta^p_{ki})\sum_{s=1}^{\infty}(\beta\theta^p_{ki})^s\mathbb{E}_t\pi_{ki,t+s}. \tag{A.76}$$

Taking conditional expectations at time $t+1$, multiplying by $\beta\theta^p_{ki}$, and rearranging terms using (A.68), we can write

$$\pi_{ki,t} = \kappa_{ki}\left(\mathrm{mc}_{ki,t} - p_{ki,t}\right) + \beta\mathbb{E}_t\pi_{ki,t+1} + u^p_{ki,t}$$

$$= \kappa_{ki}\left[(\mathrm{mc}_{ki,t} - p^C_{k,t}) - (p_{ki,t} - p^C_{k,t})\right] + \beta\mathbb{E}_t\pi_{ki,t+1} + u^p_{ki,t}$$

$$= \kappa_{ki}\left(\widehat{\mathrm{mc}}_{ki,t} - \widehat{p}_{ki,t}\right) + \beta\mathbb{E}_t\pi_{ki,t+1} + u^p_{ki,t}, \tag{A.77}$$

where $\kappa_{ki} = \Theta_{ki}(1-\theta^p_{ki})(1-\beta\theta^p_{ki})/\theta^p_{ki}$.

We assume that the sectoral price cost-push shocks $u^p_{ki,t} = \kappa_{ki}\widehat{\mu}_{ki,t} = \kappa_{ki}(\mu_{ki,t} - \mu_{ki})$, micro-founded through a time-varying elasticity of substitution $\epsilon^p_{ki,t}$ in (3), follow independent AR(1) processes:

$$u^p_{ki,t} = \rho^p_{ki} u^p_{ki,t-1} + \varepsilon^p_{ki,t}, \tag{A.78}$$

where $u^p_{ki,t} \sim \mathcal{N}\left(0, \sigma^2_{ki\,p}\right)$. The sectoral-inflation rates and sectoral-level real prices ($\widehat{p}_{ki,t}$) are related through the identity

$$\pi_{ki,t} = p_{ki,t} - p_{ki,t-1} = \widehat{p}_{ki,t} - \widehat{p}_{ki,t-1} + \pi^C_{k,t}, \tag{A.79}$$

where $\widehat{p}_{ki,t} = p_{ki,t} - p^C_{k,t}$.

Marginal costs and aggregate inflation. In this open input-output (IO) economy, the price Phillips curve (A.77) depends on the international supply network of intermediate and investment goods through the real marginal costs faced by firm i in country k, $\widehat{\mathrm{mc}}_{ki,t}$. Combining the log-linearized price indices (A.39), (A.51) and (A.63), we obtain the marginal cost equation,

(A.80)

$$\widehat{\mathrm{mc}}_{ki,t} = \frac{1-\psi_{ki}}{\psi_{ki}} \widehat{y}_{ki,t} - \frac{1}{\psi_{ki}} a_{ki,t} + \frac{\mathcal{M}^p_{ki}}{\psi_{ki}} \alpha_{ki} \widehat{w}_{k,t} + \frac{\mathcal{M}^p_{ki}}{\psi_{ki}} \varsigma_{ki} \widehat{r}_{ki,t} + \sum_{l=1}^{K} \sum_{j=1}^{I} \frac{\mathcal{M}^p_{ki}}{\psi_{ki}} \omega^X_{kl\,ij} \left(\widehat{p}^k_{l\,j,t} + \tau_{kl\,j,t}\right)$$

where, in the absence of a production subsidy, $\alpha_{ki} = \frac{W_k N_{ki}}{P_{ki} Y_{ki}} = \frac{W_k N_{ki}}{\mathcal{M}^p_{ki} \mathrm{MC}_{ki} Y_{ki}}$ denotes the (steady-state) labor income share of total sales of firm i, $\varsigma_{ki} = \frac{R_{ki} K_{ki}}{P_{ki} Y_{ki}} = \frac{R_{ki} K_{ki}}{\mathcal{M}^p_{ki} \mathrm{MC}_{ki} Y_{ki}}$ denotes the (steady-state) capital income share of total sales of firm i, $\omega^X_{kl\,ij} = \frac{P_{kl\,j} X_{kl\,ij}}{P_{ki} Y_{ki}} = \frac{P_{kl\,j} X_{kl\,ij}}{\mathcal{M}^p_{ki} \mathrm{MC}_{ki} Y_{ki}}$ denotes the (steady-state) IO expenditure share of total sales of firm i, and $\mathcal{M}^p_{ki} = \epsilon^p_{ki}/(\epsilon^p_{ki} - 1)$ denotes the steady-state markup charged by firm i.

Writing (A.14)-(A.16) in first-differences, we can obtain consumer price inflation,

$$\pi^C_{k,t} = \sum_{i=1}^{I} \sum_{l=1}^{K} \beta^C_{kl\,i} \left(\pi^k_{l\,i,t} + \Delta\tau_{kl\,i,t}\right), \tag{A.81}$$

where $\pi^k_{l\,i,t} \equiv p^k_{l\,i,t} - p^k_{l\,i,t-1}$ is origin-sector inflation expressed in country k's currency and $\Delta\tau_{kl\,i,t}$ is the log change in the buyer-price tariff wedge, approximated by the change in $\tau_{kl\,i,t}$ around the zero-tariff steady state. Equivalently, if $p^k_{l\,i,t} = p_{l\,i,t} + e^k_{l,t}$, then $\pi^k_{l\,i,t} = \pi_{l\,i,t} + \Delta e^k_{l,t}$. For domestic goods, the exchange-rate and tariff terms are zero. The expenditure weight is $\beta^C_{kl\,i} = \frac{P^k_{l\,i} C_{kl\,i}}{P^C_k C_k} = \zeta_{kl\,i} \left[\nu_{ki} \varrho^C_k \mathbb{1}_{\{i \in I_E\}} + \upsilon_{ki} (1 - \varrho^C_k) \left(1 - \mathbb{1}_{\{i \in I_E\}}\right)\right]$.

A.4. Wage–Setting

Following Erceg *et al.* (2000), wage stickiness is introduced in a way analogous to price stickiness. Labor unions specialized in any given labor type can reset their nominal wage only with probability $1-\theta_k^w$ each period, independently of the time elapsed since they last adjusted their wage. We assume that firms employ a continuum of differentiated labor services. In particular, $N_{kif,t}$ is an index of labor input used by firm f, and defined by

$$N_{kif,t} = \left(\int_0^1 N_{fgki,t}^{\frac{\epsilon_k^w - 1}{\epsilon_k^w}} dg \right)^{\frac{\epsilon_k^w}{\epsilon_k^w - 1}}, \tag{A.82}$$

where $N_{fgki,t}$ denotes the quantity of type-g labor employed by firm f in period t. Note that $\epsilon_{k,t}^w$ represents the elasticity of substitution among labor varieties. Note also the assumption of a continuum of labor types, indexed by $g \in [0, 1]$.

Let $W_{gk,t}$ denote the nominal wage for type-g labor prevailing in period t. Nominal wages are set by workers of each type (or a union representing them) and taken as given by firms. Given the wages effective at any point in time for the different types of labor services, cost minimization yields a corresponding set of demand schedules for each firm f and labor type g, given the firm's total employment $N_{fk,t}$,

$$N_{fgki,t} = \left(\frac{W_{gk,t}}{W_{k,t}} \right)^{-\epsilon_{k,t}^w} N_{kif,t}, \tag{A.83}$$

where

$$W_{k,t} \equiv \left(\int_0^1 W_{gk,t}^{1-\epsilon_{k,t}^w} dg \right)^{\frac{1}{1-\epsilon_{k,t}^w}} \tag{A.84}$$

is an aggregate wage index. Combining the previous conditions, one can obtain a convenient aggregation result, $\int_0^1 W_{gk,t} N_{fgki,t} dg = W_{k,t} N_{kif,t}$. That is, the wage bill of any given firm can be expressed as the product of the wage index and the firm's employment index.

Consider a union resetting its members' wage in period t, and let $W_{k,t}^*$ denote the newly set wage. The union chooses $W_{k,t}^*$ in a way consistent with utility maximization of its members' households, taking as given the decisions of other unions as well as the paths of aggregate consumption and prices. Specifically, the union seeks to maximize

$$\max_{W^*_{k,t}} \mathbb{E}_t \sum_{l=0}^{\infty} (\beta\theta^w_k)^l \left(\frac{C_{k,t+l}\, W^*_{k,t} N_{k,t+l|t}}{P^c_{t+l}} - \frac{N^{1+\varphi}_{k,t+l|t}}{1+\varphi} \right)$$

subject to the sequence of labor demand schedules

$$N_{k,t+l|t} = \left(\frac{W^*_{k,t}}{W_{k,t+l}} \right)^{-\epsilon^w_{k,t}} \int_0^1 N_{gk,t} dg,$$

where $N_{k,t+l|t}$ denotes the level of employment in period $t+l$ among workers that last reset their wage in period t. The first-order condition is given by

$$\sum_{l=0}^{\infty} (\beta\theta^w_k)^l \, \mathbb{E}_t \left[N_{k,t+l|t} C^{-\sigma}_{k,t+l} \left(\frac{W^*_{k,t}}{P^c_{t+l}} - \mathcal{M}^w_{k,t} \mathrm{MRS}_{k,t+l|t} \right) \right] = 0,$$

where $\mathcal{M}^w_{k,t} = \frac{\epsilon^w_{k,t}}{\epsilon^w_{k,t}-1}$, and $\mathrm{MRS}_{k,t+l|t} = C^{\sigma}_{k,t+l} N^{\varphi}_{k,t+l|t}$ denotes the marginal rate of substitution between household consumption and employment in period $t+l$ relevant to the workers resetting their wage in period t. Log-linearizing the above expression around a zero inflation steady-state yields the wage setting rule

$$w^*_{k,t} = (1-\beta\theta^w_k) \sum_{l=0}^{\infty} (\beta\theta^w_k)^l \, \mathbb{E}_t \left(\mathrm{mrs}_{t+l|t} + \mu^{w,n}_{k,t} + p_{kc,t+l} \right) \tag{A.85}$$

where $\mu^w_{k,t} = \log \mathcal{M}^w_{k,t} - \log \mathcal{M}^w_k$, $\widehat{\mathrm{mrs}}_{t+l|t} = \sigma \widehat{c}_{k,t+l} + \varphi \widehat{n}_{k,t+l|t}$, and $w^*_{k,t} = \log W^*_{k,t} - \log W_k$.

Letting $\widehat{\mathrm{mrs}}_{t+l} = \sigma \widehat{c}_{k,t+l} + \varphi \widehat{n}_{k,t+l}$ define the economy's average marginal rate of substitution, where $\widehat{n}_{k,t+l} = \log \int_0^1 \int_0^1 N_{fgk,t} df dg - \log \int_0^1 \int_0^1 N_{fgk} df dg$ denotes the log deviation of aggregate employment. Up to a first-order approximation,

$$\widehat{\mathrm{mrs}}_{t+l|t} = \widehat{\mathrm{mrs}}_{t+l} + \varphi(n_{k,t+l} - n_{k,t+l|t}) = \widehat{\mathrm{mrs}}_{t+l} - \epsilon^w_k \varphi (w^*_{k,t} - w_{k,t+l}).$$

Hence, we can write (A.85) as

$$w^*_{k,t} = \frac{1-\beta\theta^w_k}{1+\epsilon^w_k\varphi} \sum_{l=0}^{\infty} (\beta\theta^w_k)^l \, \mathbb{E}_t \left[(1+\epsilon^w_k\varphi) w_{k,t+l} - \left(w_{k,t+l} - p_{kc,t+l} - \mathrm{mrs}_{k,t+l} - \widehat{\mu}^w_{k,t+l} \right) \right] \tag{A.86}$$

where $\widehat{\mu}^w_{k,t+l} = \mu^w_{k,t+l} - \mu^w_k$.

Given the assumed wage setting structure, the evolution of the aggregate wage index is given by

$$W_{k,t} = \left(\theta_k^w W_{k,t-1}^{1-\epsilon_k^w} + (1-\theta_k^w)(W_{k,t}^*)^{1-\epsilon_k^w} \right)^{\frac{1}{1-\epsilon_k^w}}.$$

Log-linearized,

$$w_{k,t} = \theta_k^w w_{k,t-1} + (1-\theta_k^w) w_{k,t}^*.$$

Combining the last expression with (A.86), and letting $\pi_{k,t}^w = w_{k,t} - w_{k,t-1}$, we obtain the wage inflation equation:

$$\pi_{k,t}^w = \kappa_k^w \left(\sigma \widehat{c}_{k,t} + \varphi \widehat{n}_{k,t} - \widehat{w}_{k,t} \right) + \beta \mathbb{E}_t \pi_{k,t+1}^w + u_{ki,t}^w, \tag{A.87}$$

where

$$\pi_{k,t}^w = w_{k,t} - w_{k,t-1} = \widehat{w}_{k,t} - \widehat{w}_{k,t-1} + \pi_{k,t}^C \tag{A.88}$$

denotes wage inflation, with $\widehat{w}_{k,t} = w_{k,t} - p_{k,t}^C$ denoting the real wage, and $u_{k,t}^w = \kappa_k^w \widehat{\mu}_{k,t}^w$.

We assume that the wage cost-push shock, micro-founded through a time-varying elasticity of substitution in the labor demand aggregator, follows an AR(1) processes:

$$u_{k,t}^w = \rho_k^w u_{k,t-1}^w + \varepsilon_{k,t}^w \tag{A.89}$$

where $u_{k,t}^w \sim \mathcal{N}\left(0, (\sigma_k^w)^2\right)$.

A.5. Monetary Authority

The log-linearized bilateral nominal exchange rate (30) is given by $e_{k,t}^{k^{\mathrm{MU}}} = e_{k,}^{k^{\mathrm{MU}}}$, $\forall k \in K^{MU}$. In stationary terms, taking first differences, this can be written as

$$\Delta e_{k,t}^{k^{\mathrm{MU}}} = 0 \quad \forall k \in K^{MU} \tag{A.90}$$

Log-linearizing the expression for the real exchange rate (A.10) and first-differencing, we obtain

$$\Delta q_{k,t}^l = \Delta e_{k,t}^l + \pi_{l,t} - \pi_{k,t}. \tag{A.91}$$

The nominal exchange rates are cross-related by

$$e_{k,t}^l = e_{k,t}^m - e_{l,t}^m, \tag{A.92}$$

and symmetry between nominal exchange rates is satisfied,

$$e^l_{k,t} = -e^k_{l,t}. \tag{A.93}$$

Log-linearizing and first-differencing the symmetry of nominal exchange rates condition $\mathcal{E}^l_{k,t} = (\mathcal{E}^k_{l,t})^{-1}$ yields

$$\Delta e^l_{k,t} = -\Delta e^k_{l,t}. \tag{A.94}$$

A.6. Market Clearing, GDP, and Trade Balance

Market Clearing. We first consider the goods market clearing condition (A.97). Pre-multiplying by $\frac{P_{ki,t}}{P_{k,t}C_{k,t}+\sum_{i=1}^{I} P^I_{ki,t}I_{ki,t}} = \frac{P_{ki,t}}{E_{k,t}}$, and making use of (A.12),

$$\begin{aligned}
\frac{P_{ki,t}Y_{ki,t}}{E_{k,t}} &= \sum_{l=1}^{K} \frac{P_{ki,t}C_{lki,t}}{E_{k,t}} + \sum_{l=1}^{K}\sum_{j=1}^{I} \frac{P_{ki,t}X_{lkji,t}}{E_{k,t}} + \sum_{l=1}^{K}\sum_{j=1}^{I} \frac{P_{ki,t}I_{lkji,t}}{E_{k,t}} \\
&= \sum_{l=1}^{K} \frac{P_{ki,t}}{P^l_{ki,t}} \frac{P^l_{ki,t}C_{lki,t}}{E_{k,t}} + \sum_{l=1}^{K}\sum_{j=1}^{I} \frac{P_{ki,t}}{P^l_{ki,t}} \frac{P_{lj,t}Y_{lj,t}}{E_{k,t}} \frac{P^l_{lki,t}X_{lkji,t}}{P_{lj,t}Y_{lj,t}} \\
&+ \sum_{l=1}^{K}\sum_{j=1}^{I} \frac{P_{ki,t}}{P^l_{ki,t}} \frac{P_{lj,t}Y_{lj,t}}{E_{k,t}} \frac{P^l_{ki,t}I_{lkji,t}}{P_{lj,t}Y_{lj,t}} \\
&= \sum_{l=1}^{K} \frac{P_{ki,t}}{P^l_{ki,t}} \frac{E_{l,t}}{E_{k,t}} \frac{P^l_{ki,t}C_{lki,t}}{E_{l,t}} + \sum_{l=1}^{K}\sum_{j=1}^{I} \frac{P_{ki,t}}{P^l_{ki,t}} \frac{E_{l,t}}{E_{k,t}} \frac{P_{lj,t}Y_{lj,t}}{E_{l,t}} \frac{P^l_{ki,t}X_{lkji,t}}{P_{lj,t}Y_{lj,t}} \\
&+ \sum_{l=1}^{K}\sum_{j=1}^{I} \frac{P_{ki,t}}{P^l_{ki,t}} \frac{E_{l,t}}{E_{k,t}} \frac{P_{lj,t}Y_{lj,t}}{E_{l,t}} \frac{P^l_{ki,t}I_{lkji,t}}{P_{lj,t}Y_{lj,t}} \quad \forall i \in I
\end{aligned} \tag{A.95}$$

which we can write in steady-state as

$$\lambda_{ki} = \sum_{l=1}^{K} y_{lk}\beta^C_{lki} + \sum_{l=1}^{K}\sum_{j=1}^{I} y_{lk}\lambda_{lj}\left(\omega^X_{lkji} + \omega^I_{lkji}\right) \quad \forall i \in I \tag{A.96}$$

where the Domar weight for sector i in country k is $\lambda_{ki} = \frac{P_{ki}Y_{ki}}{y_k}$, the nominal GDP ratio between countries l and k is defined as $y_{lk} = \frac{P^C_l C_l+\sum_{i=1}^{I} P^I_{li}I_{li}}{P^C_k C_k+\sum_{i=1}^{I} P^I_{ki}I_{ki}} = \frac{y_l}{y_k}$, and the IO share is given by $\omega^X_{lkji} = \frac{P^l_{ki}X_{lkji}}{P_{lj}Y_{lj}} = \frac{P_{ki}X_{lkji}}{P_{lj}Y_{lj}} = \zeta^X_{lkji}\vartheta_{lj}\left[\nu^X_{lji}\varrho^X_{lj}\mathbb{1}_{\{i\in I_E\}} + \upsilon^X_{lji}(1-\varrho^X_{lj})\left(1-\mathbb{1}_{\{i\in I_E\}}\right)\right]$, where we have made use of the law of one price in steady-state, $P_{klj} = P_{lj}$. Similarly, $\omega^I_{lkji} = \frac{P^l_{ki} I_{lkji}}{P_{lj}Y_{lj}} = \frac{P_{ki} I_{lkji}}{P_{lj}Y_{lj}} = \zeta^I_{lkji}\chi_{lj}\left[\nu^I_{lji}\varrho^I_{lj}\mathbb{1}_{\{i\in I_E\}} + \upsilon^I_{lji}(1-\varrho^I_{lj})\left(1-\mathbb{1}_{\{i\in I_E\}}\right)\right]$. Notice that β^C_{lki}, ω^X_{lkji} and ω^I_{lkji} can be extracted directly from the data.

Hence, we can write the log-linearized version of the goods market clearing condition (A.97), $Y_{ki}\widehat{y}_{ki,t} = \sum_{l=1}^K \left(C_{lki}\widehat{c}_{lki,t} + \sum_{j=1}^I X_{lkji}\widehat{x}_{lkji} + \sum_{j=1}^I I_{lkji}\widehat{i}_{lkji}\right)$. Pre-multiplying the expression by P_{ki}/E_k, we can write

$$\lambda_{ki}\widehat{y}_{ki,t} = \sum_{l=1}^{K} y_{lk}\left(\beta^C_{lki}\widehat{c}_{lki,t} + \sum_{j=1}^{I}\lambda_{lj}\omega^X_{lkji}\widehat{x}_{lkji} + \sum_{j=1}^{I}\lambda_{lj}\omega^I_{lkji}\widehat{i}_{lkji}\right). \tag{A.97}$$

The log-linearized version of the labor market clearing condition (32) is given by

$$\widehat{n}_{k,t} = \sum_{i=1}^{I}\delta^N_{ki}\widehat{n}_{ki,t} \tag{A.98}$$

where $\delta^N_{ki} = \frac{N_{ki}}{N_k} = \frac{W_k N_{ki}}{P_{ki}Y_{ki}}\frac{P_{ki}Y_{ki}}{y_k}\frac{y_k}{W_k N_k} = \frac{\alpha_{ki}\lambda_{ki}}{1-\sum_{j=1}^I \frac{R_{kj}K_{kj}}{y_k} - \sum_{j=1}^I\left(1-\frac{\psi_{kj}}{\mathcal{M}_{kj}}\right)\lambda_{kj}} = \frac{\alpha_{ki}\lambda_{ki}}{1-\sum_{j=1}^I \varsigma_{kj}\lambda_{kj} - \sum_{j=1}^I\left(1-\frac{\psi_{kj}}{\mathcal{M}_{kj}}\right)\lambda_{kj}}$ can be derived using (A.103).

The NFA from the "global" country K (33) can be log-linearized to

$$\frac{1}{\beta}\sum_{k=1}^{K-1}\mathrm{nfa}^K_{k,t-1} - \sum_{k=1}^{K-1}\mathrm{nfa}^K_{k,t} = \Upsilon_K\left(\widehat{\exp}_{K,t} - \widehat{\mathrm{imp}}_{K,t} + \widehat{p}_{K\mathrm{EXP},t} - \widehat{p}_{K\mathrm{IMP},t}\right), \tag{A.99}$$

the NFA from country $k \neq K$ $k \notin$ MU, (33) can be log-linearized to

$$\mathrm{nfa}^K_{k,t} - \frac{1}{\beta}\mathrm{nfa}^K_{k,t-1} = \Upsilon_k\left(\widehat{\exp}_{k,t} - \widehat{\mathrm{imp}}_{k,t} + \widehat{p}_{k\mathrm{EXP},t} - \widehat{p}_{k\mathrm{IMP},t}\right) \tag{A.100}$$

where $\Upsilon_k = \frac{P_{k\mathrm{EXP}}\mathrm{EXP}_k}{y_k} = \frac{P_{k\mathrm{IMP}}\mathrm{IMP}_k}{y_k}$ denotes the export (or import) share of nominal GDP. The linearized export and import price deflators are given by:

$$\begin{aligned}\widehat{p}_{k\mathrm{IMP},t} &= \sum_{l\neq k}\sum_{i=1}^{I}\left[\frac{P_{li}C_{kli} + \sum_{j=1}^I P_{li}X_{klji} + \sum_{j=1}^I P_{li}I_{klji}}{P_{k\mathrm{IMP}}IMP_k}\widehat{p}_{kli,t}\right] \\ &= \sum_{l\neq k}\sum_{i=1}^{I}\Upsilon_k^{-1}\left[\beta^C_{kli} + \sum_{j=1}^{I}\lambda_{kj}(\omega^X_{klji} + \omega^I_{klji})\right]\widehat{p}_{kli,t}\end{aligned} \tag{A.101}$$

$$\widehat{p}_{k\mathrm{EXP},t} = \sum_{l\neq k}\sum_{i=1}^{I}\left[\frac{P_{ki}C_{l\,ki} + \sum_{j=1}^{I} P_{ki}X_{l\,kji} + \sum_{j=1}^{I} P_{ki}I_{l\,kji}}{P_{k,EXP}EXP_k}(q^l_{k,t} + \widehat{p}_{l\,ki,t})\right]$$

$$= \sum_{l\neq k}\sum_{i=1}^{I}\frac{y_{l\,k}}{\Upsilon_k}\left[\beta^C_{l\,ki} + \sum_{j=1}^{I}\lambda_{l\,j}(\omega^X_{l\,kji} + \omega^I_{l\,kji})\right](q^l_{k,t} + \widehat{p}_{l\,ki,t}) \tag{A.102}$$

Gross Domestic Product and Net Exports. Let us now move to nominal GDP (36). In steady state, assuming zero net exports, $P_{k\mathrm{EXP}}\mathrm{EXP}_k - P_{k\mathrm{IMP}}\mathrm{IMP}_k = 0$, we can write $y_k = P^C_k C_k + \sum_{i=1}^{I} P^I_{ki}I_{ki}$. Using the household's budget constraint (A.17) in steady state, we can write

(A.103)

$$y_k = P^C_k C_k + \sum_{i=1}^{I} p^I_{ki}I_{ki} = W_k N_k + \sum_{i=1}^{I} R_{ki}K_{ki} + \Pi_k = W_k N_k + \sum_{i=1}^{I} R_{ki}K_{ki} + \sum_{i=1}^{I}\left(1 - \frac{\psi_{ki}}{\mathcal{M}^p_{ki}}\right)P_{ki}Y_{ki}$$

where the last equality makes use of (A.38).

Log-linearizing the real GDP (38) definition,

$$\widehat{y}_{k,t} = \frac{P^C_k C_k}{y_k}\widehat{c}_{k,t} + \sum_{i=1}^{I}\frac{P^I_{ki}I_{ki}}{y_k}\widehat{i}_{ki,t} + \frac{P_{k\mathrm{EXP}}\mathrm{EXP}_k}{y_k}\widehat{\mathrm{exp}}_{k,t} - \frac{P_{k\mathrm{IMP}}\mathrm{IMP}_k}{y_k}\widehat{\mathrm{imp}}_{k,t}$$

$$= \Upsilon^C_k\widehat{c}_{k,t} + \sum_{i=1}^{I}\lambda_{ki}\chi_{ki}\widehat{i}_{ki,t} + \Upsilon_k\left(\widehat{\mathrm{exp}}_{k,t} - \widehat{\mathrm{imp}}_{k,t}\right)$$

$$= \Upsilon^C_k\widehat{c}_{k,t} + \sum_{i=1}^{I}\frac{\beta\lambda_{ki}\delta_{ki}\varsigma_{ki}}{1-\beta(1-\delta_{ki})}\widehat{i}_{ki,t} + \Upsilon_k\left(\widehat{\mathrm{exp}}_{k,t} - \widehat{\mathrm{imp}}_{k,t}\right). \tag{A.104}$$

where $\Upsilon^C_k \equiv \frac{P^C_k C_k}{y_k} = \sum_{l=1}^{K}\sum_{i=1}^{I}\beta^C_{kl\,i}$ denotes the steady-state consumption share.

The nominal exports expression (34) can be log-linearized to:

$$\widehat{\mathrm{exp}}_{k,t} = \sum_{l\neq k}\sum_{i\in I}\left(\frac{P_{ki}C_{l\,ki}}{P_{k\mathrm{EXP}}\mathrm{EXP}_k}\widehat{c}_{l\,ki,t} + \sum_{j=1}^{I}\frac{P_{ki}X_{l\,kji}}{P_{k\mathrm{EXP}}\mathrm{EXP}_k}\widehat{x}_{l\,kji,t} + \sum_{j=1}^{I}\frac{P_{ki}I_{l\,kji}}{P_{k\mathrm{EXP}}\mathrm{EXP}_k}\widehat{i}_{l\,kji,t}\right)$$

$$= \sum_{l\neq k}\sum_{i\in I}\frac{y_{l\,k}}{\Upsilon_k}\left[\beta^C_{l\,ki}\widehat{c}_{l\,ki,t} + \sum_{j=1}^{I}\lambda_{l\,j}(\omega^X_{l\,kji}\widehat{x}_{l\,kji,t} + \omega^I_{l\,kji}\widehat{i}_{l\,kji,t})\right] \tag{A.105}$$

where the export share of nominal GDP is given by

(A.106)

$$\Upsilon_k = \frac{P_{k\text{EXP}}\text{EXP}_k}{y_k} = \sum_{l\neq k}\sum_{i=1}^{I} y_{lk}\left[\beta^C_{lki} + \sum_{j=1}^{I}\lambda_{lj}(\omega^X_{lkji} + \omega^I_{lkji})\right] = \sum_{i=1}^{I}\left[\lambda_{ki} - \beta^C_{kki} - \sum_{j=1}^{I}\lambda_{kj}(\omega^X_{kkji} + \omega^I_{kkji})\right]$$

(A.107)

$$= \frac{P_{k\text{IMP}}\text{IMP}_k}{y_k} = \sum_{l\neq k}\sum_{i=1}^{I}\left[\beta^C_{kli} + \sum_{j=1}^{I}\lambda_{kj}(\omega^X_{klji} + \omega^I_{klji})\right]$$

Similarly, the nominal imports expression (35) can be log-linearized to

$$\widehat{\text{imp}}_{k,t} = \sum_{l\neq k}\sum_{i\in I}\left(\frac{P_{li}C_{kli}}{P_{k\text{IMP}}\text{IMP}_k}\widehat{c}_{kli,t} + \sum_{j=1}^{I}\frac{P_{li}X_{klji}}{P_{k\text{IMP}}\text{IMP}_k}\widehat{x}_{klji,t} + \sum_{j=1}^{I}\frac{P_{li}I_{klji}}{P_{k\text{IMP}}\text{IMP}_k}\widehat{i}_{klji,t}\right)$$

$$= \sum_{l\neq k}\sum_{i\in I}\Upsilon_k^{-1}\left(\beta^C_{kli}\widehat{c}_{kli,t} + \sum_{j=1}^{I}\lambda_{kj}(\omega^X_{klji}\widehat{x}_{klji,t} + \omega^I_{klji}\widehat{i}_{klji,t})\right) \tag{A.108}$$

Now we can combine the linearized expression for real gdp, que the expressions for real imports and exports:

$$\widehat{y}_{k,t} = \Upsilon^C_k\widehat{c}_{k,t} + \sum_{i=1}^{I}\lambda_{ki}\chi_{ki}\widehat{i}_{ki,t} + \sum_{l\neq k}\sum_{i\in I}\left[y_{lk}\beta^C_{lki}\widehat{c}_{lki,t} + \sum_{j=1}^{I}y_{lk}\lambda_{lj}(\omega^X_{lkji}\widehat{x}_{lkji,t} + \omega^I_{lkji}\widehat{i}_{lkji,t})\right.$$
$$\left. - \beta^C_{kli}\widehat{c}_{kli,t} - \sum_{j=1}^{I}\lambda_{kj}(\omega^X_{klji}\widehat{x}_{klji,t} + \omega^I_{klji}\widehat{i}_{klji,t})\right]$$
$$= \Upsilon^C_k\widehat{c}_{k,t} + \sum_{i=1}^{I}\frac{\beta\lambda_{ki}\delta_{ki}\varsigma_{ki}}{1-\beta(1-\delta_{ki})}\widehat{i}_{ki,t} + \sum_{l\neq k}\sum_{i\in I}\left[y_{lk}\beta^C_{lki}\widehat{c}_{lki,t} + \sum_{j=1}^{I}y_{lk}\lambda_{lj}(\omega^X_{lkji}\widehat{x}_{lkji,t} + \omega^I_{lkji}\widehat{i}_{lkji,t})\right.$$

(A.109)

$$\left. - \beta^C_{kli}\widehat{c}_{kli,t} - \sum_{j=1}^{I}\lambda_{kj}(\omega^X_{klji}\widehat{x}_{klji,t} + \omega^I_{klji}\widehat{i}_{klji,t})\right]$$

Aggregate investment and price of capital. Aggregate nominal investment is simply $P^I_{k,t}\text{INV}_{k,t} = \sum_{i=1}^{I}P^I_{ki,t}I_{ki,t}$. Log-linearizing this, the price and quantity aggregates are defined jointly via: $\widehat{p}^I_{k,t} + \widehat{\text{inv}}_{k,t} = \sum_{i=1}^{I}P^I_{ki}I_{ki}/(\sum_{j=1}^{I}P^I_{kj}I_{kj})(\widehat{p}^I_{ki,t} + \widehat{i}_{ki,t}) = \sum_{i=1}^{I}\chi_{ki}\lambda_{ki}/(\sum_{j=1}^{I}\chi_{kj}\lambda_{kj})(\widehat{p}^I_{ki,t} + \widehat{i}_{ki,t})$. To separate price from quantity we perform a Laspeyres decomposition that gives the aggregate price of capital and investment in country k:

$$\widehat{p}^I_{k,t} = \sum_{i=1}^{I} \frac{\chi_{ki}\lambda_{ki}}{\sum_{j=1}^{I}\chi_{kj}\lambda_{kj}} \widehat{p}^I_{ki,t} \qquad \text{and} \qquad \widehat{\text{inv}}_{k,t} = \sum_{i=1}^{I} \frac{\chi_{ki}\lambda_{ki}}{\sum_{j=1}^{I}\chi_{kj}\lambda_{kj}} \widehat{i}_{ki,t} \tag{A.110}$$

Sectoral value added and real GDP. Define sectoral nominal value added as

$$\text{VA}^{\text{nom}}_{ki,t} = P_{ki,t}Y_{ki,t} - \sum_{l=1}^{K}\sum_{j=1}^{I}(1+\tau_{klj,t})P^k_{lj,t}X_{kl\,ij,t}$$

and its associated price index as

$$P^{\text{VA}}_{ki,t} = \frac{\text{VA}^{\text{nom}}_{ki,t}}{P_{ki}Y_{ki,t} - \sum_{l=1}^{K}\sum_{j=1}^{I}P^k_{lj}X_{kl\,ij,t}}$$

where the denominator contains constant prices. Define sectoral real value added as

$$\text{VA}_{ki,t} = \frac{\text{VA}^{\text{nom}}_{ki,t}}{P^{\text{VA}}_{ki,t}} = P_{ki}Y_{ki,t} - \sum_{l=1}^{K}\sum_{j=1}^{I}P^k_{lj}X_{kl\,ij,t}$$

which, log-linearized, can be written as

$$\widehat{\text{va}}_{ki,t} = \frac{P_{ki}Y_{ki}}{P_{ki}Y_{ki} - P^X_{ki}X_{ki}}\widehat{y}_{ki,t} - \sum_{l=1}^{K}\sum_{j=1}^{I}\frac{P_{lj}X_{kl\,ij}}{P_{ki}Y_{ki} - P^X_{ki}X_{ki}}\widehat{x}_{kl\,ij,t} = \frac{1}{1-\vartheta_{ki}}\widehat{y}_{ki,t} - \sum_{l=1}^{K}\sum_{j=1}^{I}\frac{\omega^X_{kl\,ij}}{1-\vartheta_{ki}}\widehat{x}_{kl\,ij,t} \tag{A.111}$$

We are now interested in showing that $\sum_{i=1}^{I}\lambda_{ki}(1-\vartheta_{ki})\widehat{\text{va}}_{ki,t} = \widehat{y}_{k,t}$, where the RHS variable is real GDP (A.109). Using (A.97)

$$\begin{aligned}\sum_{i=1}^{I}\lambda_{ki}(1-\vartheta_{ki})\widehat{\text{va}}_{ki,t} &= \sum_{i=1}^{I}\lambda_{ki}\widehat{y}_{ki,t} - \sum_{i=1}^{I}\sum_{l=1}^{K}\sum_{j=1}^{I}\lambda_{ki}\omega^X_{kl\,ij}\widehat{x}_{kl\,ij,t} \\ &= \sum_{i=1}^{I}\sum_{l=1}^{K}y_{lk}\left(\beta^C_{lki}\widehat{c}_{lki,t} + \sum_{j=1}^{I}\lambda_{lj}\omega^X_{lkji}\widehat{x}_{lkji} + \sum_{j=1}^{I}\lambda_{lj}\omega^I_{lkji}\widehat{i}_{lkji}\right) - \sum_{i=1}^{I}\sum_{l=1}^{K}\sum_{j=1}^{I}\lambda_{ki}\omega^X_{kl\,ij}\widehat{x}_{kl\,ij,t}\end{aligned} \tag{A.112}$$

Separating domestic and foreign absorption, we can write

$$\begin{aligned}\sum_{i=1}^{I}\lambda_{ki}(1-\vartheta_{ki})\widehat{\text{va}}_{ki,t} &= \sum_{i=1}^{I}\left(\beta^{C}_{kki}\widehat{c}_{kki,t}+\sum_{j=1}^{I}\lambda_{kj}\omega^{X}_{kkji}\widehat{x}_{kkji,t}+\sum_{j=1}^{I}\lambda_{kj}\omega^{I}_{kkji}\widehat{i}_{kkji,t}\right)\\ &+\sum_{i=1}^{I}\sum_{l\neq k}^{K}y_{lk}\left(\beta^{C}_{lki}\widehat{c}_{lki,t}+\sum_{j=1}^{I}\lambda_{lj}\omega^{X}_{lkji}\widehat{x}_{lkji,t}+\sum_{j=1}^{I}\lambda_{lj}\omega^{I}_{lkji}\widehat{i}_{lkji,t}\right)\\ &-\sum_{i=1}^{I}\sum_{j=1}^{I}\lambda_{ki}\omega^{X}_{kkij}\widehat{x}_{kkij,t}-\sum_{i=1}^{I}\sum_{l\neq k}^{K}\sum_{j=1}^{I}\lambda_{ki}\omega^{X}_{klij}\widehat{x}_{klij,t}.\end{aligned}\tag{A.113}$$

Now note that, by relabeling indices, $\sum_{i=1}^{I}\sum_{j=1}^{I}\lambda_{ki}\omega^{X}_{kkij}\widehat{x}_{kkij,t} = \sum_{i=1}^{I}\sum_{j=1}^{I}\lambda_{kj}\omega^{X}_{kkji}\widehat{x}_{kkji,t}$, so domestic intermediate input use cancels out and

$$\begin{aligned}\sum_{i=1}^{I}\lambda_{ki}(1-\vartheta_{ki})\widehat{\text{va}}_{ki,t} &= \sum_{i=1}^{I}\beta^{C}_{kki}\widehat{c}_{kki,t}+\sum_{i=1}^{I}\sum_{j=1}^{I}\lambda_{kj}\omega^{I}_{kkji}\widehat{i}_{kkji,t}\\ &+\sum_{i=1}^{I}\sum_{l\neq k}^{K}y_{lk}\left(\beta^{C}_{lki}\widehat{c}_{lki,t}+\sum_{j=1}^{I}\lambda_{lj}\omega^{X}_{lkji}\widehat{x}_{lkji,t}+\sum_{j=1}^{I}\lambda_{lj}\omega^{I}_{lkji}\widehat{i}_{lkji,t}\right)\\ &-\sum_{i=1}^{I}\sum_{l\neq k}^{K}\sum_{j=1}^{I}\lambda_{ki}\omega^{X}_{klij}\widehat{x}_{klij,t}.\end{aligned}\tag{A.114}$$

Using the definition of aggregate consumption, $\Upsilon^{C}_{k}\widehat{c}_{k,t} = \sum_{l=1}^{K}\sum_{i=1}^{I}\beta^{C}_{kli}\widehat{c}_{kli,t}$, it follows that

$$\sum_{i=1}^{I}\beta^{C}_{kki}\widehat{c}_{kki,t} = \Upsilon^{C}_{k}\widehat{c}_{k,t}-\sum_{l\neq k}^{K}\sum_{i=1}^{I}\beta^{C}_{kli}\widehat{c}_{kli,t}.\tag{A.115}$$

Similarly, using $\chi_{kj} = \sum_{l=1}^{K}\sum_{i=1}^{I}\omega^{I}_{klji}$, we have

$$\sum_{i=1}^{I}\sum_{j=1}^{I}\lambda_{kj}\omega^{I}_{kkji}\widehat{i}_{kkji,t} = \sum_{j=1}^{I}\lambda_{kj}\chi_{kj}\widehat{i}_{kj,t}-\sum_{l\neq k}^{K}\sum_{i=1}^{I}\sum_{j=1}^{I}\lambda_{kj}\omega^{I}_{klji}\widehat{i}_{klji,t}.\tag{A.116}$$

Finally, relabeling indices in the imported intermediates term gives

$$\sum_{i=1}^{I}\sum_{l\neq k}^{K}\sum_{j=1}^{I}\lambda_{ki}\omega^{X}_{klij}\widehat{x}_{klij,t} = \sum_{l\neq k}^{K}\sum_{i=1}^{I}\sum_{j=1}^{I}\lambda_{kj}\omega^{X}_{klji}\widehat{x}_{klji,t}.\tag{A.117}$$

Substituting the previous expressions yields

$$\sum_{i=1}^{I}\lambda_{ki}(1-\vartheta_{ki})\widehat{\text{va}}_{ki,t} = \Upsilon_k^C\widehat{c}_{k,t} + \sum_{j=1}^{I}\lambda_{kj}\chi_{kj}\widehat{i}_{kj,t}$$

$$+\sum_{l\neq k}^{K}\sum_{i=1}^{I}\left[y_{lk}\beta^C_{lki}\widehat{c}_{lki,t} + \sum_{j=1}^{I}y_{lk}\lambda_{lj}\omega^X_{lkji}\widehat{x}_{lkji,t} + \sum_{j=1}^{I}y_{lk}\lambda_{lj}\omega^I_{lkji}\widehat{i}_{lkji,t}\right.$$

$$\left.-\beta^C_{kli}\widehat{c}_{kli,t} - \sum_{j=1}^{I}\lambda_{kj}\omega^X_{klji}\widehat{x}_{klji,t} - \sum_{j=1}^{I}\lambda_{kj}\omega^I_{klji}\widehat{i}_{klji,t}\right]. \tag{A.118}$$

But the right-hand side is precisely real GDP in (A.109). Therefore,

$$\sum_{i=1}^{I}\lambda_{ki}(1-\vartheta_{ki})\widehat{\text{va}}_{ki,t} = \widehat{y}_{k,t}. \tag{A.119}$$

Appendix B. Derivations for the Analytical Mechanism

This appendix derives the equations in Section 4. The derivations are first-order, around a zero-tariff steady state. Producer prices and exchange rates are fixed in the pass-through step, while the rental rate of capital is determined by the investment Euler equation.

Let $\tau_{ks,t}$ denote the first-order wedge from a common tariff that country k applies to all foreign origins $l \neq k$ in sector s. For a bilateral tariff, the same expressions hold after restricting the sums over l to the affected origins.

B.1. Use-specific pass-through

The exposure measures used in the main text are

$$FD_{ks} \equiv \frac{\sum_{l\neq k}\beta^C_{kls}}{\sum_{l=1}^{K}\sum_{i=1}^{I}\beta^C_{kli}}, \tag{B.1}$$

$$e^X_{kis} \equiv \frac{\sum_{l\neq k}\omega^X_{klis}}{\vartheta_{ki}}, \qquad IO_{ks} \equiv \sum_{l\neq k}\sum_{i=1}^{I}\lambda_{ki}\omega^X_{klis}, \tag{B.2}$$

$$e^I_{kis} \equiv \frac{\sum_{l\neq k}\omega^I_{klis}}{\chi_{ki}}, \qquad KI_{ks} \equiv \sum_{l\neq k}\sum_{i=1}^{I}\lambda_{ki}\omega^I_{klis}. \tag{B.3}$$

The sector-specific exposures e^X_{kis} and e^I_{kis} are expenditure weights in the sectoral intermediate-input and investment price indices. The investment matrix used in equilibrium is gross-output normalized: $\omega^{I}_{kl\,is} = \chi_{ki} s^{I}_{kl\,is}$, where $s^{I}_{kl\,is}$ is the conditional sourcing share in sector i's investment bundle. Thus e^{I}_{kis} divides by χ_{ki} to recover the conditional imported sector-s investment share. The aggregate objects IO_{ks} and KI_{ks} are Domar-weighted exposures.

PROPOSITION B1 (Use-specific pass-through). *At a zero-tariff steady state with fixed producer prices and exchange rates, a small common tariff $\tau_{ks,t}$ on imported sector-s goods implies, to first order,*

$$p^{C}_{k,t} = FD_{ks}\,\tau_{ks,t}, \tag{B.4}$$

$$p^{X}_{ki,t} = e^{X}_{kis}\,\tau_{ks,t}, \tag{B.5}$$

$$\widehat{p}^{X}_{ki,t} = \left(e^{X}_{kis} - FD_{ks}\right)\tau_{ks,t}, \tag{B.6}$$

$$p^{I}_{ki,t} = e^{I}_{kis}\,\tau_{ks,t}, \tag{B.7}$$

$$\widehat{p}^{I}_{ki,t} = \left(e^{I}_{kis} - FD_{ks}\right)\tau_{ks,t}. \tag{B.8}$$

PROOF. To first order, the log of a CES price index responds to a log buyer-price wedge by the expenditure share of the affected good, evaluated at the initial allocation. The consumption price index therefore loads on the tariff by $\sum_{l\neq k}\beta^{C}_{kl\,s}/\sum_{l=1}^{K}\sum_{i=1}^{I}\beta^{C}_{kl\,i} = FD_{ks}$, which gives (B.4). The sector-i intermediate-input price index has imported sector-s expenditure weight $\sum_{l\neq k}\omega^{X}_{kl\,is}/\vartheta_{ki} = e^{X}_{kis}$, which gives (B.5). Subtracting the consumption-price response gives (B.6). The same argument applied to the sector-i investment price index gives (B.7); subtracting the consumption-price response gives (B.8). □

The relative-price formulas identify the two non-consumption channels. A tariff raises sector i's intermediate-input price relative to the consumption numeraire when $e^{X}_{kis} > FD_{ks}$. It raises the relative price of investment, and therefore creates a capital-accumulation wedge, when $e^{I}_{kis} > FD_{ks}$.

B.2. User cost of capital

Households satisfy the sectoral investment Euler equation

$$P^{I}_{ki,t}U'(C_{k,t}) = \beta\,\mathbb{E}_t\left\{U'(C_{k,t+1})\left[R_{ki,t+1} + P^{I}_{ki,t+1}(1-\delta_{ki})\right]\right\}. \tag{B.9}$$

Combining its log-linear approximation with the stochastic discount factor gives

$$\left[1-\beta(1-\delta_{ki})\right]\mathbb{E}_t\widehat{r}_{ki,t+1} = \mathfrak{r}_{k,t} + \widehat{p}^{I}_{ki,t} - \beta(1-\delta_{ki})\mathbb{E}_t\,\widehat{p}^{I}_{ki,t+1}, \tag{B.10}$$

where $\widehat{r}_{ki,t} = r_{ki,t} - p^{C}_{k,t}$, $\widehat{p}^{I}_{ki,t} = p^{I}_{ki,t} - p^{C}_{k,t}$, and $\widehat{\mathfrak{r}}_{k,t} \equiv -\mathbb{E}_t\widehat{m}_{k,t+1}$ is the real risk-free return implied by the household stochastic discount factor.

Define

$$\mathcal{A}_{ki}(\rho) \equiv \frac{1-\beta(1-\delta_{ki})\rho}{1-\beta(1-\delta_{ki})}. \tag{B.11}$$

PROPOSITION B2 (Direct tariff-induced user-cost wedge). *Suppose the tariff innovation follows* $\mathbb{E}_t \tau_{ks,t+1} = \rho^{\tau}_{ks} \tau_{ks,t}$. *Under Proposition B1,*

$$\frac{\partial\, \mathbb{E}_t \widehat{r}_{ki,t+1}}{\partial \tau_{ks,t}} = \frac{1}{1-\beta(1-\delta_{ki})} \frac{\partial \widehat{\mathfrak{r}}_{k,t}}{\partial \tau_{ks,t}} + \mathcal{A}_{ki}(\rho^{\tau}_{ks})\left(e^{I}_{kis} - FD_{ks}\right). \tag{B.12}$$

Holding the real risk-free return fixed, the direct investment-price component is

$$\left.\frac{\partial\, \mathbb{E}_t \widehat{r}_{ki,t+1}}{\partial \tau_{ks,t}}\right|_{\widehat{\mathfrak{r}}_{k,t}} = \mathcal{A}_{ki}(\rho^{\tau}_{ks})\left(e^{I}_{kis} - FD_{ks}\right). \tag{B.13}$$

PROOF. Equation (B.8) gives $\widehat{p}^{I}_{ki,t} = (e^{I}_{kis} - FD_{ks})\tau_{ks,t}$. The assumed persistence of the tariff implies $\mathbb{E}_t \widehat{p}^{I}_{ki,t+1} = \rho^{\tau}_{ks} \widehat{p}^{I}_{ki,t}$. Substituting these two expressions into (B.10) and dividing by $\tau_{ks,t}$ yields (B.12); holding $\widehat{\mathfrak{r}}_{k,t}$ fixed gives (B.13). □

For a permanent tariff, $\mathcal{A}_{ki}(1) = 1$. For $\rho < 1$, $\mathcal{A}_{ki}(\rho) > 1$: a less persistent tariff raises the user cost more because it makes current investment expensive relative to expected future investment.

B.3. Investment demand

The user-cost wedge lowers investment by reducing desired future capital. The following corollary combines the user-cost equation with sectoral capital demand and the capital law of motion.

COROLLARY B1 (Sectoral investment response). *Holding the real risk-free return, expected output, and marginal costs fixed, the impact response of sectoral investment to a tariff in sector s is*

$$\frac{\partial \widehat{i}_{ki,t}}{\partial \tau_{ks,t}} = -\frac{\psi}{\delta_{ki}} \mathcal{A}_{ki}(\rho^{\tau}_{ks})\left(e^{I}_{kis} - FD_{ks}\right), \tag{B.14}$$

where ψ is the elasticity of substitution between capital and other inputs in production.

PROOF. With the real risk-free return, expected output, and marginal costs fixed, the capital demand condition implies that desired future capital falls by ψ times the direct expected real rental-rate increase: $\widehat{k}_{ki,t+1} = -\psi\, \mathbb{E}_t \widehat{r}_{ki,t+1}$. The capital law of motion implies $\widehat{i}_{ki,t} = \widehat{k}_{ki,t+1}/\delta_{ki}$ on impact. Combining these two relationships with Proposition B2 gives (B.14). □

The quantitative model adds the endogenous movements in output, marginal costs, exchange rates, monetary policy, and the real risk-free return. The corollary isolates the direct investment-price channel.

B.4. Aggregate investment exposure

Aggregate relative investment-price pass-through is obtained by weighting sectoral investment-price responses by $\lambda_{ki}\chi_{ki}$:

$$\widehat{p}^{I}_{k,t} = \frac{\sum_{i=1}^{I} \lambda_{ki}\chi_{ki}\, \widehat{p}^{I}_{ki,t}}{\sum_{i=1}^{I} \lambda_{ki}\chi_{ki}}.$$

Using Proposition B1,

$$\widehat{p}^{I}_{k,t} = \left(\frac{KI_{ks}}{\sum_{i=1}^{I} \lambda_{ki}\chi_{ki}} - FD_{ks}\right)\tau_{ks,t}. \tag{B.15}$$

Multiplying both sides by $\sum_i \lambda_{ki}\chi_{ki}$ shows that the aggregate investment-price wedge is ranked by

$$KI_{ks} - FD_{ks}\sum_{i=1}^{I} \lambda_{ki}\chi_{ki}. \tag{B.16}$$

Thus sectors with high investment-goods exposure and low final-demand exposure generate the largest relative investment-price increases. If the investment-demand amplification term $\psi\mathcal{A}_{ki}(\rho^{\tau}_{ks})/\delta_{ki}$ is similar across destination sectors, the same statistic ranks the direct aggregate investment response.

B.5. Propagation through the investment network

Investment enters goods-market clearing. The component of investment demand from destination country k to origin sector (l, s) is proportional to

$$\sum_{i=1}^{I} \lambda_{ki}\omega^{I}_{kl\,is}\,\widehat{i}_{ki,t}.$$

A tariff that raises the relative price of investment therefore reduces not only sectoral investment in the destination economy, but also demand for the foreign sectors that supply investment goods. This is the network propagation channel emphasized in the sectoral simulations.

Appendix C. Counterfactual Economies

In this section, we outline the implementation of the different counterfactual economies.

C.1. No Investment

We consider a counterfactual version of the model in which physical capital is removed from production, but we aim to preserve as much as possible the steady-state cost structure of the baseline economy. In particular, we seek to eliminate the capital block while keeping unchanged (i) the labor share and (ii) the steady-state profit share, or equivalently the markup, at the sector-country level.

Baseline with capital. In the quantitative model, sectoral production combines labor, capital, and intermediate inputs according to the CES technology in equation (19), and the corresponding log-linear marginal cost equation is given by equation (A.39). In steady state, the relevant cost shares are the labor share α_{ki}, the capital share ς_{ki}, and the intermediate-input share ϑ_{ki}. In the calibration, these objects satisfy the accounting relation

$$\mathcal{M}^{p}_{ki} = \frac{\psi_{ki}}{\alpha_{ki} + \varsigma_{ki} + \vartheta_{ki}}, \tag{C.1}$$

where $\mathcal{M}^{p}_{ki}$ is the gross markup and ψ_{ki} is the returns-to-scale object used in the calibration. Therefore, holding $\mathcal{M}^{p}_{ki}$ fixed is equivalent to holding fixed the sum of effective cost shares entering the denominator of (C.1).

In the baseline calibration with capital, the investment expenditure share χ_{ki} and the capital income share ς_{ki} are linked by the steady-state Euler equation for capital accumulation:

$$\varsigma_{ki} = \frac{1-\beta(1-\delta_{ki})}{\beta\delta_{ki}}\chi_{ki}. \tag{C.2}$$

This distinction is important below: the input-output matrix for investment goods delivers χ_{ki} directly, whereas the marginal-cost object that matters for production is ς_{ki}.

Objective of the counterfactual. A naive no-investment counterfactual would simply remove the capital block and recalibrate markups using only labor and intermediate-input shares: $\mathcal{M}^{p,\mathrm{nc}}_{ki} = \frac{\psi_{ki}}{\alpha^{\mathrm{nc}}_{ki}+\vartheta^{\mathrm{nc}}_{ki}}$. However, this approach generally changes both the markup and the implied steady-state profit share relative to the baseline, because the capital share disappears from the denominator of (C.1). Furthermore, it decimates the impact of tariffs, since it would reduce the effective number of goods being subject to tariffs.

Our goal is instead to construct a counterfactual satisfying $\alpha^{\text{nc}}_{ki} = \alpha_{ki}$, $\mathcal{M}^{p,\text{nc}}_{ki} = \mathcal{M}^{p}_{ki}$, for all countries k and sectors i. To achieve this, the capital share that disappears from production must be reallocated to another production margin. We choose to reallocate it entirely to the intermediate-input block, in order to maintain the effective tariff in the counterfactual economy.

Cost-share absorption through a rescaled investment network. We reassign the capital block to intermediate inputs *after rescaling the investment network* so that its row sums equal the capital income share ς_{ki} rather than the investment expenditure share χ_{ki}. Specifically, for each destination pair (k, i) we define $\widetilde{\omega}^{I}_{kl\,ij} = \omega^{I}_{kl\,ij}\frac{\varsigma_{ki}}{\chi_{ki}}$, if $\chi_{ki} > 0$, and $\widetilde{\omega}^{I}_{kl\,ij} = 0$ whenever $\chi_{ki} = 0$. The counterfactual intermediate-input network is then $\widetilde{\omega}^{X}_{kl\,ij} = \omega^{X}_{kl\,ij} + \widetilde{\omega}^{I}_{kl\,ij}$. By construction, $\sum_{l=1}^{K}\sum_{j=1}^{I} \widetilde{\omega}^{I}_{kl\,ij} = \varsigma_{ki}$, and therefore the new intermediate-input share satisfies $\overline{\vartheta}_{ki} = \vartheta_{ki} + \varsigma_{ki}$. If we also keep the baseline labor share and markup fixed, $\overline{\alpha}_{ki} = \alpha_{ki}$, $\overline{\mathcal{M}}_{ki} = \mathcal{M}^{p}_{ki}$, then $\overline{\alpha}_{ki} + \overline{\vartheta}_{ki} = \alpha_{ki} + \vartheta_{ki} + \varsigma_{ki}$, so that the denominator of (C.1) is exactly preserved. Equivalently, the steady-state profit share is unchanged sector by sector.

Recalculation of steady-state objects. Once the counterfactual matrix $\widetilde{\omega}^{X}_{kl\,ij}$ has been constructed, all objects associated with the intermediate-input nest must be recomputed from it. In particular, the bilateral sourcing shares become $\widetilde{\zeta}^{X}_{kl\,ij} = \widetilde{\omega}^{X}_{kl\,ij}/(\sum_{l'=1}^{K}\widetilde{\omega}^{X}_{kl'\,ij})$, whenever the denominator is positive. The total intermediate-input share is given by $\overline{\vartheta}_{ki} = \sum_{l,j}\widetilde{\omega}^{X}_{kl\,ij}$, and the decomposition between energy and non-energy inputs is recomputed in the usual way:

$$\widetilde{\varrho}^{X}_{ki} = \frac{\sum_{l=1}^{K}\sum_{j\in\mathcal{I}_E}\widetilde{\omega}^{X}_{kl\,ij}}{\sum_{l=1}^{K}\sum_{j=1}^{I}\widetilde{\omega}^{X}_{kl\,ij}}, \quad \widetilde{\nu}^{X}_{kij} = \frac{\sum_{l=1}^{K}\widetilde{\omega}^{X}_{kl\,ij}}{\sum_{l=1}^{K}\sum_{m\in\mathcal{I}_E}\widetilde{\omega}^{X}_{kl\,im}}, j\in\mathcal{I}_E, \quad \widetilde{\upsilon}^{X}_{kij} = \frac{\sum_{l=1}^{K}\widetilde{\omega}^{X}_{kl\,ij}}{\sum_{l=1}^{K}\sum_{m\in\mathcal{I}_M}\widetilde{\omega}^{X}_{kl\,im}}, j\in\mathcal{I}_M.$$

Thus, the entire intermediate-input block remains internally consistent after the reallocation.

Model equations. We elimate all model equations related to capital. Namely, we eliminate the investment Euler equation (A.24), the capital demand curve (A.34), the investment inputs demand curves, baskets, and price indices (A.43)-(A.51).

The definition of the steady-state Domar weights (A.96) is modified to $\widetilde{\lambda}_{ki} = \sum_{l=1}^{K}\mathcal{Y}_{l\,k}\beta^{C}_{l\,ki} + \sum_{l=1}^{K}\sum_{j=1}^{I}\mathcal{Y}_{l\,k}\widetilde{\lambda}_{lj}\widetilde{\omega}^{X}_{l\,kji}$, the relative labor share δ^{N}_{ki} in (A.98) is modified to $\delta^{N}_{ki} = \alpha_{ki}\widetilde{\lambda}_{ki}/\left[1-\sum_{j=1}^{I}\left(1-\frac{\psi_{kj}}{\mathcal{M}_{kj}}\right)\widetilde{\lambda}_{kj}\right]$, the steady-state nominal GDP (A.103) is modified to $\mathcal{Y}_{k} = P^{C}_{k}C_{k} = W_{k}N_{k} + \Pi_{k} = W_{k}N_{k} + \sum_{i=1}^{I}\left(1-\frac{\psi_{ki}}{\mathcal{M}^{p}_{ki}}\right)P_{ki}Y_{ki}$, and the steady-state export (or import) share of nominal GDP (A.107) is modified to $\widetilde{\Upsilon}_{k} = \sum_{l\neq k}\sum_{i=1}^{I}\left[\beta^{C}_{kl\,i} + \sum_{j=1}^{I}\widetilde{\lambda}_{kj}\widetilde{\omega}^{X}_{kl\,ji}\right]$.

Furthermore, the production function (A.36) is modified to

$$\widehat{y}_{fki,t} = a_{ki,t} + \mathcal{M}^{p}_{ki}\alpha_{ki}\widehat{n}_{fki,t} + \mathcal{M}^{p}_{ki}\overline{\vartheta}_{ki}\widehat{x}_{fki,t}, \tag{C.3}$$

the marginal cost function (A.39) is modified to

$$\widehat{\mathrm{mc}}_{ki,t} = \frac{1-\psi_{ki}}{\psi_{ki}}\widehat{y}_{ki,t} - \frac{1}{\psi_{ki}}a_{ki,t} + \frac{\mathcal{M}_{ki}}{\psi_{ki}}\alpha_{ki}\widehat{w}_{k,t} + \frac{\mathcal{M}_{ki}}{\psi_{ki}}\overline{\vartheta}_{ki}\widehat{p}^{X}_{ki,t}, \tag{C.4}$$

the goods' market clearing condition (A.97) is modified to

$$\widetilde{\lambda}_{ki}\widehat{y}_{ki,t} = \sum_{l=1}^{K} y_{lk}\left(\beta^{C}_{lki}\widehat{c}_{lki,t} + \sum_{j=1}^{I}\widetilde{\lambda}_{lj}\widetilde{\omega}^{X}_{lkji}\widehat{x}_{lkji}\right), \tag{C.5}$$

the import price deflator (A.101) is modified to

$$\widehat{p}_{k\mathrm{IMP},t} = \sum_{l\neq k}\sum_{i=1}^{I}\widetilde{\Upsilon}_{k}^{-1}\left[\beta^{C}_{kli} + \sum_{j=1}^{I}\widetilde{\lambda}_{kj}\widetilde{\omega}^{X}_{klji}\right]\widehat{p}_{kli,t}, \tag{C.6}$$

the export price deflator (A.102) is modified to

$$\widehat{p}_{k\mathrm{EXP},t} = \sum_{l\neq k}\sum_{i=1}^{I}\frac{y_{lk}}{\widetilde{\Upsilon}_{k}}\left[\beta^{C}_{lki} + \sum_{j=1}^{I}\widetilde{\lambda}_{lj}\widetilde{\omega}^{X}_{lkji}\right](q^{l}_{k,t} + \widehat{p}_{lki,t}), \tag{C.7}$$

the real GDP (A.104) is modified to

$$\widehat{y}_{k,t} = \widehat{c}_{k,t} + \widetilde{\Upsilon}_{k}\left(\widehat{\exp}_{k,t} - \widehat{\mathrm{imp}}_{k,t}\right), \tag{C.8}$$

the exports (A.105) are modified to

$$\widehat{\exp}_{k,t} = \sum_{l\neq k}\sum_{i\in I}\frac{y_{lk}}{\widetilde{\Upsilon}_{k}}\left[\beta^{C}_{lki}\widehat{c}_{lki,t} + \sum_{j=1}^{I}\widetilde{\lambda}_{lj}\widetilde{\omega}^{X}_{lkji}\widehat{x}_{lkji,t}\right], \tag{C.9}$$

and the imports (A.108) are modified to

$$\widehat{\mathrm{imp}}_{k,t} = \sum_{l\neq k}\sum_{i\in I}\widetilde{\Upsilon}_{k}^{-1}\left(\beta^{C}_{kli}\widehat{c}_{kli,t} + \sum_{j=1}^{I}\widetilde{\lambda}_{kj}\widetilde{\omega}^{X}_{klji}\widehat{x}_{klji,t}\right). \tag{C.10}$$

C.2. No Investment Network

We consider a counterfactual version of the model in which physical capital remains in production, but the investment network is shut down and the sector-specific capital block is replaced by an aggregate one at the country level. Relative to the baseline, this counterfactual removes both the capital-goods network and the sector-specific dynamics of capital accumulation. In that sense, it is closer to the framework in Peri *et al.* (2026), where households purchase one aggregate investment good, accumulate one aggregate capital stock, and sectoral capital services are allocated from aggregate capital through a reallocation margin.

Baseline with sector-specific capital accumulation. In the baseline quantitative model, sectoral production combines labor, capital, and intermediate inputs according to equation (19), and the corresponding log-linear marginal cost equation is given by equation (A.39). Capital accumulation is sector specific: each country-sector pair (k, i) features its own investment bundle $I_{ki,t}$, investment price $P^{I}_{ki,t}$, capital stock $K_{ki,t}$, and Euler equation, as summarized by equations (5), (9), (A.20), (A.24), and (C.11). In addition, the capital-goods network is encoded by the nested investment structure in equations (A.43)-(A.51), which allocates sectoral investment demand across energy and non-energy goods, source sectors, and source countries.

Objective of the counterfactual. The goal of this counterfactual is to isolate the propagation role of the investment network while preserving the role of capital in production. Unlike the baseline economy, investment and capital accumulation are no longer sector specific.

Conditions removed. Relative to the baseline, the counterfactual removes three sets of conditions.

First, we eliminate the *investment-network block*, namely the investment-input demand curves, baskets, and price-index equations in (A.43)-(A.51). These are the conditions that determine how each destination sector (k, i) allocates investment demand across source sectors j and source countries l.

Second, we eliminate the *sector-specific Euler equations for capital accumulation*. Hence, the system of sectoral intertemporal conditions summarized by equations (A.20) and (A.24), one for each country-sector pair (k, i), is no longer part of the model.

Third, we eliminate the *sector-specific law of motion for capital*. Thus, the sector-by-sector capital accumulation equation (C.11) is removed from the model. Equivalently, the dynamic state variable is no longer the collection of sectoral capital stocks $\{K_{ki,t}\}_{i=1}^{I}$, but a single aggregate capital stock for each country.

Conditions that remain unchanged. Capital remains an input in firms' production functions and marginal costs. Accordingly, equation (19) and the log-linear marginal cost equation (A.39) are preserved. What changes is not the use of capital in production, but the dynamic structure through which capital is accumulated and allocated across sectors.

Aggregate investment and capital block. In place of the removed sector-specific capital block, we introduce one aggregate investment good and one aggregate capital stock for each country.

First, households in country k purchase a single aggregate investment bundle, denoted $\text{INV}_{k,t}$, at price $P^I_{k,t}$. This replaces the collection of sector-specific investment bundles $\{I_{ki,t}\}_{i=1}^I$.

Second, country k accumulates a single aggregate capital stock $K_{k,t}$ according to $K_{k,t} = (1-\delta^K_k)K_{k,t-1} + \text{INV}_{k,t-1}$, where δ^K_k denotes the aggregate depreciation rate in country k. The log-linearized law of motion of aggregate capital is given by

$$\widehat{k}_{k,t} = \delta^K_k \widehat{\text{inv}}_{k,t-1} + (1-\delta^K_k)\widehat{k}_{k,t-1}, \tag{C.11}$$

Third, households satisfy a single aggregate Euler equation for capital accumulation, $P^I_{k,t}U'(C_{k,t}) = \beta E_t\left[U'(C_{k,t+1})\left(R_{k,t+1} + P^I_{k,t+1}(1-\delta^K_k)\right)\right]$, where $R_{k,t}$ denotes the aggregate rental rate of capital in country k. Its log-linearized version is given by

$$\widehat{c}_{k,t} = -\frac{1}{\sigma}\left\{\beta(1-\delta^K_k)\mathbb{E}_t\widehat{p}^I_{k,t+1} + [1-\beta(1-\delta^K_k)]\mathbb{E}_t\widehat{r}_{k,t+1} - \widehat{p}^I_{k,t} + \mathbb{E}_t\pi^C_{k,t+1}\right\} + \mathbb{E}_t\widehat{c}_{k,t+1} \tag{C.12}$$

Capital reallocation across sectors. Although capital accumulation is aggregate at the country level, sectoral capital services remain relevant for production. We therefore introduce a static reallocation block that maps aggregate country capital into sectoral capital services. A convenient implementation is a CES aggregator of the form

$$K_{k,t} = \left(\sum_{i=1}^{I}(\omega^K_{ki})^{1/\nu_K}K_{ki,t}^{(\nu_K-1)/\nu_K}\right)^{\nu_K/(\nu_K-1)},$$

where ω^K_{ki} denotes the sectoral capital-allocation weight and ν_K governs the elasticity of substitution across sectoral capital services. We set $\nu_K = 1$ following Peri *et al.* (2026). The associated optimality condition implies $K_{ki,t} = \omega^K_{ki}\left(\frac{R_{ki,t}}{R_{k,t}}\right)^{-\nu_K}K_{k,t}$, which yields the following log-linear reallocation equation:

$$\widehat{k}_{ki,t} = \widehat{k}_{k,t} - \nu_K\left(\widehat{r}_{ki,t} - \widehat{r}_{k,t}\right). \tag{C.13}$$

The implied condition for the aggregate rental rate is $R_{k,t} = \left(\sum_{i=1}^{I} \omega_{ki}^{K} R_{ki,t}^{1-\nu_K}\right)^{1/(1-\nu_K)}$, whose log-linear approximation is

$$\widehat{r}_{k,t} = \sum_{i=1}^{I} \omega_{ki}^{K} \widehat{r}_{ki,t}. \tag{C.14}$$

Calibration of the new steady-state objects. The counterfactual preserves the baseline sectoral production-side shares α_{ki}, ς_{ki}, ϑ_{ki}, as well as the baseline markups $\mathcal{M}_{ki}^{p}$. Hence, the firm-side production and marginal-cost blocks remain unchanged.

On the demand side, the detailed investment network is replaced by aggregate investment shares. For each destination country k, source country l, and source sector i, define

$$\beta_{kl\,i}^{I} \equiv \sum_{m=1}^{I} \lambda_{km} \omega_{kl\,mi}^{I},$$

that is, the GDP-normalized aggregate investment expenditure share of destination country k spent on goods produced by sector i in country l, after collapsing the destination-sector dimension of the baseline investment network. The aggregate investment expenditure share of country k is then

$$\chi_{k}^{K} = \sum_{l=1}^{K} \sum_{i=1}^{I} \beta_{kl\,i}^{I} = \sum_{m=1}^{I} \lambda_{km} \chi_{km}.$$

The capital-reallocation weights are calibrated from the baseline steady state as

$$\omega_{ki}^{K} = \frac{\lambda_{ki} \chi_{ki}}{\sum_{m=1}^{I} \lambda_{km} \chi_{km}},$$

so that sectors that account for a larger share of baseline investment expenditure within country k receive a larger steady-state weight in the aggregate-capital block. In this sense, the aggregate-capital object in the counterfactual is constructed to be comparable to the aggregate-capital measure used in the baseline, which weights sectoral capital by the same investment-expenditure shares. Using these weights, the aggregate depreciation rate and aggregate capital-income share are defined as

$$\delta_{k}^{K} = \sum_{i=1}^{I} \omega_{ki}^{K} \delta_{ki}, \qquad \varsigma_{k}^{K} = \sum_{i=1}^{I} \omega_{ki}^{K} \varsigma_{ki}.$$

Aggregate investment demand and price index. Since the detailed investment network is removed, aggregate investment no longer enters goods-market clearing through the baseline sector-specific objects $I_{lkji,t}$. Instead, for each destination country k, aggregate investment demand is allocated across origin countries and source sectors through the reduced-form expenditure shares β^I_{kli}. The aggregate investment price is therefore defined directly as

$$\widehat{p}^I_{k,t} = (\chi^K_k)^{-1} \sum_{l=1}^{K} \sum_{i=1}^{I} \beta^I_{kli} \left(\widehat{p}^k_{kli,t} + \tau_{kli,t} \right). \tag{C.15}$$

Modifications to equilibrium conditions. The removal of the investment network changes all equilibrium conditions in which the baseline economy contains the term $\sum_{j=1}^{I} \lambda_{lj} \omega^I_{lkji} \widehat{i}_{lkji,t}$. In the counterfactual, this term is replaced by $\beta^I_{lki} \widehat{\text{inv}}_{l,t}$, where $\widehat{\text{inv}}_{l,t}$ denotes aggregate investment in country l. Likewise, all steady-state trade-share objects involving $\sum_{j=1}^{I} \lambda_{kj} \omega^I_{klji}$ are replaced by β^I_{kli}. This modification applies to goods-market clearing, the export equation, the import equation, and the associated trade-weight and trade-price blocks.

Accordingly, the log-linear goods-market-clearing condition (A.97) becomes

$$\lambda_{ki} \widehat{y}_{ki,t} = \sum_{l=1}^{K} y_{lk} \left[\beta^C_{lki} \widehat{c}_{lki,t} + \sum_{j=1}^{I} \lambda_{lj} \omega^X_{lkji} \widehat{x}_{lkji,t} + \beta^I_{lki} \widehat{\text{inv}}_{l,t} \right]. \tag{C.16}$$

Similarly, exports (A.105) and imports (A.108) become

$$\widehat{\exp}_{k,t} = \sum_{l \neq k} \sum_{i=1}^{I} \frac{y_{lk}}{\Upsilon_k} \left[\beta^C_{lki} \widehat{c}_{lki,t} + \sum_{j=1}^{I} \lambda_{lj} \omega^X_{lkji} \widehat{x}_{lkji,t} + \beta^I_{lki} \widehat{\text{inv}}_{l,t} \right], \tag{C.17}$$

$$\widehat{\text{imp}}_{k,t} = \sum_{l \neq k} \sum_{i=1}^{I} \frac{1}{\Upsilon_k} \left[\beta^C_{kli} \widehat{c}_{kli,t} + \sum_{j=1}^{I} \lambda_{kj} \omega^X_{klji} \widehat{x}_{klji,t} + \beta^I_{kli} \widehat{\text{inv}}_{k,t} \right]. \tag{C.18}$$

The import price index (A.101) is modified to

$$\widehat{p}^{\text{IMP}}_{k,t} = \sum_{l \neq k} \sum_{i=1}^{I} \frac{1}{\Upsilon_k} \left[\beta^C_{kli} + \sum_{j=1}^{I} \lambda_{kj} \omega^X_{klji} + \beta^I_{kli} \right] \widehat{p}_{kli,t}, \tag{C.19}$$

and the corresponding export-price index (A.102) is modified analogously,

$$\widehat{p}^{\text{EXP}}_{k,t} = \sum_{l \neq k} \sum_{i=1}^{I} \frac{y_{lk}}{\Upsilon_k} \left[\beta^C_{lki} + \sum_{j=1}^{I} \lambda_{lj} \omega^X_{lkji} + \beta^I_{lki} \right] (q^l_{k,t} + \widehat{p}_{lki,t}). \tag{C.20}$$

Finally, the real GDP equation (A.104) becomes

$$\widehat{y}_{k,t} = \Upsilon_k^C \widehat{c}_{k,t} + \chi_k^K \widehat{\text{inv}}_{k,t} + \Upsilon_k \left(\widehat{\exp}_{k,t} - \widehat{\text{imp}}_{k,t} \right), \tag{C.21}$$

where the sectoral investment contribution of the baseline is replaced by aggregate country-level investment.

Appendix D. Construction of the Investment Input-Output Matrix: The Asset-to-Sector Bridge

This appendix documents the bridge between the asset classification of EU KLEMS and the producing sectors of OECD ICIO that is used to construct the investment input-output matrix of Section 2.1. The bridge first delivers conditional sourcing shares $s^I_{kl\,ij}$ and then scales them by investment expenditure shares to obtain the model object $\omega^I_{kl\,ij} = \chi_{ki} s^I_{kl\,ij}$. The bridge is the conceptual hinge between the two data sources: national accounts identify the asset in which each sector-country invests, but not the country and sector that produced it, while international input-output tables identify the producing sector-country and the receiving country, but aggregate over receiving sectors. The mapping below assigns to each producing sector in ICIO at most one asset class, so that the cross-border flows recorded in ICIO can be reinterpreted as flows of specific assets and then allocated across destination sectors using the within-country asset composition observed in EU KLEMS.

Asset categories. We work with thirteen operational asset categories. Starting from EU KLEMS asset codes, we keep computing equipment (IT), communications equipment (CT), software and databases (SOFT_DB), other machinery and equipment (OMACH), cultivated assets (CULT), and research and development (RD) as separate classes. We aggregate other construction and residential structures into CONS, since their producing sector in ICIO is the same. We split transport equipment into motor vehicles (TRAEQ), ships (SHIP), and aircraft/other transport equipment (PLANE). We split other intellectual-property products into publishing and media assets (PUBLI), mining-related intellectual property (MINING), and pharmaceutical capital (PHARMA). The labels IT and CT are inherited from the source asset taxonomy; Table D.1 reports the actual ICIO producer-sector assignments used in the bridge.

Mapping. Table D.1 reports the assignment from ICIO sectors to asset classes. Sectors that do not produce a capital good are listed with a blank asset entry; they appear as suppliers in the intermediate-input matrix but not in the investment-sourcing matrix. The mapping is one-to-one in the direction sector → asset: each producing sector supplies at most one asset class. The reverse direction is one-to-many: an asset can be produced by several sectors. For example, the asset class OMACH (other machinery and equipment) is supplied by C25 (fabricated metal products), C28 (machinery and equipment n.e.c.), C31_33 (furniture and other manufacturing), and G (wholesale and retail trade, which delivers the capital good as a service).

ICIO sector	Asset class	Description
A01	CULT	Crops, animal production, hunting
B09	MINING	Mining support services (mineral exploration)
C16	CONS	Wood and products of wood (construction materials)
C21	PHARMA	Pharmaceuticals
C25	OMACH	Fabricated metal products
C26	CT	Computer, electronic and optical products
C27	IT	Electrical equipment
C28	OMACH	Machinery and equipment n.e.c.
C29	TRAEQ	Motor vehicles, trailers and semi-trailers
C301	SHIP	Building of ships and boats
C302T309	PLANE	Other transport equipment (aircraft, railway, etc.)
C31_33	OMACH	Furniture and other manufacturing
F	CONS	Construction
G	OMACH	Wholesale and retail trade
J58T60	PUBLI	Publishing, audiovisual and broadcasting
J62_63	SOFT_DB	Computer programming and information services
L	CONS	Real estate activities
M	RD	Professional, scientific and technical services
P	RD	Education

TABLE D.1. Bridge between producing sector (ICIO) and asset class (EU KLEMS).

Product-by-destination refinements. Three asset classes use additional information about the using sector. The category PLANE captures investment in aircraft, which in EU KLEMS is recorded inside TRAEQ but in ICIO is produced by C302T309. We allocate PLANE to the using sector H51 (air transport), which is the dominant industrial user of aircraft. Analogously, SHIP is allocated to H50 (water transport). The category PHARMA is allocated to C21 (pharmaceuticals), which is both the producer and, given the specificity of pharmaceutical equipment and intellectual property, the dominant user. These refinements affect how the share of each asset is distributed across using sectors within a country, but they do not change which producing sector appears as supplier in the matrix.

Computation of within-country shares. For each asset class a, country k, and using sector i, EU KLEMS provides the share w_{ki}^{a} of national investment in asset a that is undertaken by sector i. By construction, $\sum_i w_{ki}^{a} = 1$ for each (k, a). These shares are computed as four-year averages over 2015–2019, using current-price gross fixed capital formation by asset. Where current-price investment in a given asset-sector is missing or non-positive, we impute it using the asset-specific depreciation rate applied to the corresponding capital stock, following the standard perpetual-inventory identity. Where an entire sector is missing in a country's EU KLEMS record, the shares of its parent NACE aggregate are used. Where a country is missing entirely from EU KLEMS, the within-country shares are imputed from the cross-country median of countries for which EU KLEMS coverage is available.

From shares to flows. The matrix is constructed as follows. Let $F_{lj,k}$ denote the cross-border GFCF flow from sector j in country l to country k, taken directly from ICIO, and let $a(j)$ denote the asset class produced by sector j according to Table D.1. The investment-goods flow that enters the capital bundle of sector i in country k from sector j in country l is

$$X_{kl\,ij}^{I} \;=\; w_{ki}^{a(j)} \cdot F_{lj,k},$$

which is zero whenever j is not assigned to any asset, consistent with the fact that the sector does not supply capital goods. The procedure preserves total bilateral country-to-country GFCF flows from ICIO, $\sum_i X_{kl\,ij}^{I} = F_{lj,k}$ for each origin pair (l, j) and destination k, allocating these totals across destination sectors using the within-country asset composition observed in EU KLEMS. Normalizing by the total capital-good purchases of destination sector (k, i) gives the conditional sourcing share

$$s_{kl\,ij}^{I} = \frac{X_{kl\,ij}^{I}}{\sum_{l'} \sum_{j'} X_{kl'\,ij'}^{I}}, \qquad \sum_{l} \sum_{j} s_{kl\,ij}^{I} = 1.$$

The object used in the model is the gross-output-normalized investment share

$$\omega_{kl\,ij}^{I} = \chi_{ki} s_{kl\,ij}^{I}, \qquad \sum_{l} \sum_{j} \omega_{kl\,ij}^{I} = \chi_{ki},$$

where χ_{ki} is investment expenditure by sector (k, i) relative to its gross sales. Thus s^I describes the composition of the capital bundle, while ω^I is the expenditure-share object that enters the price indices, market clearing, and exposure measures.

Depreciation rates. Asset-specific depreciation rates used to discipline the law of motion of capital are computed as median implied depreciation rates from EU KLEMS over 2016–2019, separately by asset class. They are then assigned to each ICIO sector through the bridge of Table D.1. Sectors that do not produce a capital good are assigned the cross-asset median depreciation rate of 0.10, which is used only as a numerical default and plays no role in equilibrium.

Appendix E. Additional Figures and Tables

This appendix reports additional figures and tables in the order in which they are used in the main text. Appendix Table E.1 lists the sectoral classification used in the calibration. Appendix Figures E1–E5 complete the uniform-tariff model comparisons. Appendix Figures E6–E14 report the full country-sector incidence panels. Appendix Tables E.2 and E.3 and Appendix Figures E15–E17 report the sectoral-characteristics and counterfactual diagnostics. Appendix Tables E.4–E.5 and Appendix Figures E18–E19 report the strategic-incidence rankings of Section 6.3.

TABLE E.1. NACE Rev. 2 sectors used in the analysis

NACE Code	**Sector Name**
A.01–02	Agriculture, forestry and fishing
A.03	Fishing and aquaculture
B.05–06	Mining of coal, lignite, crude petroleum and natural gas
B.07–08	Mining of metal ores and other mining and quarrying
B.09	Mining support service activities
C.10–12	Manufacture of food, beverages and tobacco
C.13–15	Manufacture of textiles, wearing apparel and leather products
C.16	Manufacture of wood and products of wood and cork
C.17–18	Manufacture of paper, and printing and media reproduction
C.19	Manufacture of coke and refined petroleum products
C.20	Manufacture of chemicals and chemical products

Continued on next page

NACE Code	Sector Name
C.21	Manufacture of basic pharmaceutical products
C.22	Manufacture of rubber and plastic products
C.23	Manufacture of other non-metallic mineral products
C.24	Manufacture of basic metals
C.25	Manufacture of fabricated metal products
C.26	Manufacture of computer, electronic and optical products
C.27	Manufacture of electrical equipment
C.28	Manufacture of machinery and equipment n.e.c.
C.29	Manufacture of motor vehicles, trailers and semi-trailers
C.30	Manufacture of other transport equipment
C.31–33	Other manufacturing and repair and installation of machinery
D.35	Electricity, gas, steam and air conditioning supply
E.36–39	Water supply; sewerage, waste management and remediation
F.41–43	Construction
G.45–47	Wholesale and retail trade; repair of motor vehicles
H.49	Land transport and transport via pipelines
H.50	Water transport
H.51	Air transport
H.52	Warehousing and support activities for transportation
H.53	Postal and courier activities
I.55–56	Accommodation and food service activities
J.58–60	Publishing, audiovisual and broadcasting activities
J.61	Telecommunications
J.62–63	IT and other information services
K.64–66	Financial and insurance activities
L.68	Real estate activities
M.69–75	Professional, scientific and technical activities
N.77–82	Administrative and support service activities
O.84	Public administration and defence; compulsory social security
P.85	Education
Q.86–88	Human health and social work activities
R.90–93	Arts, entertainment and recreation

Continued on next page

NACE Code	Sector Name
S.94–96	Other personal service activities

Macroeconomic counterfactuals across regions

Appendix Figures E1–E5 extend the uniform-tariff model comparison of Figure 1 to the Euro Area and China and report the US robustness check. Figures E1 and E2 compare the baseline with the No Investment economy. As in the United States, removing capital roughly halves the contraction in real GDP and employment: the spillover to these economies operates through the same investment margin, because a US tariff that depresses investment demand also depresses trade in the goods used to build capital. Figures E4 and E5 compare the baseline with the No Investment Network economy; consistent with the aggregate result in the main text, removing only the detailed investment network leaves the aggregate paths close to the baseline while reallocating the response across sectors. Figure E3 shows that the US No Investment Network results are robust to raising the cross-sector reallocation elasticity from $\nu_K = 1$ to $\nu_K = 1000$: a near-perfectly substitutable aggregate capital good leaves the aggregate dynamics essentially unchanged, confirming that the network matters for incidence rather than for the aggregate.

Sectoral tariff incidence across outcomes

Appendix Figures E6–E14 report the full 44-sector incidence panels behind Figure 2, one outcome at a time and for all regions. Three regularities recur and line up with the analytical mapping of Section 4. First, the real-activity outcomes—real GDP (Figure E6), investment (Figure E8), and employment (Figure E9)—are dominated by the same investment-supplying sectors (electronics, motor vehicles, machinery, other transport equipment), reflecting the investment-goods exposure KI_{ks} and the user-cost channel in equation (45). The investment-price panel (Figure E11) makes this transparent: the sectors that move the investment price are those with high $e^I_{kis} - FD_{ks}$. Second, CPI inflation (Figure E7) and consumption (Figure E10) are organized instead by final-demand exposure FD_{ks}, so the sectoral ranking differs from the real-activity panels. Third, the trade outcomes (Figures E12–E14) show that the sectors whose tariffs contract imports the most are the heavily traded capital- and intermediate-goods sectors, consistent with the network-propagation term $\sum_i \lambda_{ki}\omega^I_{kl\,is}\,\widehat{\imath}_{ki,t}$ of Section 4.

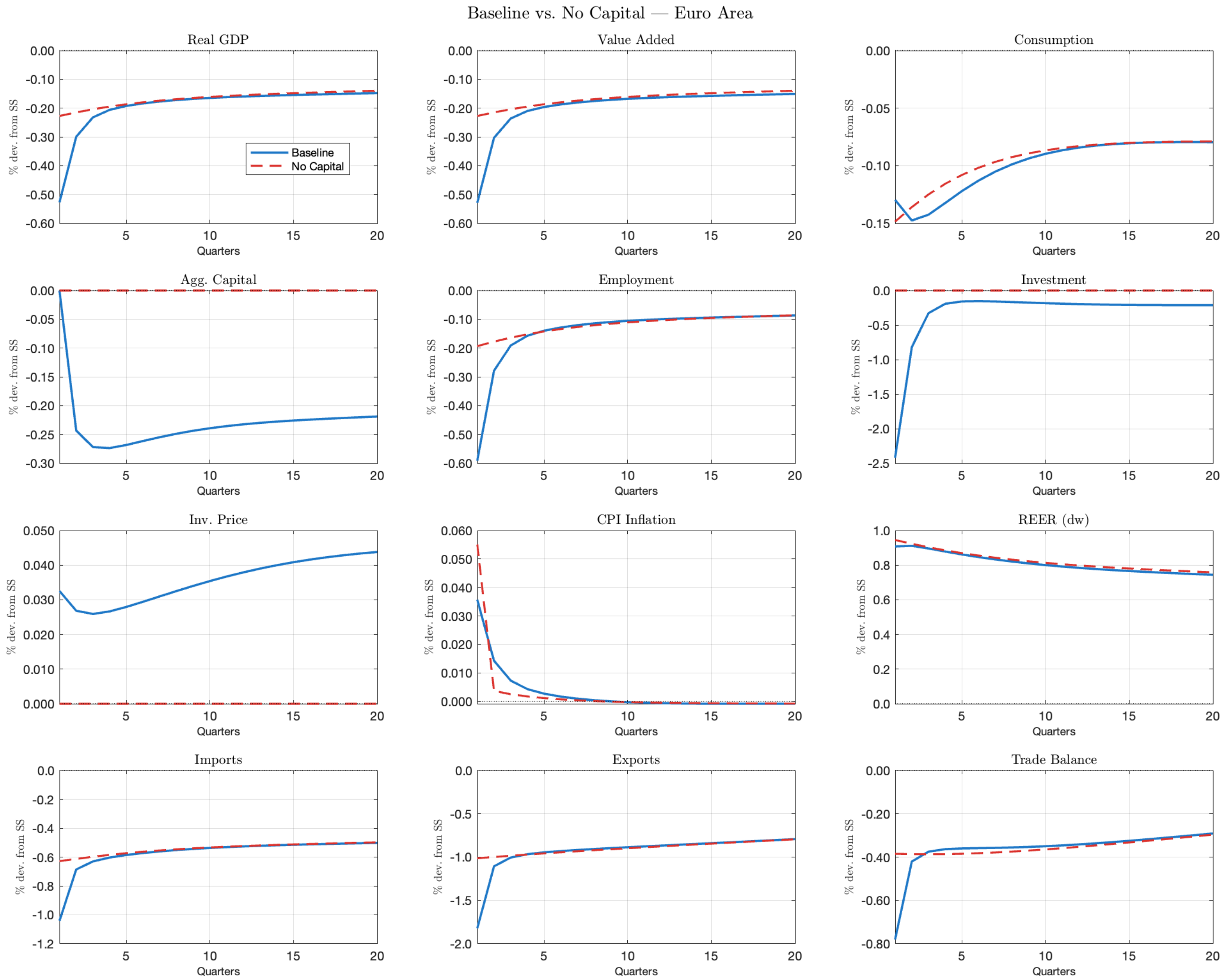

FIGURE E1. Baseline versus No Investment: Euro Area.

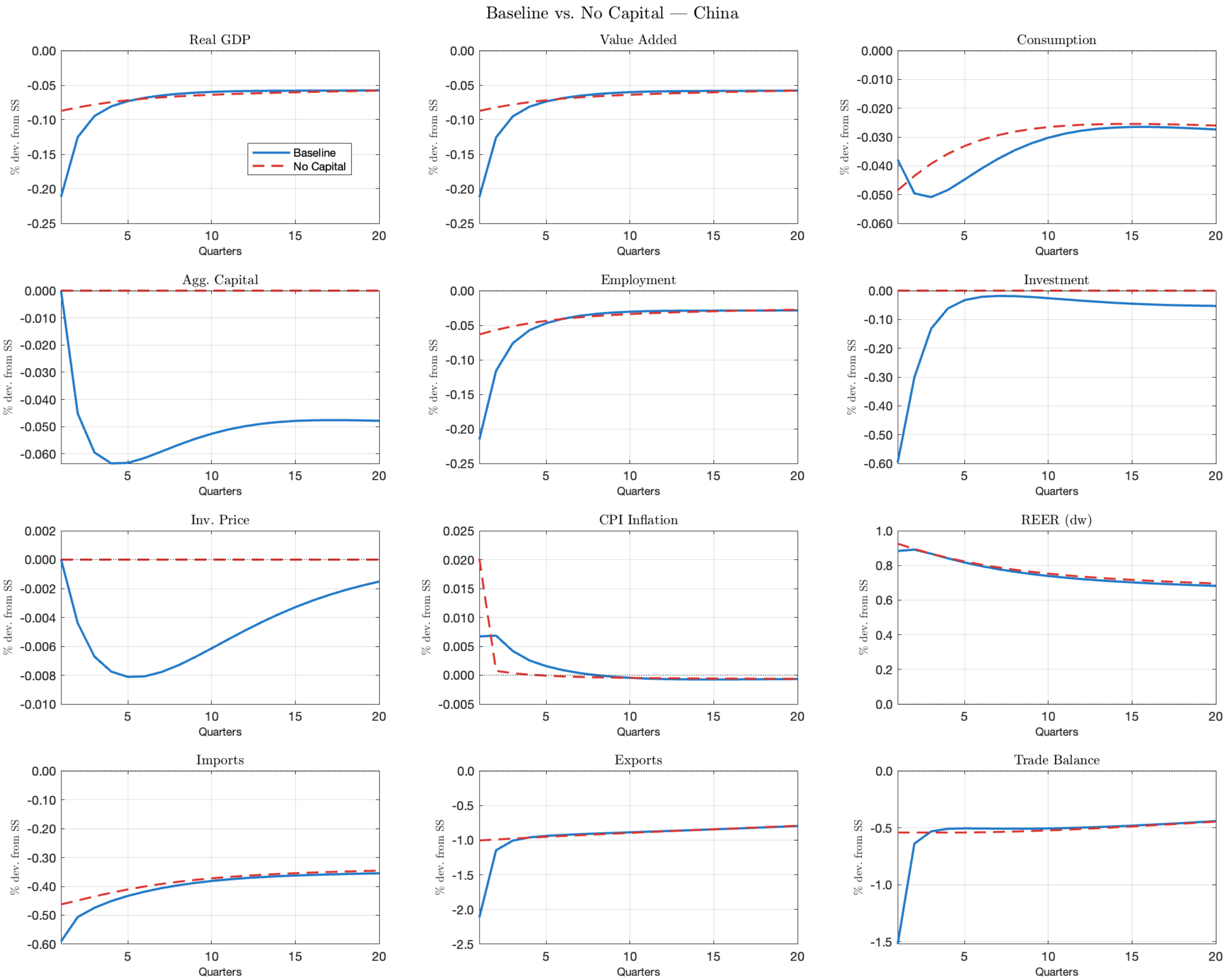

FIGURE E2. Baseline versus No Investment: China.

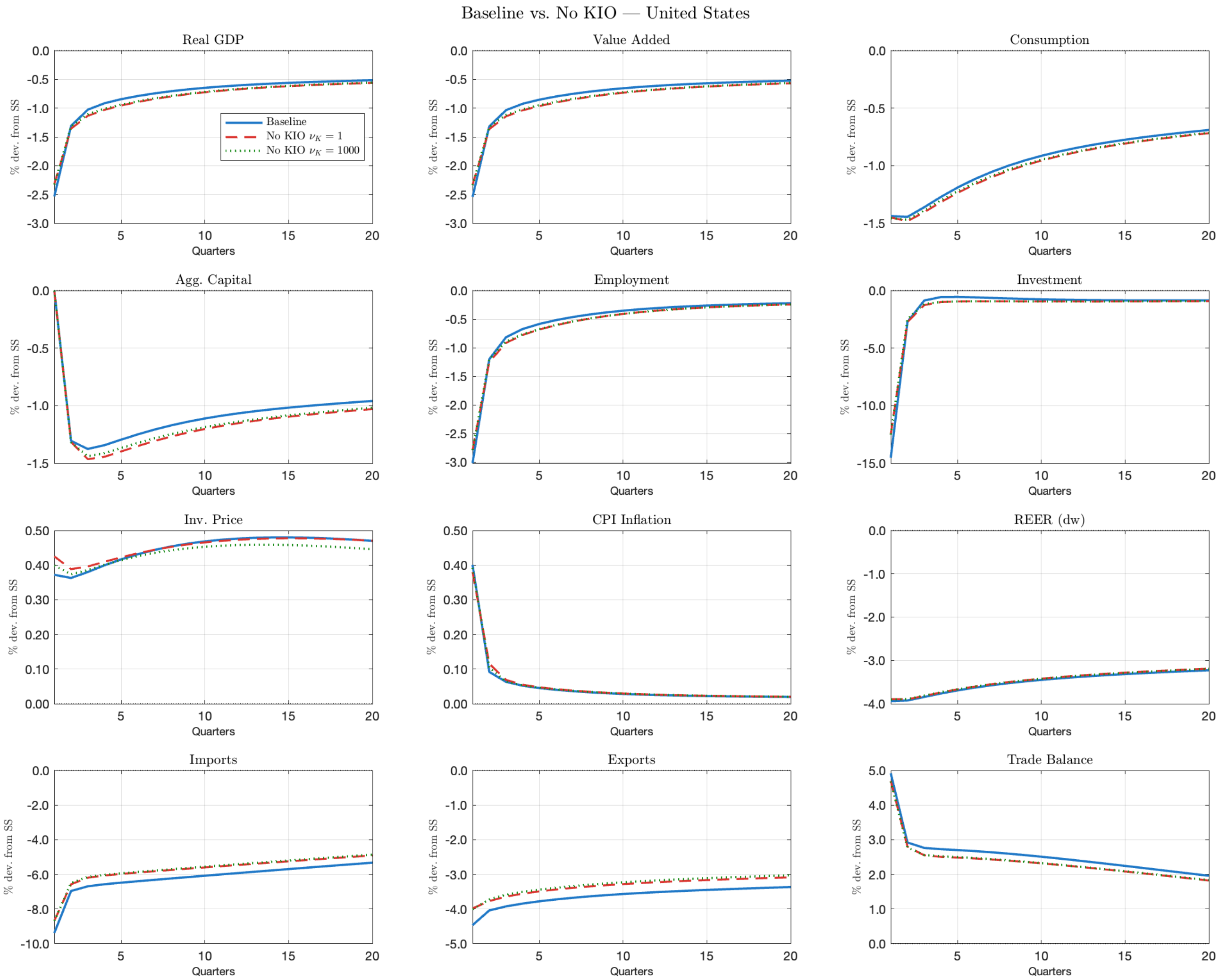

FIGURE E3. No Investment Network robustness: United States. The figure compares the baseline with No Investment Network economies under $\nu_K = 1$ and $\nu_K = 1000$.

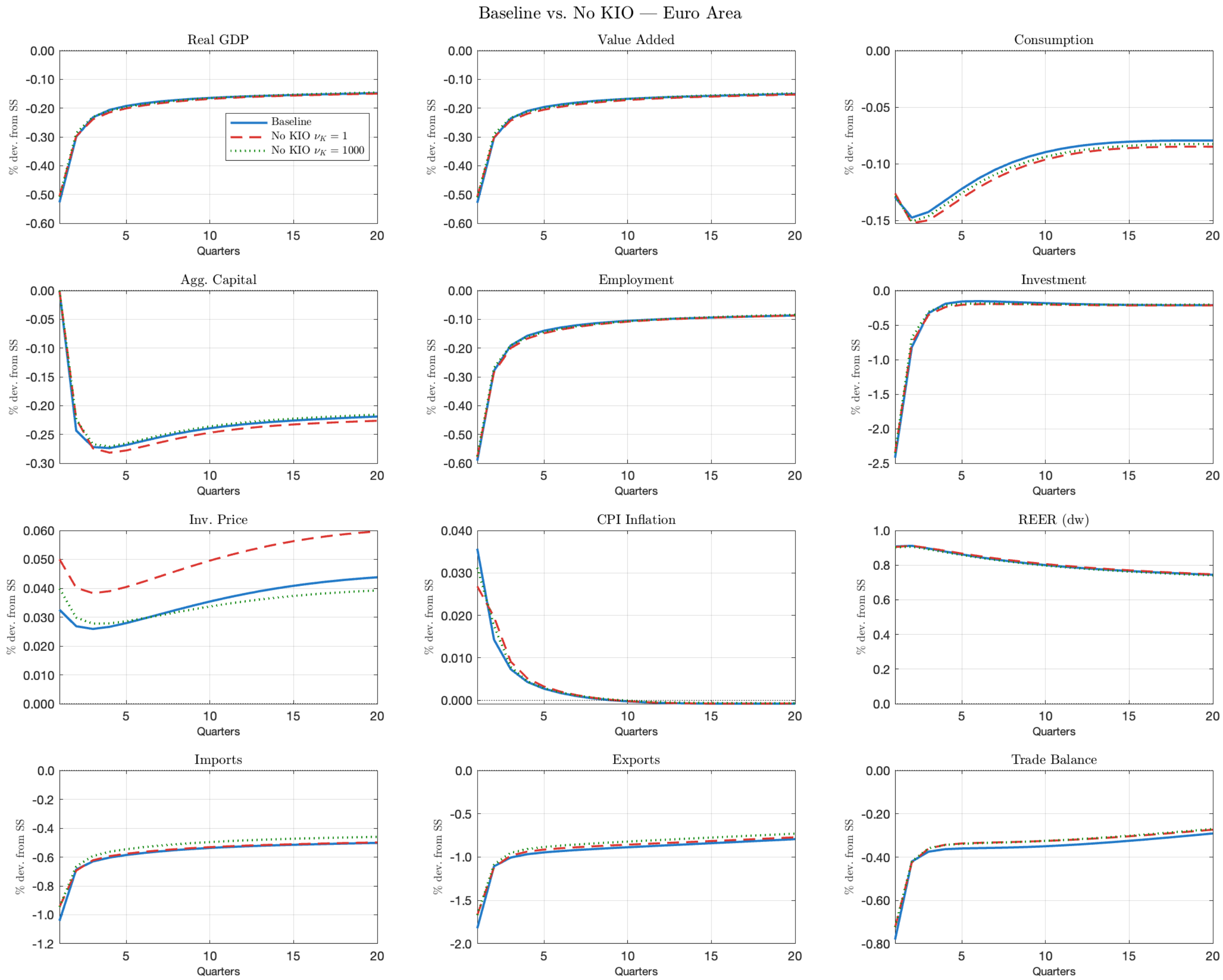

FIGURE E4. Baseline versus No Investment Network: Euro Area.

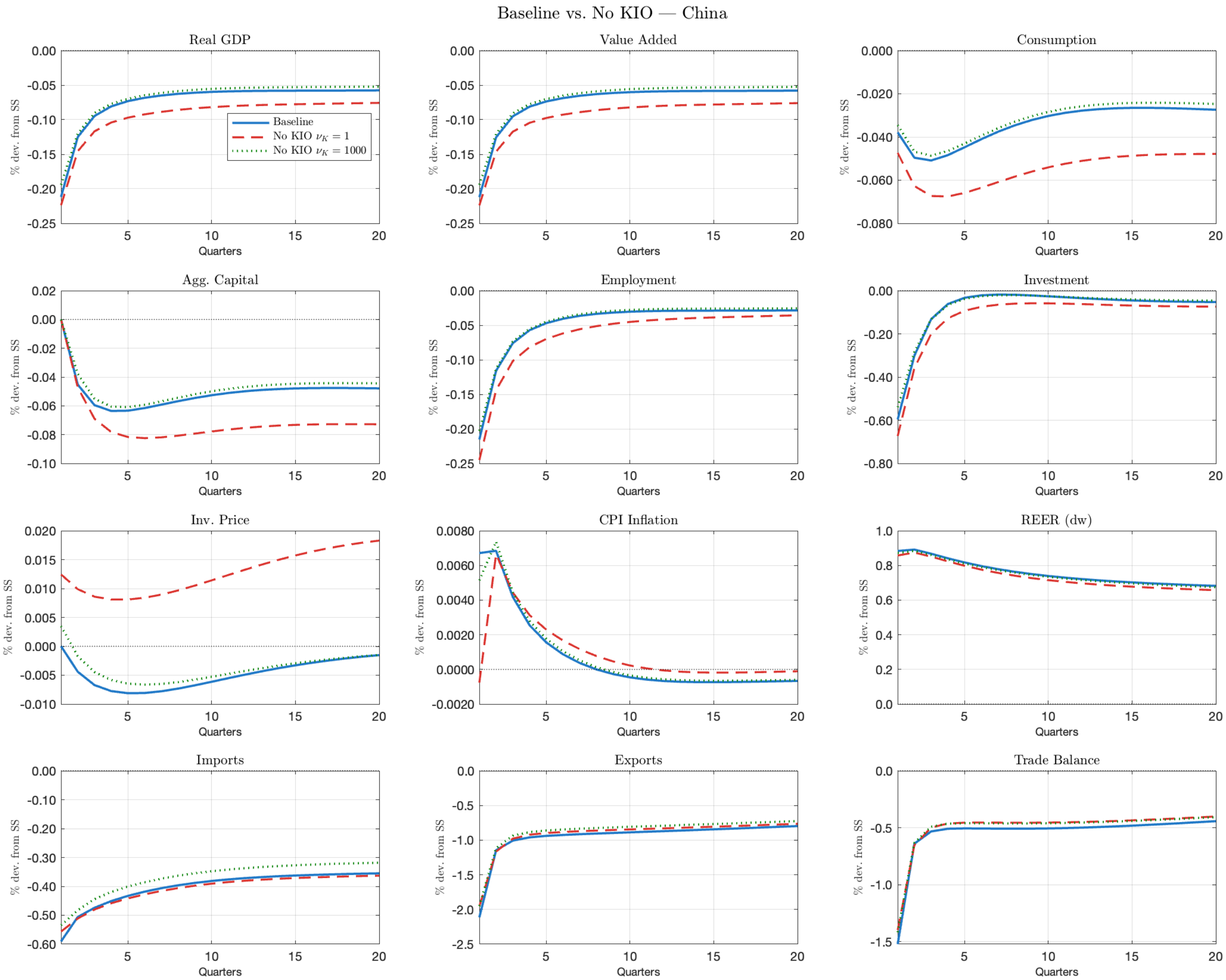

FIGURE E5. Baseline versus No Investment Network: China.

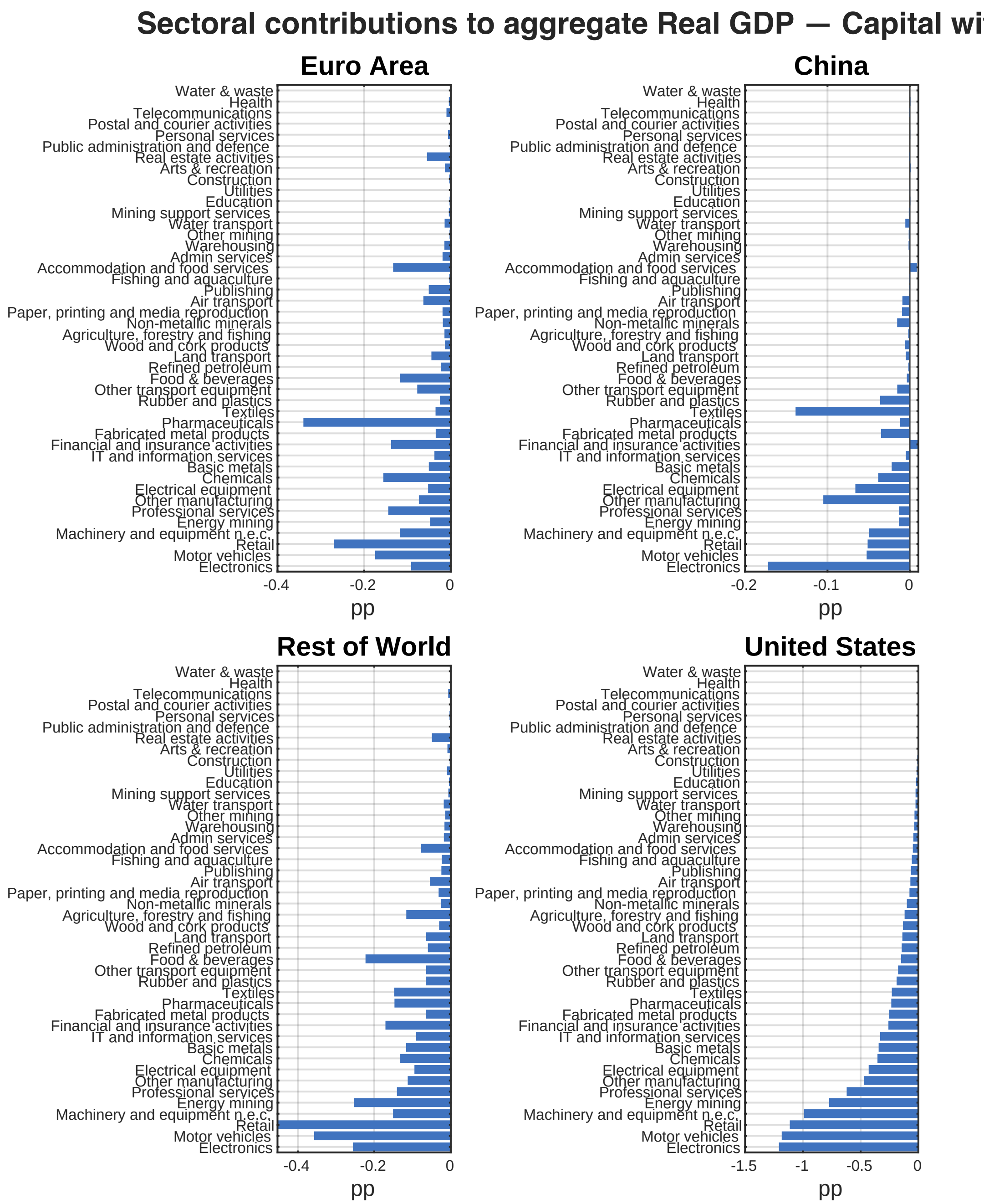

FIGURE E6. Sectoral tariff incidence on real GDP. Bars report 12-quarter cumulative contributions from sector-specific US import tariff shocks.

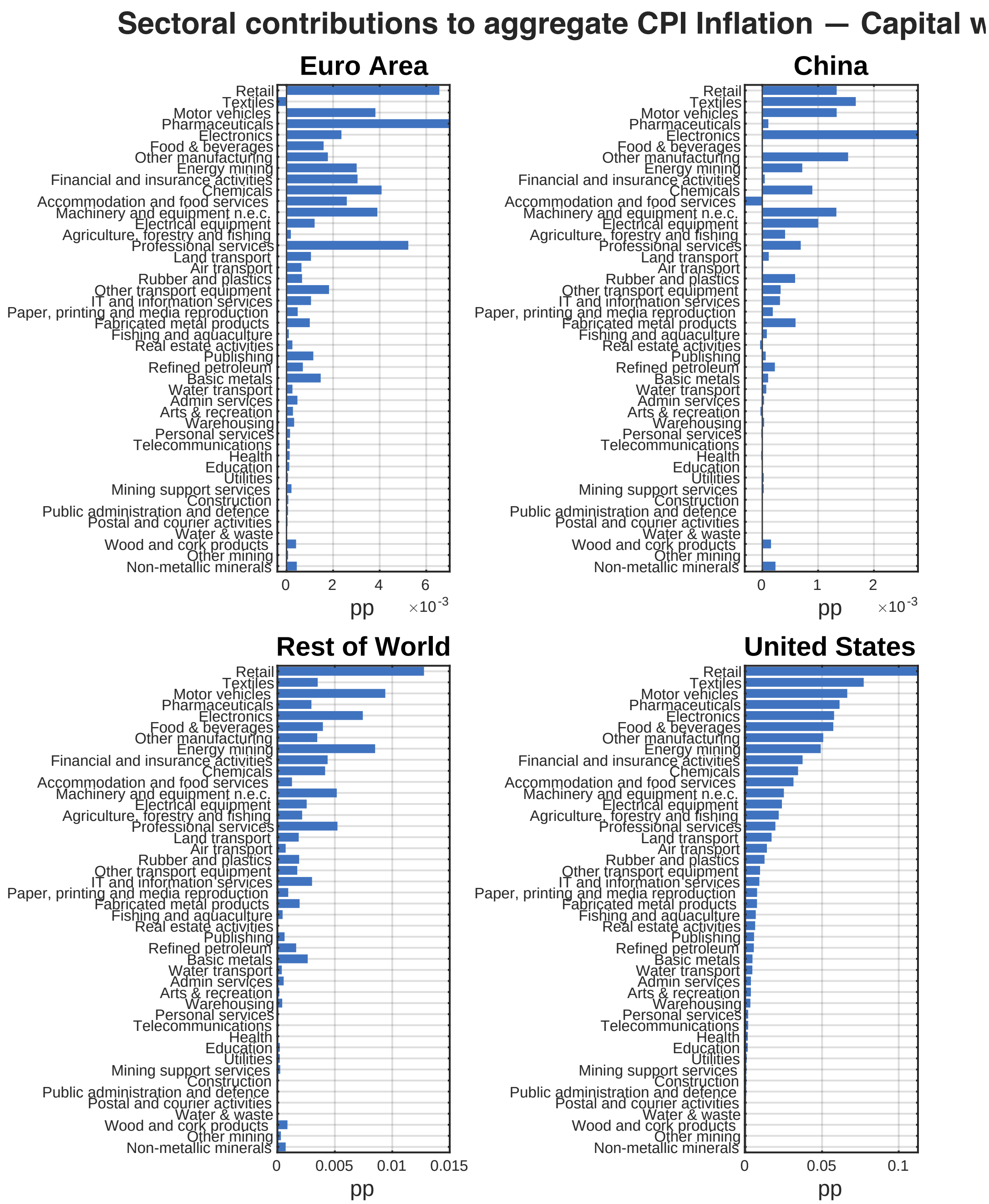

FIGURE E7. Sectoral tariff incidence on CPI inflation. Bars report 12-quarter cumulative contributions from sector-specific US import tariff shocks.

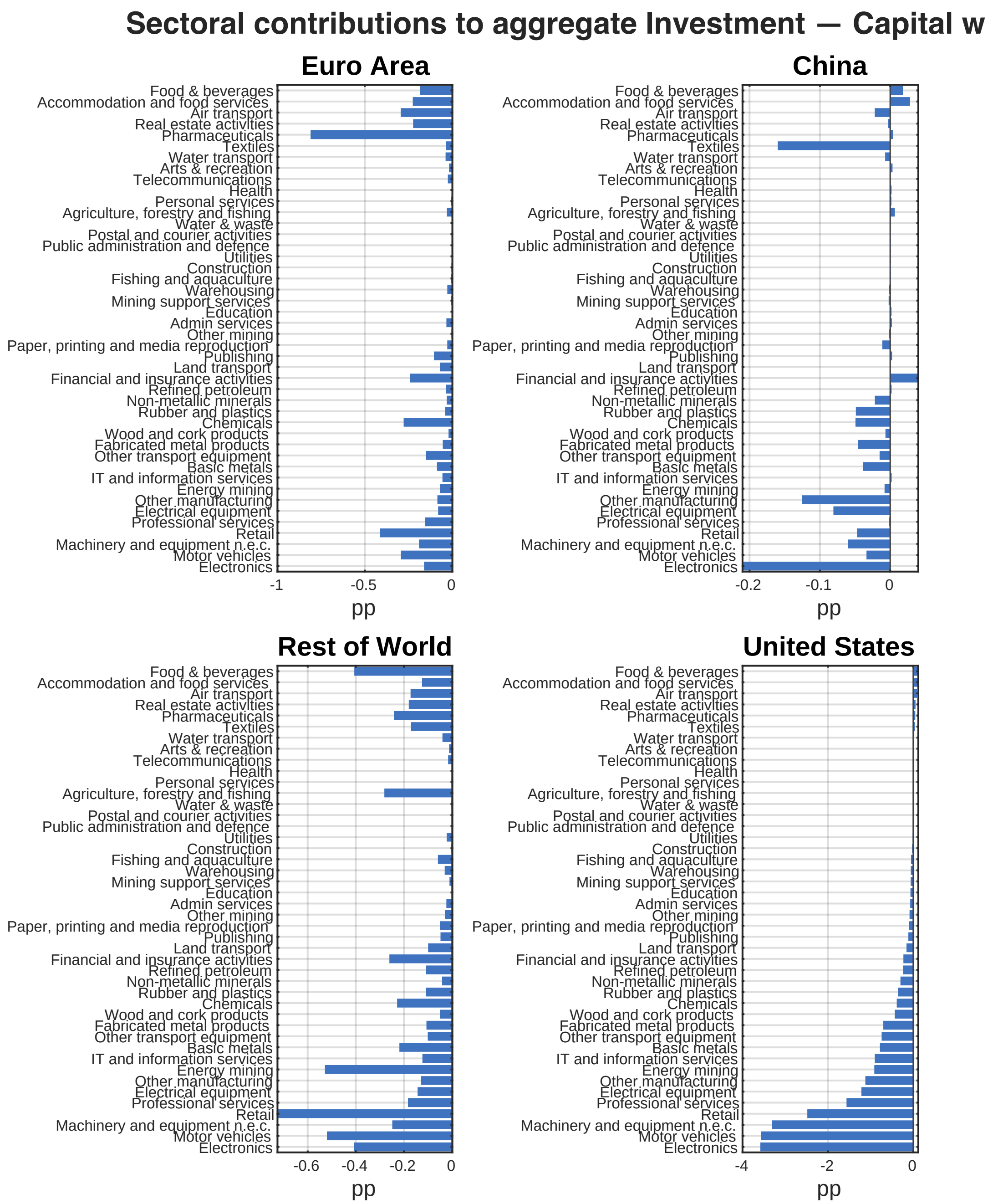

FIGURE E8. Sectoral tariff incidence on investment. Bars report 12-quarter cumulative contributions from sector-specific US import tariff shocks.

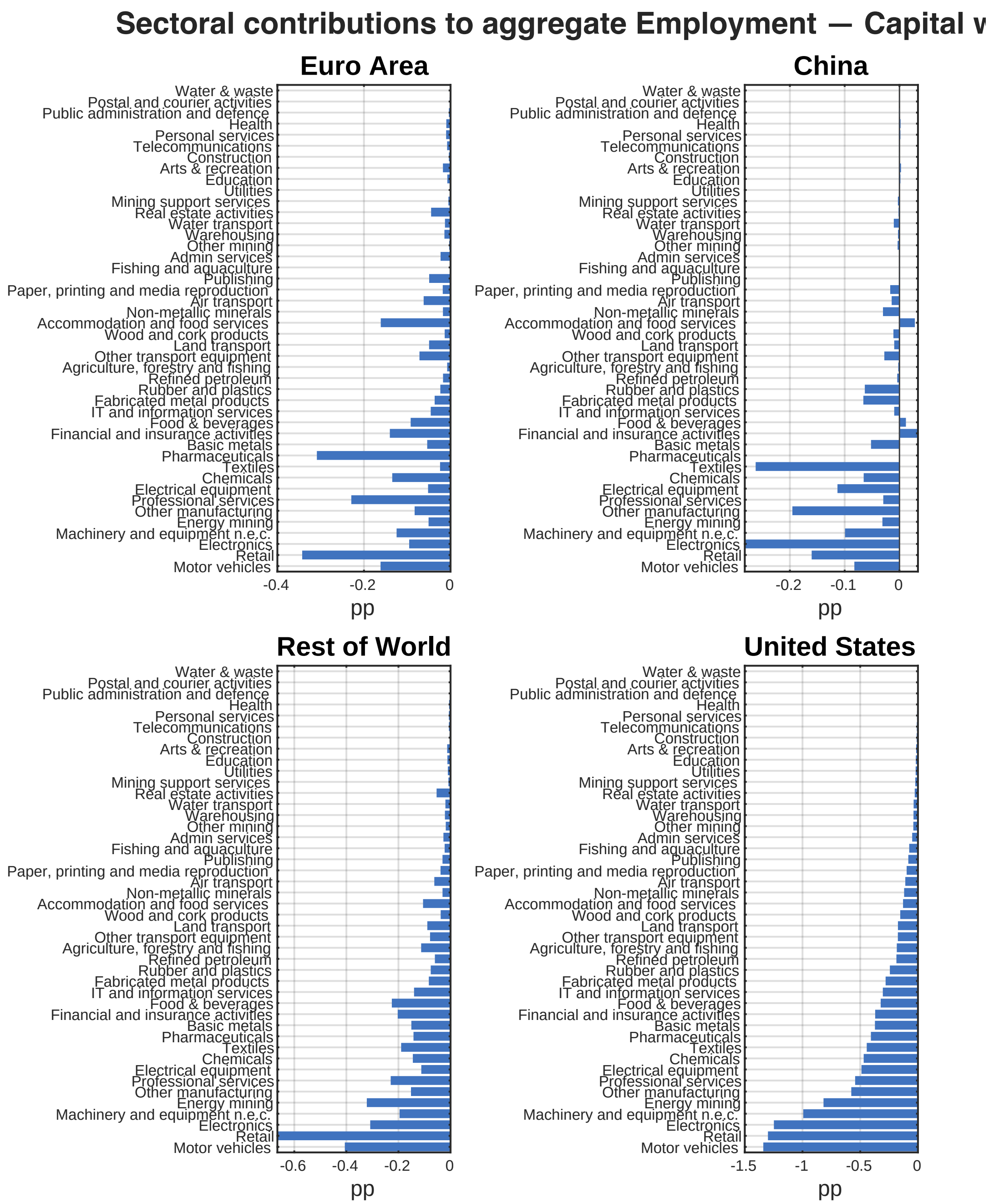

FIGURE E9. Sectoral tariff incidence on employment.

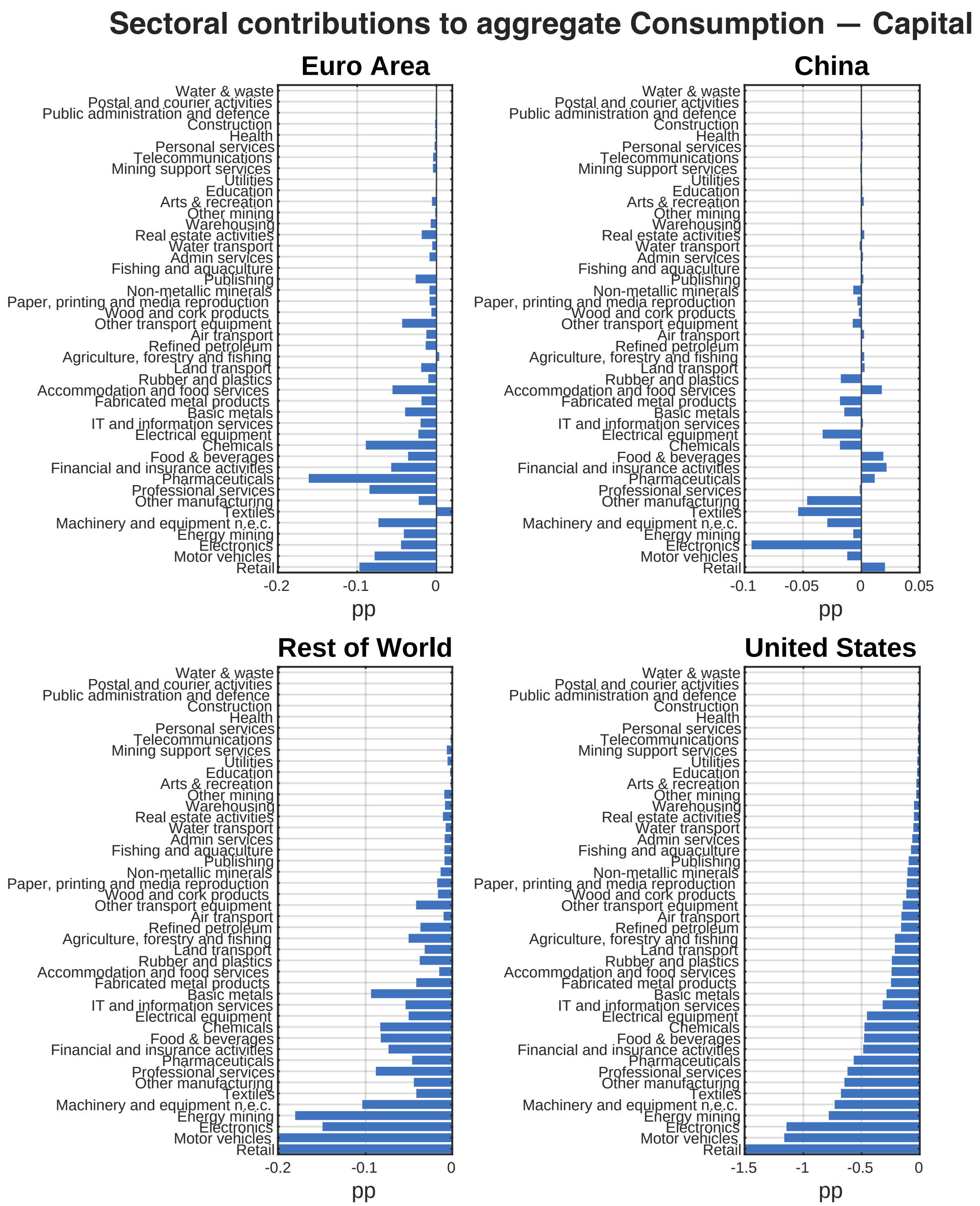

FIGURE E10. Sectoral tariff incidence on consumption.

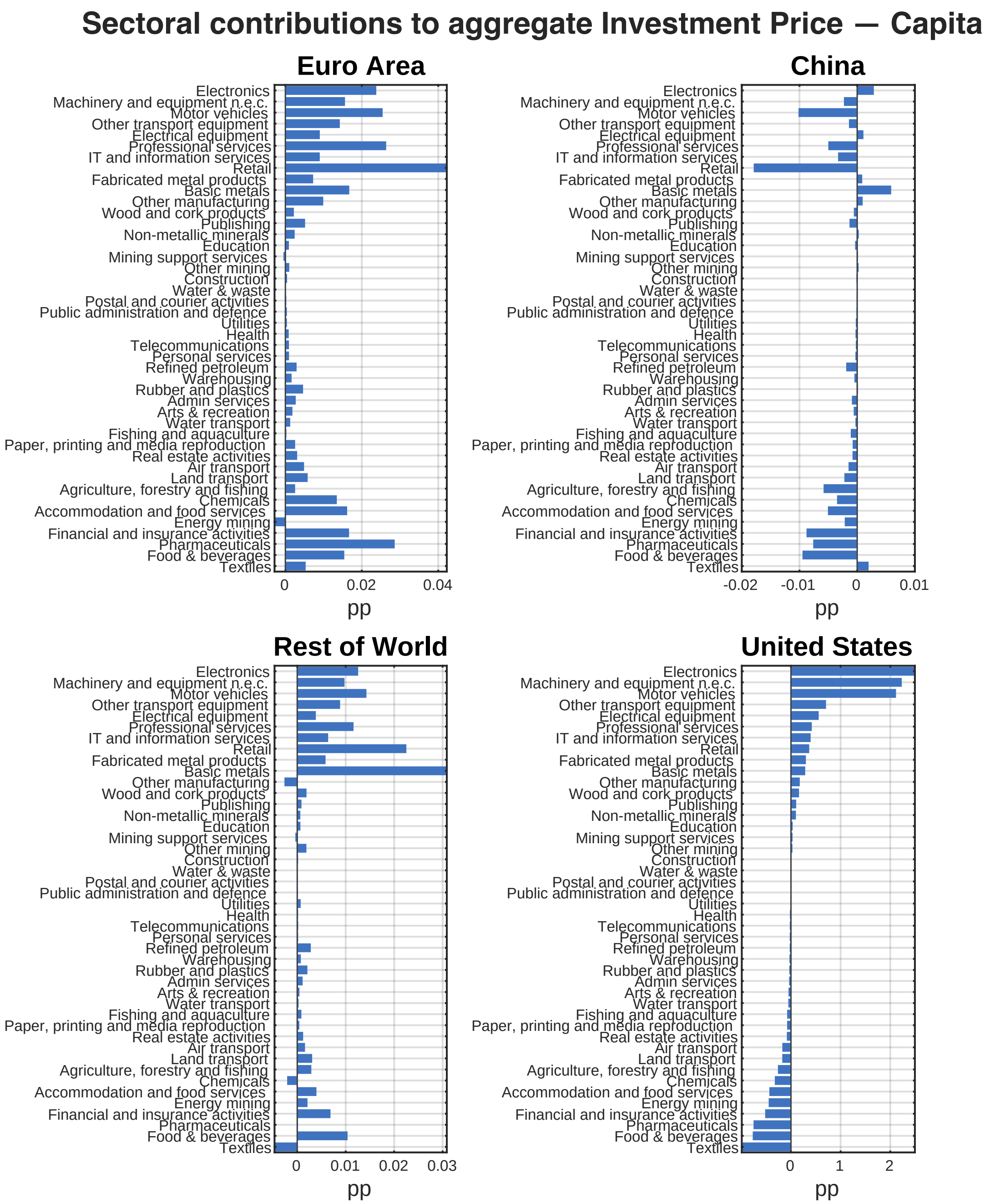

FIGURE E11. Sectoral tariff incidence on the investment price.

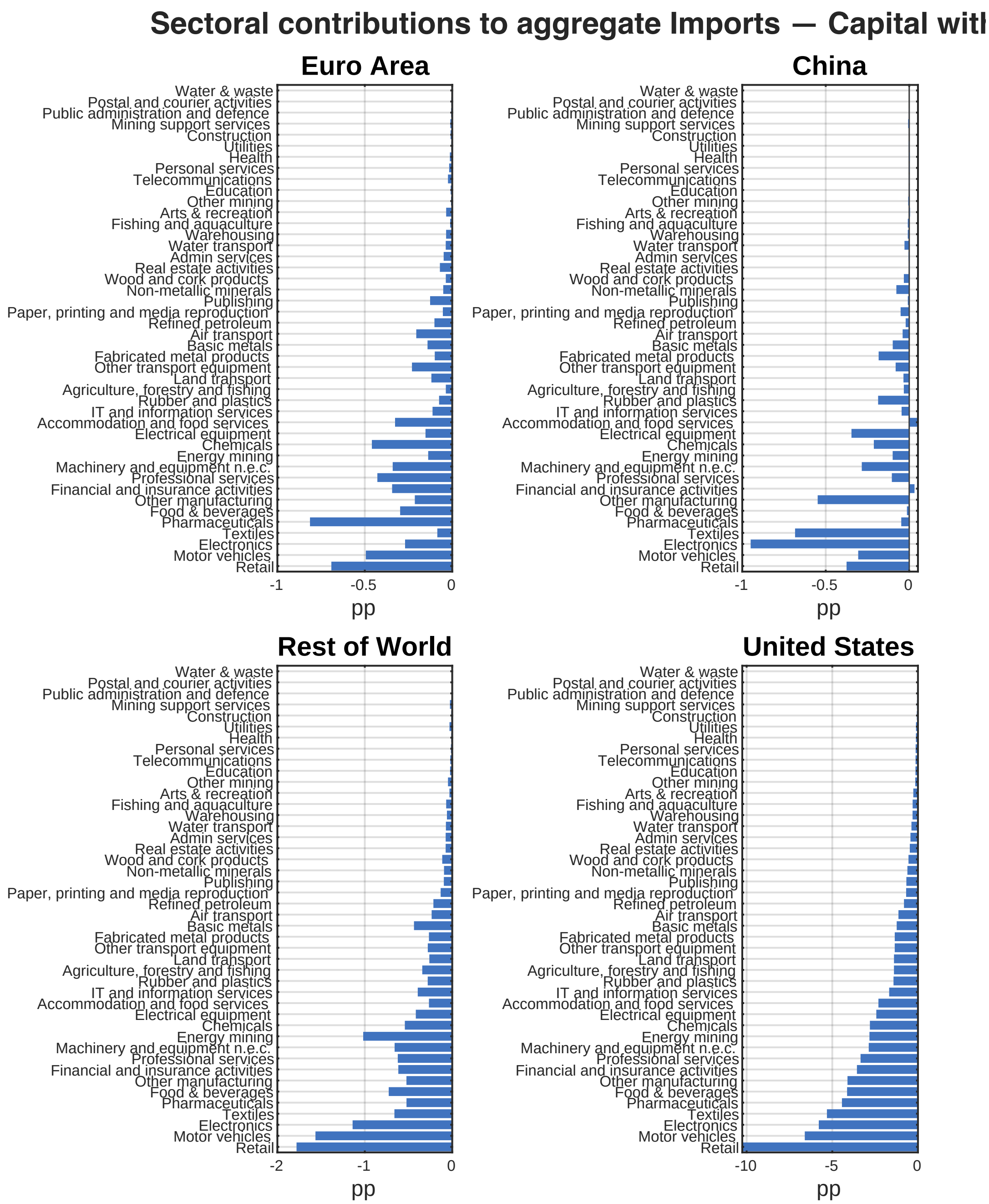

FIGURE E12. Sectoral tariff incidence on imports.

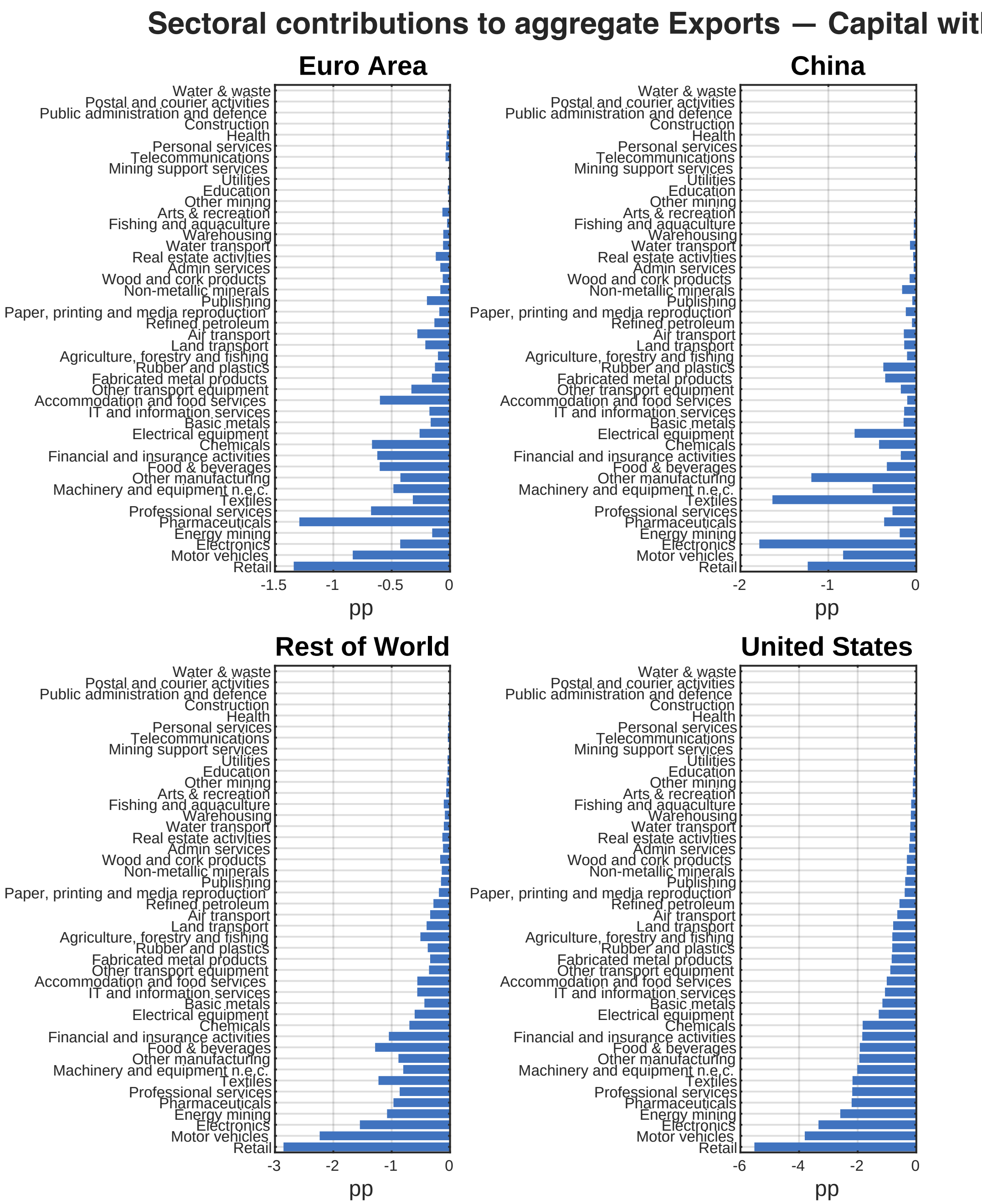

FIGURE E13. Sectoral tariff incidence on exports.

FIGURE E14. Sectoral tariff incidence on the trade balance.

Sectoral characteristics and counterfactual diagnostics

Appendix Table E.2 quantifies the visual message of the scatter plots: it reports the R^2 from univariate regressions of each 12-quarter cumulative US sectoral response on each exposure measure. Investment-goods exposure alone accounts for 77 percent of the cross-sector variation in real GDP and 92 percent in investment, whereas final-demand exposure accounts for 87 percent of the variation in CPI inflation; the off-diagonal entries are small, confirming that the three exposure measures of Section 4 are not interchangeable. Appendix Table E.3 reports the 12-quarter cumulative counterfactual gaps that complement the impact gaps in Table 2. Over this horizon the No Investment gaps are largest for the capital-goods sectors (machinery, electronics, motor vehicles), while the No Investment Network gaps switch sign for the same sectors—the economy without the detailed network eventually produces the larger cumulative investment contraction—confirming that the investment network reallocates propagation across sectors and over time rather than uniformly scaling it.

Structural exposure and sectoral incidence by region

Appendix Figures E15–E17 report, region by region, the scatter plots that underlie Figure 3, relating each sectoral response to the three structural exposure measures of Section 4. The pattern documented for the United States holds across regions: real GDP and investment line up with investment-goods exposure KI_{ks}, whereas CPI inflation lines up with final-demand exposure FD_{ks}. The use-specific reading of tariff incidence is therefore not specific to the United States.

Strategic incidence rankings

Appendix Table E.4 reports the sectoral rankings behind Figure 8 when the domestic-cost weight is one. Appendix Table E.5 reports the sector that maximizes the GDP-based strategic score for alternative domestic-cost weights. Appendix Figures E18 and E19 plot the top twelve GDP-based scores for each bilateral direction.

Appendix F. The Optimal Composition of a Tariff Package: Details

This appendix collects the formal statement, the analytical characterization, and the alternative objectives behind Section 6.3.2. Throughout, we consider the set of tariff packages with a fixed import-weighted average tariff $\bar{\tau}$, equation (48): nonnegative destination-origin-sector wedges $\tau_{kl\,i}$, capped at $\tau^{\max}$, weighted by the steady-state import shares $\omega_{l\,i}^{\mathrm{IMP}}$. We deliberately do not solve for the welfare-optimal level of tariffs but instead focus on the composition

Outcome	Final demand	Intermediate inputs	Investment goods	Total exposure
Real GDP	0.25	0.56	0.77	0.69
Investment	0.12	0.32	0.92	0.47
CPI inflation	0.87	0.35	0.26	0.93

TABLE E.2. Exposure and US sectoral incidence

Notes: Entries are R^2 statistics from univariate regressions of the 12-quarter cumulative US sectoral contribution on each exposure measure. Total exposure is the sum of final-demand, intermediate-input, and investment-goods exposure.

Counterfactual gap	**Outcome**	**Horizon**	**Largest US sectoral gaps**
Baseline minus No Investment	Real GDP	12-quarter cumulative	Machinery (–0.294), electronics (–0.277), motor vehicles (–0.256)
Baseline minus No Investment	Employment	12-quarter cumulative	Motor vehicles (–0.591), electronics (–0.580), retail (–0.547)
Baseline response (No Investment absent)	Investment	12-quarter cumulative	Electronics (–3.582), motor vehicles (–3.567), machinery (–3.313)
Baseline minus No Investment Network	Real GDP	12-quarter cumulative	Electronics (+0.340), motor vehicles (+0.222), other transport equipment (+0.219)
Baseline minus No Investment Network	Employment	12-quarter cumulative	Electronics (+0.411), motor vehicles (+0.248), other transport equipment (+0.243)
Baseline minus No Investment Network	Investment	12-quarter cumulative	Electronics (+0.690), other transport equipment (+0.410), motor vehicles (+0.396)

Notes: Entries are 12-quarter cumulative percentage-point contributions. A negative gap means that the baseline model is more contractionary than the counterfactual for that sectoral tariff. A positive gap means that the counterfactual is more contractionary. The investment row for the No Investment economy is a baseline response, not a subtraction, because investment is absent in that counterfactual.

TABLE E.3. Sectoral counterfactual comparisons in the United States: 12-quarter cumulative gaps

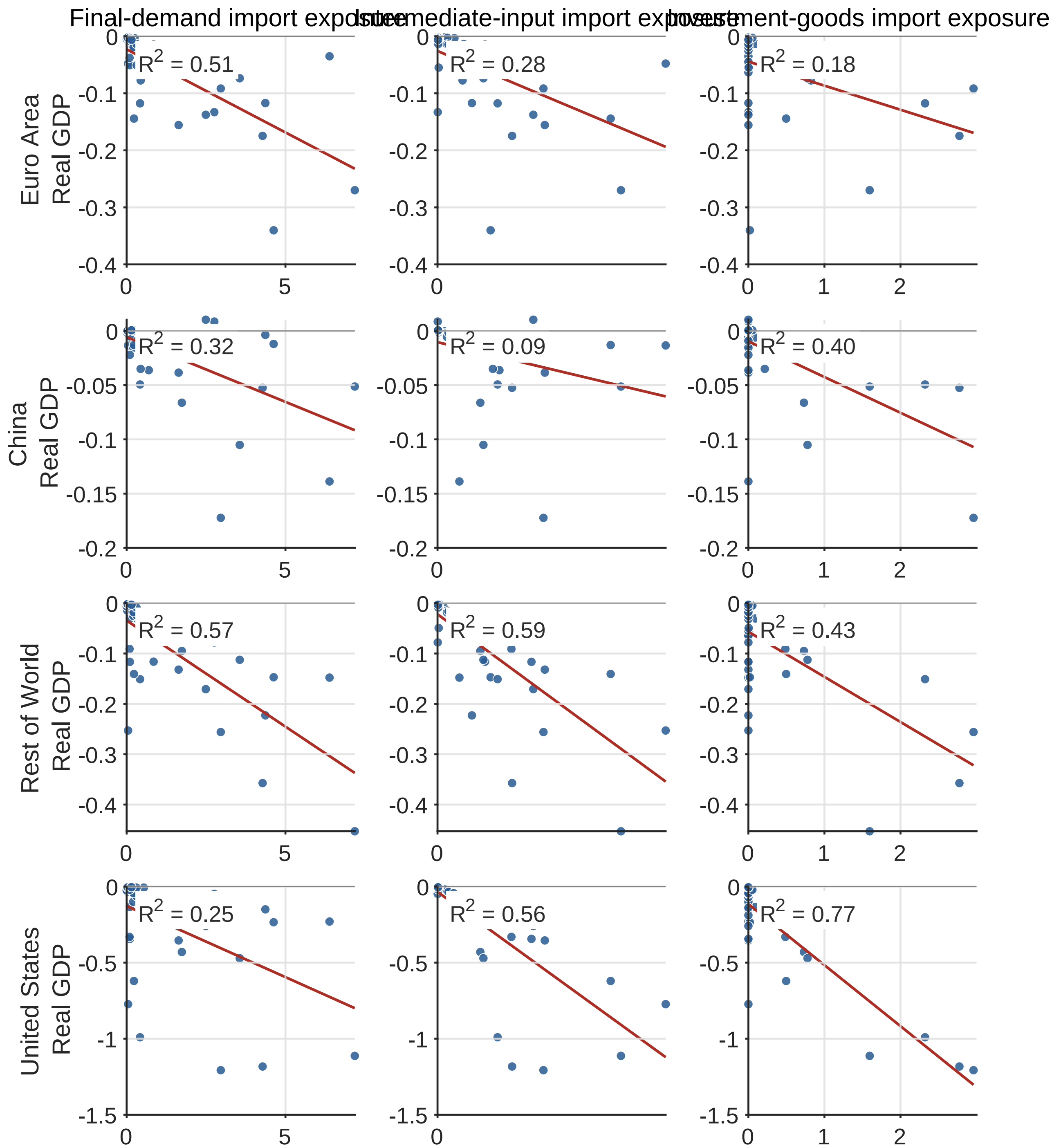

FIGURE E15. Structural exposure and sectoral GDP incidence. Each point is a sector; rows plot the sectoral real GDP contribution against final-demand, intermediate-input, and investment-goods exposure, by region.

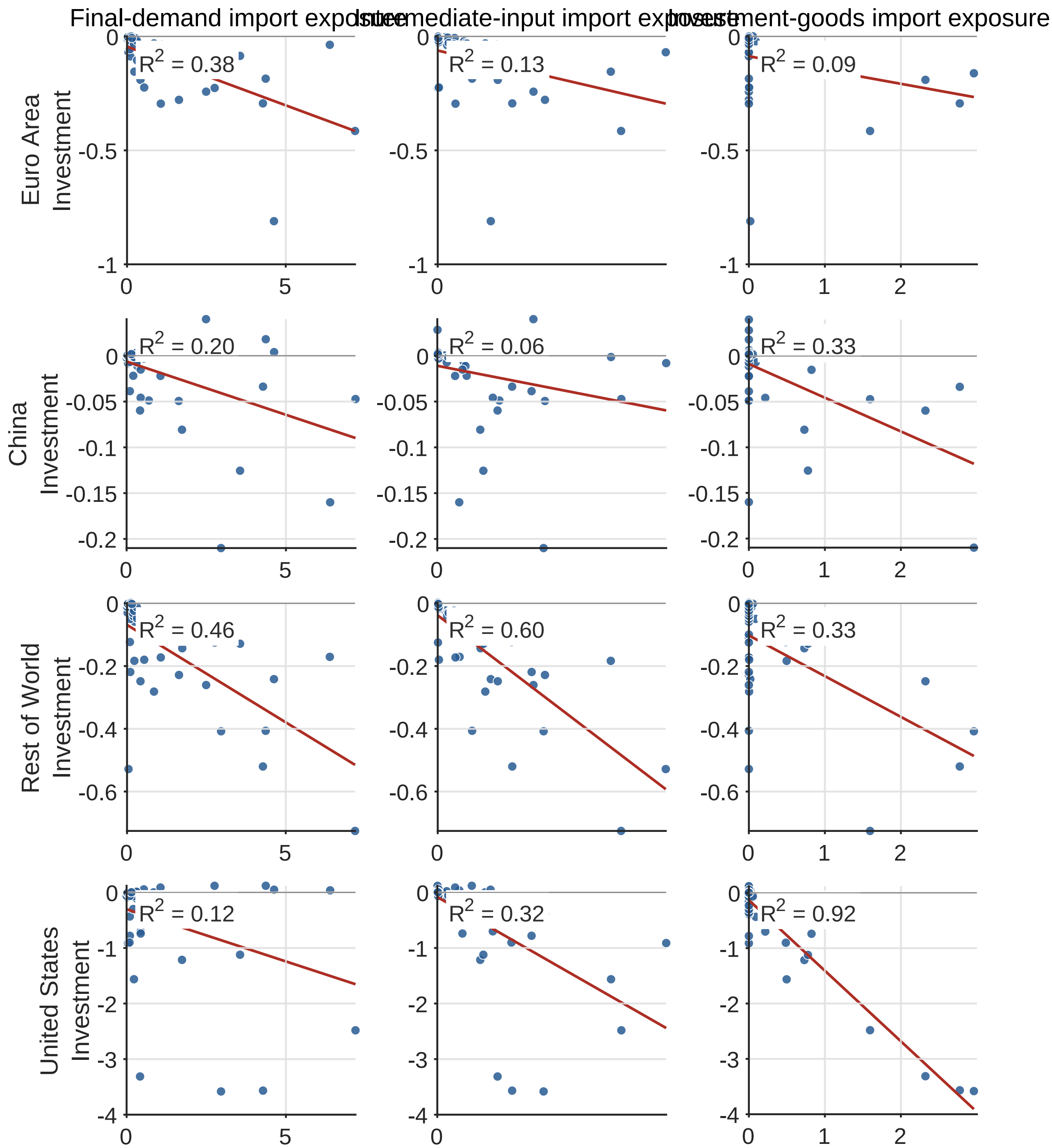

FIGURE E16. Structural exposure and sectoral investment incidence. Each point is a sector; rows plot the sectoral investment contribution against final-demand, intermediate-input, and investment-goods exposure, by region.

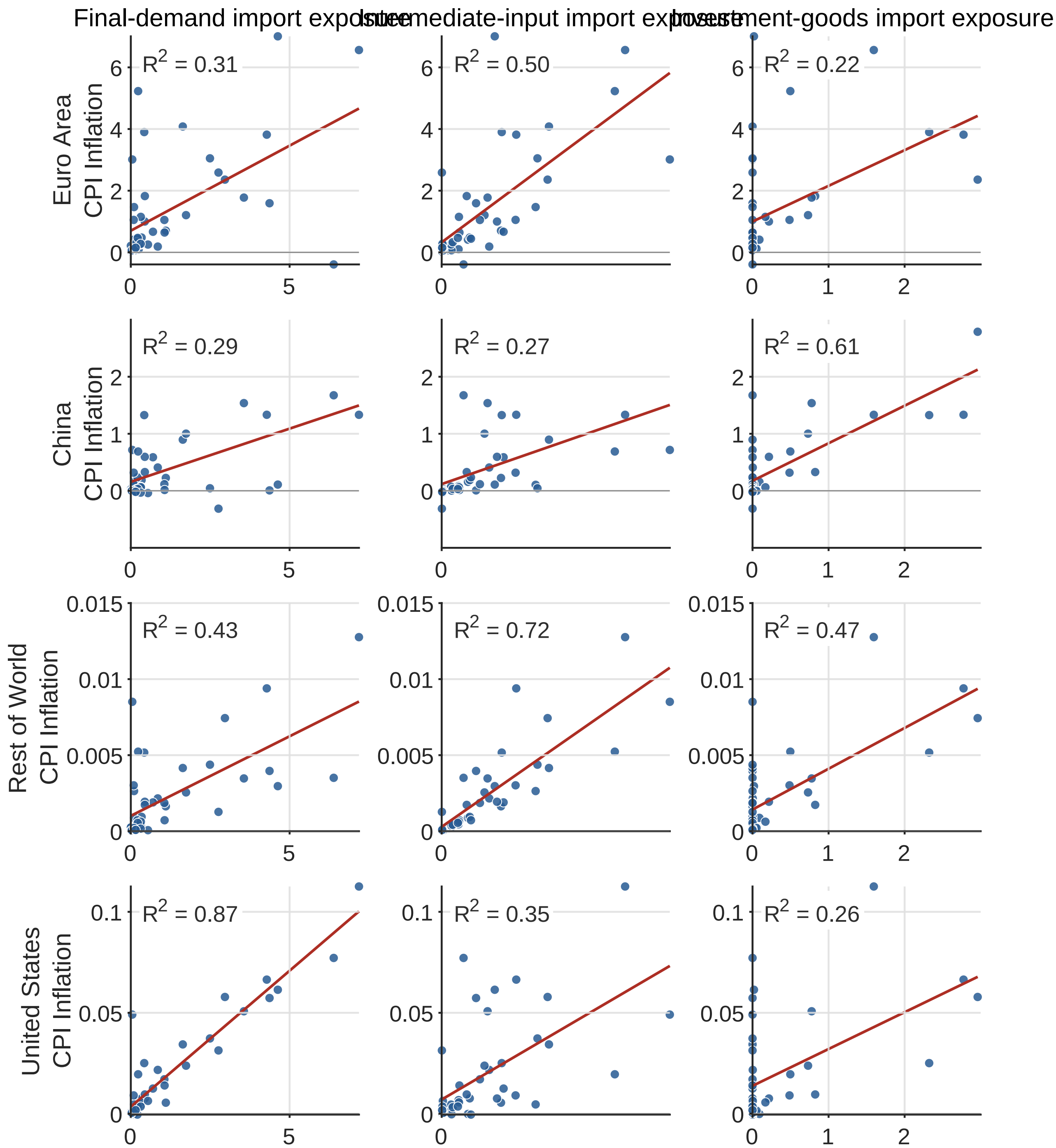

FIGURE E17. Structural exposure and sectoral CPI incidence. Each point is a sector; rows plot the sectoral CPI inflation contribution against final-demand, intermediate-input, and investment-goods exposure, by region.

Direction	Rank	Sector	Home cost	Target damage	Score	Ratio
US tariff on China goods	1	C13_15 Textiles, apparel and leather	0.093	0.153	0.060	1.640
	2	C21 Pharmaceuticals	0.012	0.026	0.014	2.106
	3	C10_12 Food, beverages and tobacco	0.008	0.018	0.011	2.320
	4	H51 Air transport	0.005	0.008	0.003	1.639
	5	L68 Real estate activities	0.000	0.002	0.002	9.151
	6	I55_56 Accommodation and food services	0.001	0.002	0.001	3.149
	7	A01_02 Agriculture, forestry and fishing	0.003	0.003	0.001	1.281
	8	H50 Water transport	0.005	0.005	0.000	1.066
China tariff on US goods	1	A01_02 Agriculture, forestry and fishing	0.068	0.207	0.139	3.024
	2	P85 Education	0.000	0.109	0.109	∞
	3	I55_56 Accommodation and food services	0.000	0.099	0.099	∞
	4	G45_47 Wholesale and retail trade	0.161	0.248	0.087	1.540
	5	C10_12 Food, beverages and tobacco	0.003	0.078	0.075	23.779
	6	J58_60 Publishing and broadcasting	0.006	0.051	0.044	7.850
	7	K64_66 Financial and insurance activities	0.003	0.041	0.038	12.939
	8	H49 Land transport	0.009	0.046	0.037	5.026

Notes: The table ranks sectors by target GDP damage minus home GDP cost over twelve quarters after a 10 percentage-point bilateral tariff innovation. With imposing country k, targeted origin l, and tariffed sector i, home cost is $C_i^k(k,l) = \max\{-\Delta y_i^k(k,l), 0\}$ and target damage is $D_i^l(k,l) = \max\{-\Delta y_i^l(k,l), 0\}$. The ratio is target damage divided by home cost. The ranking is descriptive and is not a welfare calculation.

TABLE E.4. Top GDP strategic-incidence rankings

Direction	η	Top sector	Home cost	Target damage	Score
US tariff on China goods	0.000	C26 Computer, electronic and optical products	0.417	0.175	0.175
	0.500	C13_15 Textiles, apparel and leather	0.093	0.153	0.107
	1.000	C13_15 Textiles, apparel and leather	0.093	0.153	0.060
	2.000	C10_12 Food, beverages and tobacco	0.008	0.018	0.003
	5.000	L68 Real estate activities	0.000	0.002	0.001
China tariff on US goods	0.000	G45_47 Wholesale and retail trade	0.161	0.248	0.248
	0.500	A01_02 Agriculture, forestry and fishing	0.068	0.207	0.173
	1.000	A01_02 Agriculture, forestry and fishing	0.068	0.207	0.139
	2.000	P85 Education	0.000	0.109	0.109
	5.000	P85 Education	0.000	0.109	0.109

Notes: For each domestic-cost weight η, the table reports the sector that maximizes target GDP damage minus η times home GDP cost. All entries are twelve-quarter cumulative responses to a 10 percentage-point bilateral tariff innovation.

TABLE E.5. Sensitivity of GDP strategic-incidence rankings to domestic-cost weights

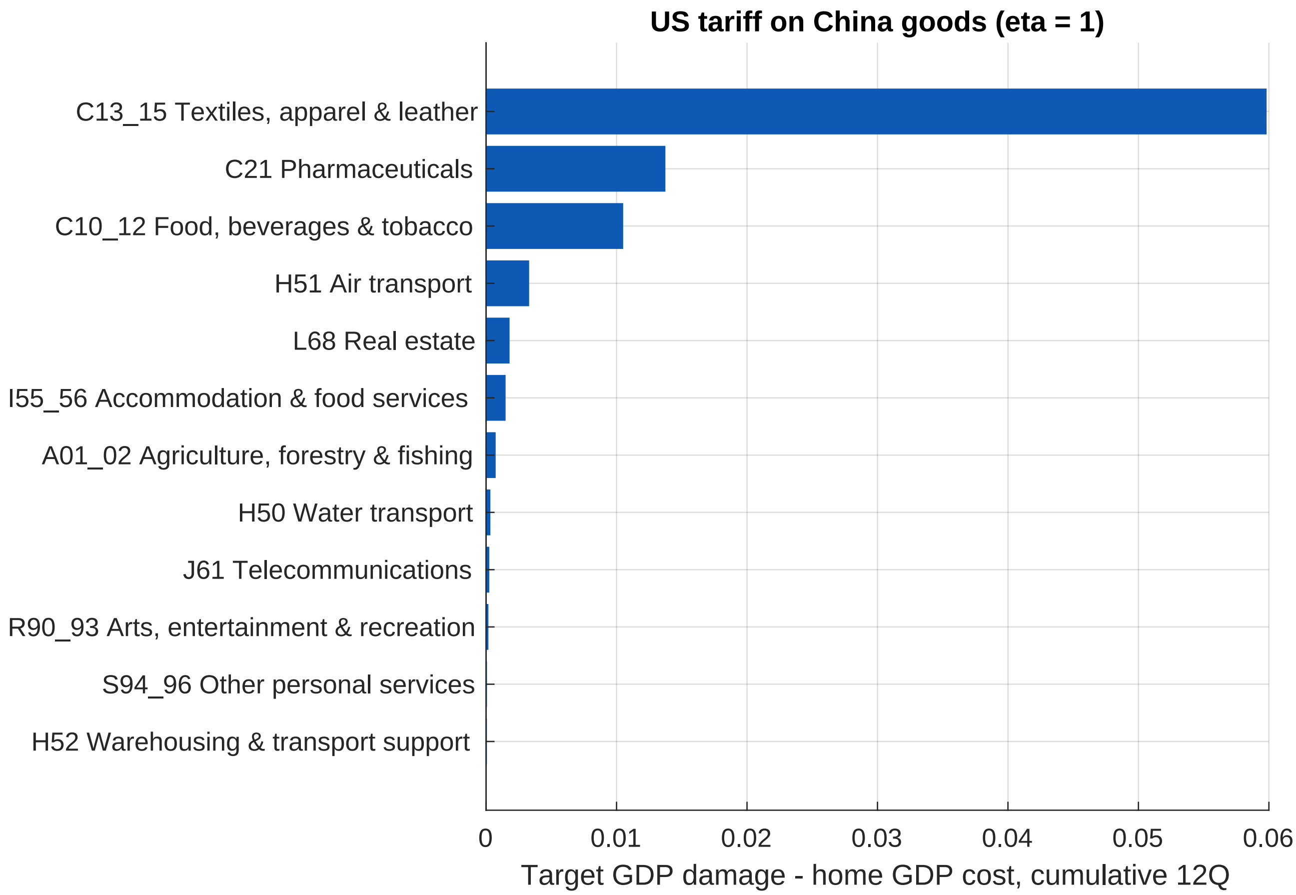

FIGURE E18. GDP strategic-incidence ranking: US tariff-equivalent wedges on Chinese sectors. Bars report target-country GDP damage minus home GDP cost over twelve quarters after a 10 percentage-point bilateral tariff-equivalent wedge, with domestic-cost weight equal to one.

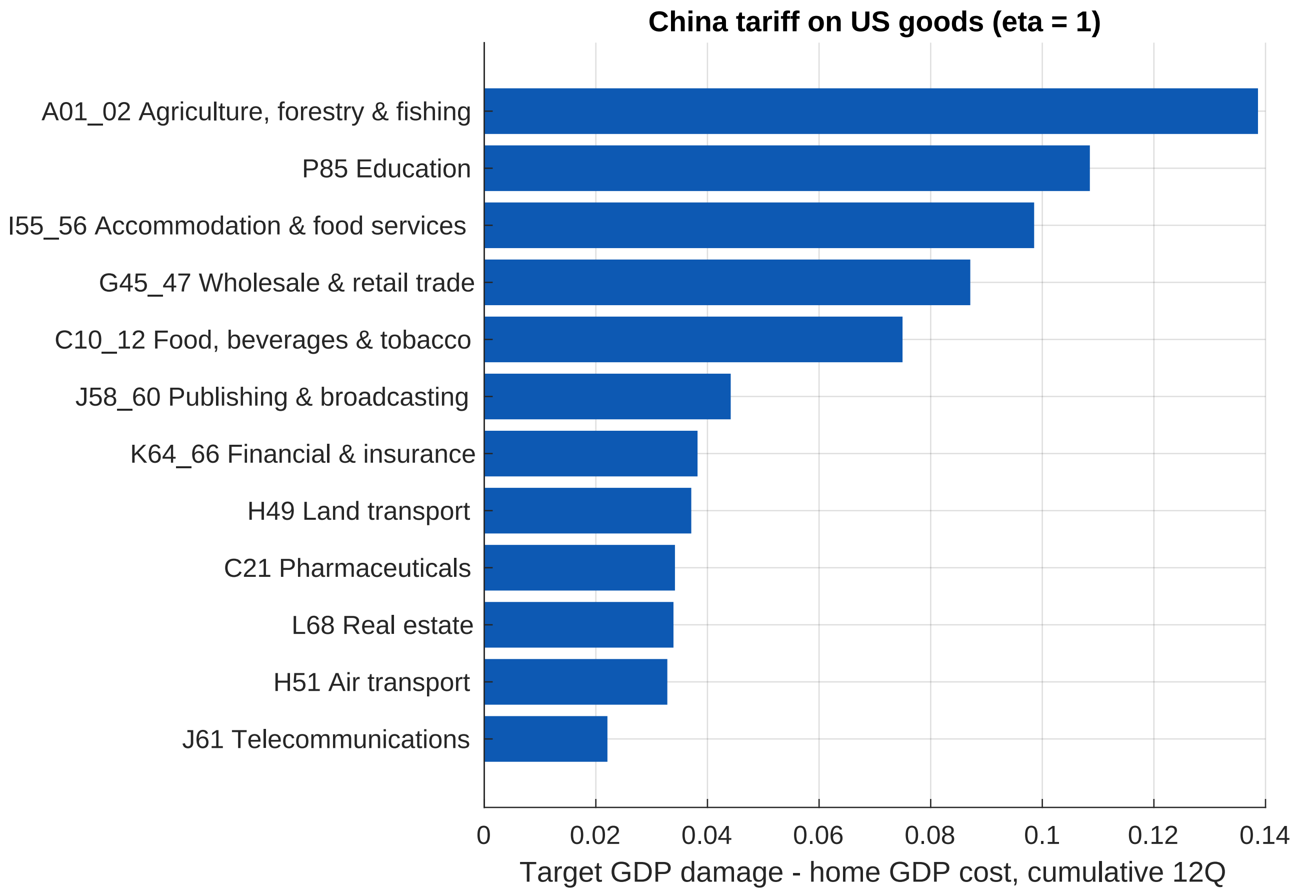

FIGURE E19. GDP strategic-incidence ranking: Chinese tariff-equivalent wedges on US sectors. Bars report target-country GDP damage minus home GDP cost over twelve quarters after a 10 percentage-point bilateral tariff-equivalent wedge, with domestic-cost weight equal to one.

question, which is both directly affected by the investment matrix and exactly identified in the model.

F.1. An approximate composition rule from the exposure statistics

Although the optimal compositions are computed numerically, the sectors they select are pinned down by the exposure statistics of Section 4, as the following corollary of the pass-through and user-cost propositions of Appendix B shows.

PROPOSITION F1 (Approximate composition rule). *Maintain the assumptions of Propositions B1 and B2. A tariff on origin sector s lowers aggregate investment on impact in proportion to*

$$\sum_i \lambda_{ki}\chi_{ki}\,\frac{\psi}{\delta_{ki}}\,\mathcal{A}_{ki}(\rho)\left(e^I_{kis} - FD_{ks}\right).$$

When $\psi\mathcal{A}_{ki}(\rho)/\delta_{ki}$ is common across using sectors i, this equals $(\psi\bar{\mathcal{A}}(\rho)/\bar{\delta})\left(KI_{ks} - FD_{ks}\sum_i \lambda_{ki}\chi_{ki}\right)$, with $\bar{\mathcal{A}}(\rho)$ and $\bar{\delta}$ the $\lambda\chi$-weighted average amplification factor and depreciation rate, so investment falls with the sector's net investment-goods exposure $KI_{ks} - FD_{ks}\sum_i \lambda_{ki}\chi_{ki}$. Real-activity damage adds the intermediate-input channel IO_{ks}; per unit of average tariff it is ranked across sectors by

$$\frac{\bar{\mathcal{A}}(\rho)\left(KI_{ks} - FD_{ks}\sum_i \lambda_{ki}\chi_{ki}\right) + IO_{ks}}{\omega^{IMP}_s}, \qquad \omega^{IMP}_s \equiv \sum_{l\neq k}\omega^{IMP}_{ls}, \tag{F.1}$$

while CPI incidence per unit is FD_{ks}/ω^{IMP}_s.

The optimal composition therefore taxes the sectors with the lowest real-activity damage per dollar of average tariff, which in this case are those with the smallest values of (F.1). Because that statistic is built from calibration data alone, the design problem collapses to a ranking, and taxing sectors in this order up to the budget attains 93 percent of the damage reduction of the exact *cost-minimizing* composition. This is the design counterpart of the paper's cross-sectional finding that the same statistics explain the incidence of sectoral tariffs (Table E.2).

The amplification factor also implies an impact persistence margin. In this case $\bar{\mathcal{A}}(0.99) = 1.10$. A permanent tariff by contrast ($\rho = 1$) carries the smallest impact amplification, $\mathcal{A}_{ki}(1) = 1$, and so raises the impact user cost of capital by less per dollar of today's average tariff than a transitory one; final-demand incidence, on the other hand is independent of persistence (Proposition B1). Cumulative domestic damage depends on the full tariff path, since a more persistent tariff also keeps the distortion in place for longer.

F.2. A stabilization-loss objective

To let inflation enter the objective, we use a dual-mandate stabilization loss, not a utility approximation:

$$\mathbb{L}_k = \sum_{t=0}^{\infty} \beta^t \left[\widehat{y}_{k,t}^2 + \lambda_\pi \, (\pi_{k,t}^C)^2 + \lambda_\pi \, (\pi_{k,t}^w)^2 \right], \tag{F.2}$$

with the wage-inflation term required by the sticky-wage environment (Guerrieri *et al.* 2025) and $\lambda_\pi \in \{0.048, 0.25, 1\}$ spanning the welfare-based, dual-mandate, and welfare-optimized weights surveyed in Debortoli *et al.* (2019). The objective is a convex quadratic in the tariff vector, so the optimizer is well defined, but the budget, nonnegativity, and cap constraints still determine which instruments bind. At the baseline cap, the loss-minimizing package is spread across fifteen positive sectoral tariffs, interior to the caps, rather than at a vertex of the constraint set like the GDP-based composition. The loss-minimizing composition cuts the US stabilization loss by 74–75 percent, and the damage-maximizing composition raises China's stabilization loss 5.5-fold at unchanged US loss (Table F.2, Panel B).

Robustness to the loss weights. Equation (F.2) ties the two inflation terms to a common weight λ_π and fixes the output weight at one, and the results are stable across its three values. To show that they do not turn on this parametrization at all, we let the three terms carry free weights. Because the loss-minimizing composition is unchanged when the loss is multiplied by a positive constant, every weighting is fully described by the shares $w_y + w_C + w_W = 1$ of total weight placed on output, consumer-price inflation, and wage inflation. We sweep all weightings with $w_y \geq 0.05$ and, at each one, re-solve for the composition that minimizes *that* loss under the same budget and caps as before. Figures F1 and F2 then answer three questions about these packages.

First, does composition still matter? Figure F1 plots the loss of each weighting's optimal package as a percentage of the uniform package's loss, both evaluated under that same weighting. The ratio lies between 24 and 40 percent throughout the sweep and is close to one-quarter at the three weightings of equation (F.2): at equal weights, recomposition lowers the loss from 7.5 to 1.9, the equivalent of shrinking a permanent blended output-and-inflation gap from 0.27 to 0.14 percentage points. Whatever the weights, recomposing the same average tariff eliminates at least sixty percent of the stabilization loss.

Second, do these packages preserve the paper's GDP result? The cumulative-GDP criterion of Section 6.3.2 is the linear, undiscounted twelve-quarter counterpart of the $w_y = 1$ corner—related to, but not nested in, the quadratic family—so agreement between the two metrics must be verified rather than assumed. Panel (a) of Figure F2 does so by reporting the cumulative

US GDP loss that each weighting's package generates: between 4.9 and 6.2 percentage points everywhere in the sweep, against 11.4 under the uniform package and 3.5 at the GDP optimum. Most of the paper's headline GDP margin is preserved no matter which of these weightings the package is designed for.

Third, are they the same package? Panel (b) of Figure F2 compares the designs in instrument space: each package places 45 to 66 percent of its tariff dollars on the same goods as the GDP-based composition, against 24 percent for the uniform package. The remaining distance to the GDP-based design largely reflects the curvature of the objective rather than disagreement about which goods to spare: the convex-quadratic loss spreads a similar core basket across more sectors than the linear GDP criterion, whose optimum is a corner solution.

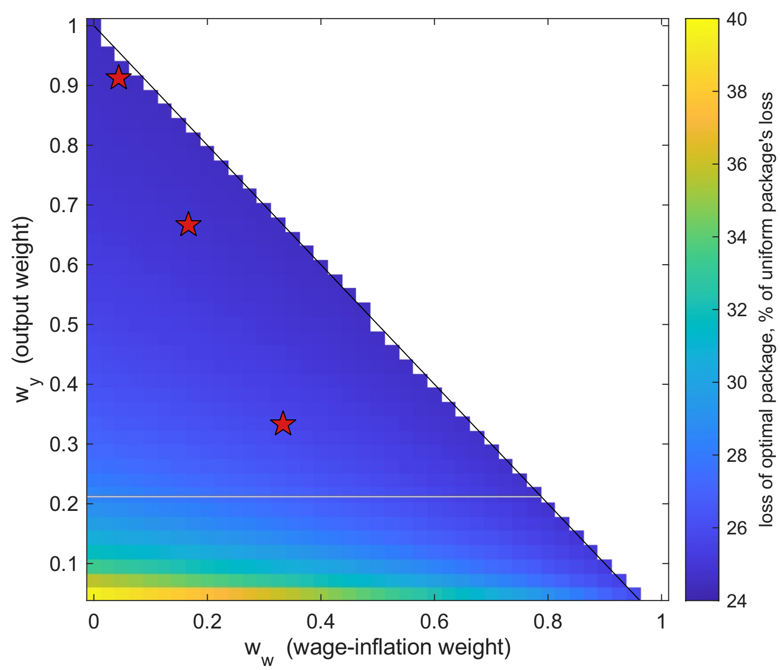

FIGURE F1. The value of recomposition under alternative stabilization-loss weights. Each point is a weighting of the loss $\mathbb{L} = \sum_t \beta^t \left[w_y \widehat{y}_t^2 + w_C(\pi_t^C)^2 + w_W(\pi_t^W)^2 \right]$, normalized so the weights sum to one: the vertical axis is the output weight w_y, the horizontal axis the wage-inflation weight w_W, and the consumer-price-inflation weight $w_C = 1 - w_y - w_W$ is implied (the blank upper-right triangle is infeasible; the output-weight axis begins at 0.05). At each weighting we compute the composition that minimizes that loss, holding fixed the 10 percentage-point average tariff and the 50 percentage-point caps, and plot its loss as a percentage of the uniform package's loss under the same weighting; a lower ratio means recomposition removes more of the loss, and a ratio of zero would eliminate it entirely. Blue marks the lower, more favorable ratios. Stars mark the three λ_π weightings of equation (F.2).

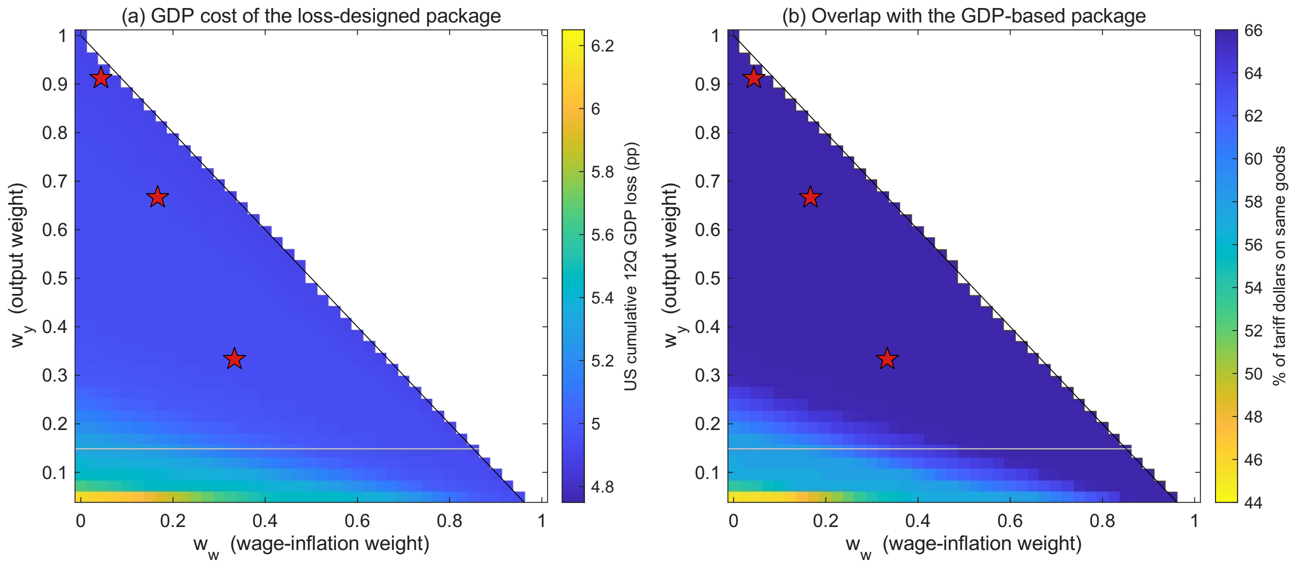

FIGURE F2. The loss-designed packages against the paper's GDP-based benchmarks. The triangle of weightings and the loss-minimizing compositions are those of Figure F1. Panel (a): the cumulative twelve-quarter US real GDP loss that each weighting's package generates, in percentage points, to be read against 11.4 under the uniform package and 3.5 at the GDP optimum; blue marks the lower losses. Panel (b): the share of each package's tariff dollars placed on the same goods as the GDP-based cost-minimizing composition of Section 6.3.2, against 24 percent for the uniform package; blue marks the greater overlap. Blue is the favorable direction in both panels, and each color scale spans the range observed across the sweep. Stars mark the three λ_π weightings of equation (F.2).

F.3. The realized 2025–26 composition and retaliation

Where does the realized 2025–26 package sit? Section 6.2 found the realized path of tariff changes less contractionary than its uniform-equivalent path; but that comparison rewards the timing and reversals of the announcements. Restricting the exercise to US tariffs on China, we fix the package size at the episode's average increase of 22.5 percentage points—so that the realized schedule is itself a feasible package. The realized package is the average tariff-level change relative to the 2025Q1 base, applied as a one-time AR(1)-decaying shock.

The realized composition performs no better than a flat tariff on all Chinese goods (a cumulative US loss of 4.0 versus 3.9 percentage points), while the cost-minimizing composition of the same size cuts the loss by 62 percent relative to the flat tariff as shown in Table F.3. Table F.3 reports the US-on-China component only, before adding the fixed China-on-US background path. The realized package's advantage came from its timing; its composition left the available gains unused.

China's own tariffs on US goods enter as a fixed background path. Because the model is linear, this path adds the same amount to every package's outcome—–0.59 percent to US GDP on impact, fading cumulatively as the tariffs de-escalate—and so does not change which composition is optimal or the 62 percent reduction reported in Table F.3. An endogenous retaliation response would, but characterizing that response requires a strategic equilibrium beyond the scope of the paper.

Cost-minimizing

Sector	Import share	τ
Coke & petroleum	2.18	50.0
Electricity & gas	0.25	50.0
Food & beverages	5.02	50.0
Textiles & apparel	6.83	50.0
Pharmaceuticals	5.64	14.0
Accommodation & food	2.79	50.0
Telecom	0.17	50.0
Real estate	0.57	50.0
Health & social work	0.14	50.0
Arts & recreation	0.33	50.0
Other services	0.16	50.0

Notes: Sectoral (all-origin) instruments, 10 percentage-point average tariff, 50 percentage-point cap. Import share is the sector's share of the US steady-state import base in percent; τ is the tariff assigned by the composition, in percentage points. Sectors not listed receive zero tariffs.

Damage-maximizing

Sector	Import share	τ
Textiles & apparel	6.83	50.0
Non-metallic minerals	0.74	50.0
Fabricated metals	1.66	50.0
Electronics	7.87	50.0
Other manufacturing	5.20	28.0

TABLE F.1. Active sectors of the optimal compositions

Panel A: cap on individual tariffs (GDP objective)		
cap = 25pp	US loss -5.66	dividend 50%
cap = 50pp	US loss -3.51	dividend 69%
cap = 100pp	US loss -2.65	dividend 77%
Panel B: stabilization loss, inflation weight λ_π (cap 50pp)		
λ_π = 0.048	cut 75%	China multiple $\times$5.5
λ_π = 0.25	cut 75%	China multiple $\times$5.5
λ_π = 1.0	cut 74%	China multiple $\times$5.5

Notes: All packages hold the 10 percentage-point import-weighted average tariff fixed. Panel A varies the per-instrument cap under the cumulative-GDP objective. Panel B uses the stabilization loss $\mathbb{L} = \sum_t \beta^t[\hat{y}_t^2 + \lambda_\pi(\pi_t^C)^2 + \lambda_\pi(\pi_t^W)^2]$; the China multiple is the factor by which the damage-maximizing composition raises China's loss at unchanged US loss.

TABLE F.2. Composition frontier: sensitivity to caps, objectives, and weights

Package	Avg. tariff	US GDP	China GDP	Max τ
Uniform on Chinese goods	22.5	-3.89	-2.48	22.5
Realized composition	22.5	-4.04	-2.49	39.3
Cost-minimizing	22.5	-1.49	-2.51	150.0
Damage-maximizing	22.5	-3.89	-2.77	150.0

Notes: US tariffs on Chinese goods only, holding the China-import-weighted average tariff at the realized episode average of 22.5 percentage points; cap 150 percentage points. The realized composition is the import-day-weighted average tariff-level *change* relative to the 2025Q1 pre-episode base, applied as a one-time AR(1)-decaying shock. GDP entries are cumulative twelve-quarter responses in percentage points. Under the alternative standing-structure definition (levels including the pre-episode base, average 36.8pp), the ranking is unchanged: realized -6.51 versus uniform -6.37 and cost-minimizing -2.59.

TABLE F.3. The realized 2025–26 composition within the US-on-China direction