Tariffs, Production Networks, and Spillovers: the Case of a US-China Trade War

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Abstract

We study the short-run macroeconomic transmission of a US–China tariff war in an open economy multi-sector New Keynesian model with input–output linkages, sectoral nominal rigidities, and heterogeneous currency invoicing. A reciprocal 10 percentage-point tariff increase generates asymmetric incidence: the tariff-imposing country bears more of the inflationary burden, while the targeted country experiences the larger output contraction. Production networks amplify this contraction by propagating the shock beyond the directly tariffed bilateral margin. Currency invoicing further shapes transmission. Under heterogeneous invoicing, dollar-priced border prices weaken the expenditure-switching role of exchange rates, deepening the contraction in China relative to producer-currency pricing and altering third-country spillovers. The EA response is small in the aggregate, but only because positive trade-diversion margins are offset by weaker demand from China and multilateral adjustments. We then exploit the model's sectoral structure by imposing tariffs on one Chinese sector at a time. Sectoral incidence is highly concentrated, but aggregate effects cannot be inferred from the directly tariffed sector alone: domestic propagation offsets own-sector gains in the US, reinforces own-sector losses in China, and leaves the EA as a net object shaped by opposing trade margins. The results show that tariff incidence depends jointly on where the tariff lands, how the shock propagates through production networks, and how invoicing governs border-price adjustment. A framework that combines these margins delivers a materially different assessment from one built on bilateral trade shares alone.

Keywords: Tariffs, Trade, DSGE, Multicountry, Networks, Dominant Currency Pricing.

JEL Classifications: E31, E32, E52, F13, F41, F42.

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1. Introduction

Tariffs have returned as a central instrument of economic policy. Their appeal is often expressed in bilateral terms: a tariff raises the price of foreign goods, protects domestic producers, and redirects expenditure toward home production. But modern trade is not bilateral in the relevant economic sense. Goods are produced through domestic and international input–output linkages, traded under heterogeneous invoicing regimes, and priced in markets where nominal rigidities constrain the speed of adjustment. A tariff aimed at one country and one set of sectors can therefore generate inflation, output losses, and third-country spillovers through channels that are not visible from bilateral trade shares alone. This paper asks how a US–China tariff war is transmitted in such an environment, and which model mechanisms account for the cross-country and sectoral incidence of the shock.

To answer this question, we develop a multi-country and multi-sector New Keynesian model with bilateral tariffs, domestic and international input–output linkages, sectoral nominal rigidities, and heterogeneous currency invoicing. The world economy consists of the US, China, the EA, and the rest of the world (ROW). Each economy contains 44 sectors, of which 20 are tradeable manufacturing sectors subject to the tariff experiments. Firms combine domestic labor with intermediate inputs sourced domestically and abroad. Households consume sectoral composites that differ by origin. Prices and wages are sticky, monetary policy follows country-specific rules, and trade prices may be set in producer currency, local currency, or dollars. The model is calibrated using input–output tables, bilateral trade flows, sectoral price-adjustment frequencies, and evidence on invoicing regimes.

Our framework belongs to a recent open-economy production-network macro literature that extends the disaggregated Keynesian approach of Baqaee and Farhi (2022) to multi-country settings with global input–output linkages. Closely related contributions include Cakmakli *et al.* (2021), who study pandemic-related labor-supply shocks in an epidemiological model with international production networks; di Giovanni *et al.* (2022), who analyze global supply-chain pressures, international trade, and inflation in the context of the 2022 ECB Forum on Central Banking; di Giovanni *et al.* (2023), who develop a multi-country, multi-sector New Keynesian model of pandemic-era inflation and global spillovers; and Kalemli-Özcan *et al.* (2025), who study tariffs in a New Keynesian open-economy model with global production networks and monetary-policy responses. Relative to these papers, we focus on the short-run transmission of a bilateral US–China tariff war in a framework that combines detailed sectoral IO propagation with heterogeneous currency invoicing at the border.

The structure of the model is essential for the question we ask. In a plain-vanilla trade model, the transmission of a tariff is organized mainly by final-goods expenditure switching. That margin is present here, but it is incomplete. Tariffs also change the cost of imported intermediate goods, and those cost changes propagate through downstream sectors. Similarly, when foreign demand for a targeted sector falls, the shock is transmitted to upstream suppliers and to other sectors connected through intermediate-input demand. The output cost of a tariff therefore depends not only on the sector directly hit by the policy, but also on the position of that sector in the domestic and international production network.

The invoicing structure introduces a second reason why standard bilateral accounting is insufficient. If border prices were fully flexible, exchange-rate movements would do much of the work of reallocating demand after the tariff shock. With sticky border prices, however, the currency of invoicing determines which exchange rate matters for pass-through. Under producer-currency pricing, a depreciation of the targeted country lowers its foreign-currency export price and cushions the fall in demand. Under dominant-currency pricing, dollar-invoiced prices are sticky in dollars, so the same depreciation provides less short-run support. The currency in which trade is priced therefore affects not only the magnitude of the adjustment, but also its distribution across countries.

These mechanisms complement the standard insights in the literature. The empirical evidence on recent tariffs emphasizes substantial pass-through into prices paid by importers and a reallocation of trade across origins (Amiti *et al.* 2019; Fajgelbaum *et al.* 2024). Quantitative trade models show that tariffs and trade barriers can have large aggregate effects once general-equilibrium linkages are taken into account (Baqaee and Farhi 2024). The international macro literature on dominant-currency pricing shows that dollar invoicing weakens the expenditure-switching role of exchange rates and changes the international transmission of shocks (Gopinath *et al.* 2020; Egorov and Mukhin 2023). Our contribution is to bring these mechanisms together in a single short-run, multi-country, multi-sector New Keynesian environment with explicit tariff shocks.

The first result is that the targeted country bears the larger output loss, and production networks shape that loss in a non-uniform way. In the benchmark US–China tariff war, the tariff-imposing country bears more of the short-run inflationary burden because tariff-inclusive import prices enter directly into domestic consumption and intermediate-input costs. The targeted country, by contrast, is hit primarily through the contraction in foreign demand. Input–output linkages then propagate this demand loss through the rest of the production structure. The counterfactuals show that this propagation is not a single mechanism: removing international IO propagation outside the directly tariffed US–China input margin slightly strengthens the Chinese contraction, while also removing domestic propagation substantially attenuates it. The net amplification result is therefore driven mainly by domestic production-network propagation, with wider international linkages partly offsetting the China effect.

The second result is that currency invoicing shapes the magnitude and geography of the adjustment. Relative to producer-currency pricing, the benchmark heterogeneous-invoicing regime delivers a larger Chinese contraction because the renminbi depreciation does less to restore Chinese competitiveness when a sizable share of border prices is sticky in dollars. Invoicing also affects third-country transmission. For the EA, the aggregate response is small on average, but the composition of trade changes across invoicing regimes: the positive diversion margin vis-à-vis the US, the negative margin vis-à-vis China, and the margin vis-à-vis the rest of the world move differently when border-price pass-through is governed by dollar pricing rather than producer-currency pricing.

The third result is that sectoral incidence is a net object. When the US tariffs one Chinese sector at a time, a small set of sectors accounts for most of the aggregate response. But the aggregate contribution of a sector is not the same as what happens inside that sector. In the US, protected sectors often expand on their own, but cross-sector losses elsewhere in the domestic economy offset part of those gains. In China, cross-sector propagation typically reinforces the own-sector contraction. In the EA, domestic propagation is weaker, and the relevant object is the balance between stronger exports to the US and weaker exports to China. Thus, even at the sectoral level, the incidence of tariffs cannot be inferred from direct bilateral exposure alone.

Related literature. The paper first contributes to the literature on production networks and aggregate fluctuations. Seminal contributions by Gabaix (2011), Acemoglu *et al.* (2012), and Baqaee and Farhi (2019, 2022) show that sectoral shocks can have aggregate consequences when firms are linked through input–output networks. More recent work embeds production networks in New Keynesian environments with nominal rigidities, emphasizing how network structure shapes shock propagation and the slope of aggregate Phillips curves (Pasten *et al.* 2020; Rubbo 2023). Our paper is particularly close to the open-economy extension of this agenda. Cakmakli *et al.* (2021) develop a global production-network model to study pandemic-related supply shocks and the economic case for global vaccinations. di Giovanni *et al.* (2022) and di Giovanni *et al.* (2023) use multi-country, multi-sector frameworks with global input–output linkages to study supply-chain pressures, inflation, and international spillovers during the pandemic. Kalemli-Özcan *et al.* (2025) develop a New Keynesian open-economy framework with trade and production networks, sectoral price rigidities, country heterogeneity in monetary policy, and tariff shocks. We build on this open-economy production-network tradition by studying the short-run macroeconomic and sectoral incidence of a US–China tariff war, with particular emphasis on the interaction between IO propagation and heterogeneous currency invoicing.

Second, the paper contributes to open-economy macroeconomics and the literature on currency invoicing. Devereux and Engel (2003) show that the currency of price setting changes exchange-rate pass-through and the role of monetary policy in open economies. Gopinath *et al.* (2020) formalize the dominant-currency paradigm and show that dollar invoicing alters international shock transmission. Egorov and Mukhin (2023) study optimal policy under dollar pricing. We build on this work by showing how invoicing interacts with tariffs and production networks: the invoicing regime determines how much exchange-rate adjustment cushions the targeted economy and how strongly third-country spillovers emerge.

Third, the paper contributes to the literature on tariffs and trade policy. Recent empirical work studies the price and welfare effects of the US tariff episode and the reallocation of trade across countries (Amiti *et al.* 2019; Fajgelbaum *et al.* 2024). Quantitative trade models analyze the welfare and output consequences of trade barriers in environments with rich cross-country linkages (Baqaee and Farhi 2024). We complement this work by focusing on short-run macroeconomic dynamics—inflation, output, trade balances, exchange rates, and sectoral incidence—in a New Keynesian model with production networks and heterogeneous invoicing.

Roadmap. The rest of the paper is organized as follows. Section 2 presents the model. Section 3 describes the calibration. Section 4 studies the aggregate transmission of a US–China tariff war, the role of production networks, the role of invoicing, and robustness to trade elasticities and the exchange-rate regime. Section 5 turns to sectoral incidence, decomposes aggregate responses into own-sector and cross-sector terms, and studies the EA as a third-country net object. Section 6 concludes.

2. Theoretical Model

We extend the theoretical framework in Aguilar *et al.* (2026) to incorporate country- and sector-specific tariffs and currency invoicing.[1] The world economy composed of K countries, indexed by k. The core of our model is a production structure characterized by national and international production networks through input-output (IO) linkages. Namely, each country is comprised of I production sectors, possibly heterogeneous within and between countries. Within each sector, there is a unit mass of monopolistically competitive firms indexed by $f \in (0,1)$ that produce using labor and intermediate goods produced by other domestic and foreign sectors. In addition, we allow for heterogeneous nominal price rigidities at the sectoral and country levels.

[1] The open-economy production-network block is closely related to Cakmakli *et al.* (2021), di Giovanni *et al.* (2022), di Giovanni *et al.* (2023), and Kalemli-Özcan *et al.* (2025), who extend the disaggregated Keynesian production-network framework to multi-country settings with international input–output linkages.

2.1. Households

There is a representative household in each country k that derives utility from consumption and disutility from labor according to the following per-period utility function,

$$U_t = C_{k,t}^{1-\sigma}/(1-\sigma) - \int_0^1 \mathcal{N}_{gk,t}^{1+\varphi}/(1+\varphi)\, dg, \tag{1}$$

which is extended to accommodate nominal wage rigidities (Erceg *et al.* 2000), where $\mathcal{N}_{gk,t}$ denotes the labor supply of labor service g, $C_{k,t}$ denotes aggregate consumption, σ denotes the inverse of the intertemporal elasticity of substitution, and φ denotes the inverse of the Frisch elasticity.[2]

Consumption Aggregators. Aggregate consumption is defined as a constant-returns-to-scale (CRS) composite of sectoral consumption, each of which is itself a CRS aggregation of country-specific consumption goods. This nested structure is summarized by

$$C_{k,t} = \mathcal{C}_k\left(\{C_{ki,t}\}_{i=1}^{I}\right) \quad \text{and} \quad C_{ki,t} = \mathcal{C}_{k,i}\left(\{C_{kl\,i,t}\}_{l=1}^{K}\right), \tag{2}$$

where $C_{ki,t}$ is consumption of sector i at time t, and $C_{kl\,i,t}$ is country k's household's consumption of goods produced by industry i in country l. Given the quantitative relevance of energy sectors in recent periods, we introduce a layer to distinguish between energy and non-energy goods,[3]

$$C_{k,t} = \left[\widetilde{\beta}_k^{\frac{1}{\gamma}} C_{kE,t}^{\frac{\gamma-1}{\gamma}} + \left(1-\widetilde{\beta}_k\right)^{\frac{1}{\gamma}} C_{kM,t}^{\frac{\gamma-1}{\gamma}}\right]^{\frac{\gamma}{\gamma-1}}, \tag{3}$$

where $C_{kE,t}$ and $C_{kM,t}$ denote the consumption of energy and non-energy goods, respectively, with steady-state shares β_k and $(1-\beta_k)$, and γ denotes the elasticity of substitution between energy and non-energy goods. These are given by:

$$C_{kE,t} = \left[\sum_{i\in I_E} \widetilde{\nu}_{ki}^{\frac{1}{\eta}} C_{ki,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}, \qquad C_{kM,t} = \left[\sum_{i\in I_M} \widetilde{\upsilon}_{ki}^{\frac{1}{\iota}} C_{ki,t}^{\frac{\iota-1}{\iota}}\right]^{\frac{\iota}{\iota-1}}, \tag{4}$$

[2] Each household takes as given labor income since wages are set by labor unions and employment is decided by firms. The only decisions made by the household are the optimal allocation of consumption expenditures among different good varieties across different sectors and countries, and the optimal intertemporal allocation of consumption.

[3] This specification allows us to introduce a specific elasticity of substitution for energy consumption that does not necessarily need to be equal to the elasticity of substitution between the rest of goods and services.

where I_E and I_M denote the sets of sectors producing energy and non-energy goods, respectively, $C_{ki,t}$ denotes the consumption of goods from sector i, with steady-state shares ν_{ki} and υ_{ki}, and η and ι denote the elasticity of substitution between goods in energy and non-energy sectors, respectively. The aggregation over sectoral goods produced in different countries is given by CES aggregators:

$$C_{ki,t} = \left[\sum_{l=1}^{K} \tilde{\zeta}_{kl\,i}^{\frac{1}{\delta}} C_{kl\,i,t}^{\frac{\delta-1}{\delta}}\right]^{\frac{\delta}{\delta-1}}, \tag{5}$$

where $C_{kl\,i,t}$ denotes the consumption of goods from sector i in country l, with steady-state shares $\zeta_{kl\,i}$, and δ denotes the Armington (1969) trade elasticity of substitution between goods in sectors in different countries.

Finally, we let $C_{kl\,i,t}$ be a Dixit and Stiglitz (1977) aggregator over differentiated goods produced by firms in sector i in country l:

$$C_{kl\,i,t} = \left(\int_0^1 C_{kl\,if,t}^{(\epsilon_{ki}^{p}-1)/\epsilon_{ki}^{p}} df\right)^{\epsilon_{ki}^{p}/(\epsilon_{ki}^{p}-1)} \tag{6}$$

where ϵ_{ki}^{p} denotes the sectoral constant elasticity of substitution between good varieties.

Intertemporal Household Problem. International financial markets are incomplete, with households in each country only having access to two risk-free bonds. The household in country k has access to a domestic bond $B_{k,t}$, and an internationally traded bond, $B_{k,t}^{K}$, issued, without loss of generality, by country K and denominated in country K's currency. The agent maximizes the present discounted value of per-period utility flows (1), with discount factor β, subject to her budget constraint,

$$\begin{aligned} &P_{k,t}^{C} C_{k,t} + B_{k,t} + B_{k,t}^{K}\left[1-\Gamma(\mathrm{NFA}_{k,t}^{K})\right]^{-1} \mathcal{E}_{kK,t} + \Xi_{k,t} \leq \\ &B_{k,t-1}(1+i_{k,t-1}) + B_{k,t-1}^{K}\mathcal{E}_{kK,t}(1+i_{K,t-1}) + \int_0^1 W_{gk,t}\mathcal{N}_{gk,t}dg + \Pi_{k,t} - T_{k,t} \end{aligned} \tag{7}$$

where $P_{k,t}^{C}$ denotes the consumption price index, $\int_0^1 W_{gk,t}\mathcal{N}_{gk,t}dg$ is the nominal labor income received by the representative household, $\mathrm{NFA}_{k,t}^{K} = B_{k,t}^{K}\mathcal{E}_{kK,t}$ is the net foreign asset position of households in country k, and where $\Gamma(x) = \gamma_{*}\left(\exp\left\{x/\mathcal{Y}_{k,t}\right\}-1\right)$ is an external financial intermediary premium that depends on the economy-wide net holdings of internationally traded foreign bonds as a ratio to the national nominal GDP $\mathcal{Y}_{k,t}$, with $\gamma_* > 0$.[4] The incurred intermediation premium is rebated to households in a lump-sum manner through the fiscal instrument $\Xi_{k,t}$. Finally, $T_{k,t}$ denotes government transfers, also rebated to households in lump sum.

For the allocation of consumption across sectors, countries, and differentiated goods:

$$(8) \qquad \frac{\partial \mathcal{C}_{k,t}}{\partial C_{ki,t}} = \frac{P^C_{ki,t}}{P^C_{k,t}}, \quad \frac{\partial \mathcal{C}_{k,i}}{\partial C_{kl\,i,t}} = \frac{(1+\tau_{kl\,i,t})P^k_{kl\,i,t}}{P^C_{ki,t}}, \quad \text{and} \quad C_{kl\,if,t} = \left(\frac{P^k_{kl\,if,t}}{P^k_{kl\,i,t}}\right)^{-\epsilon^p_{ki}} C_{kl\,i,t},$$

where $P^C_{ki,t}$ denotes the consumer price index of sector i faced by the household in country k, and $P^k_{kl\,i,t} \equiv \left[1 \int_0^1 (P^k_{kl\,if,t})^{(\epsilon^p_{ki}-1)/\epsilon^p_{ki}} df \right]^{\epsilon^p_{ki}/(\epsilon^p_{ki}-1)}$, where $P^m_{kl\,i,t}$ is the price of a good produced in sector i and country l, purchased by country k, in the currency of country m.

Wage-Setting. We follow Erceg *et al.* (2000) and assume that labor unions specialized in any given labor type can reset their nominal wage only with probability $1-\theta^w_k$ each period, independently of the time elapsed since they last adjusted their wage. The labor union chooses the optimal wage to maximize the discounted sum of households' welfare

$$(9) \qquad \max_{W^*_{k,t}} \mathbb{E}_t \sum_{l=0}^{\infty} (\beta\theta^w_k)^l \left(\frac{C^{-\sigma}_{k,t+l} W^*_{k,t} N_{k,t+l|t}}{P^C_{k,t+l}} - \frac{N^{1+\varphi}_{k,t+l|t}}{1+\varphi} \right)$$

subject to the sequence of labor demand schedules

$$(10) \qquad N_{k,t+l|t} = \left(\frac{W^*_{k,t}}{W_{k,t+l}} \right)^{-\epsilon_{wk,t}} \int_0^1 N_{gk,t} dg,$$

where $N_{k,t+l|t}$ denotes the level of employment in period $t+l$ among workers that last reset their wage in period t, and $\epsilon_{wk,t}$ denotes the elasticity of substitution between different labor varieties.

2.2. Firms

There are I industries in each economy, indexed by $i \in \{1, 2, \ldots, I\}$, and within each industry there is a unit mass of firms.

[4]The role of this intermediation premium is to stabilize the net foreign asset position in response to transitory shocks, a common practice in open economies with incomplete financial markets (Schmitt-Grohe and Uribe 2003). Furthermore, this specification guarantees that, in the non-stochastic steady state, households have no incentive to hold foreign bonds and the economy's net foreign asset position is zero.

Production. Each firm f, in sector i and country k, produces a differentiated good $Y_{kif,t}$ via a CRS production function F_{ki} that combines labor $N_{kif,t}$ and a basket of intermediate goods $X_{kif,t}$ as inputs:

$$Y_{kif,t} = F_{ki}(N_{kif,t}, X_{kif,t}). \tag{11}$$

We consider the following CES production function:

$$F_{ki}(N_{kif,t}, X_{kif,t}) = \left[\widetilde{\alpha}_{ki}^{\frac{1}{\psi}} N_{kif,t}^{\frac{\psi-1}{\psi}} + \widetilde{\vartheta}_{ki}^{\frac{1}{\psi}} X_{kif,t}^{\frac{\psi-1}{\psi}}\right]^{\frac{\psi}{\psi-1}}. \tag{12}$$

Both factors are combined under a constant elasticity of substitution ψ; $\widetilde{\alpha}_{ki}$ can be interpreted as a measure of labor-biased demand in production in sector i and country k, and $\widetilde{\vartheta}_{ki}$ can be interpreted as a measure of input-biased demand in production in sector i and country k.

Input Baskets. The bundle of intermediate goods $X_{kif,t}$ is defined similarly to the household's consumption basket, given by a CRS aggregator of sectoral intermediate goods, which are themselves defined as a CRS aggregator of country-specific intermediate goods:

$$X_{kif,t} = \mathcal{X}_{ki}\left(\{X_{kijf,t}\}_{j=1}^{I}\right) \quad \text{and} \quad X_{kijf,t} = \mathcal{X}_{kij}\left(\{X_{kl\,ijf,t}\}_{l=1}^{K}\right), \tag{13}$$

where $X_{kijf,t}$ is firm f's demand in sector i of country k for goods produced in sector j, and $X_{kl\,ijf,t}$ is firm f's demand in sector i of country k for goods produced in sector j in country l. Similarly, we assume a CES structure for the intermediate good aggregators in (13). As on the household side, we introduce an additional layer to distinguish between energy and non-energy goods,

$$X_{ki,t} = \left[\widetilde{\beta}_{ki}^{\frac{1}{\phi}} X_{kiE,t}^{\frac{\phi-1}{\phi}} + (1-\widetilde{\beta}_{ki})^{\frac{1}{\phi}} X_{kiM,t}^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}}, \tag{14}$$

where $X_{kiE,t}$ and $X_{kiM,t}$ denote sector i intermediate goods' demand for energy and non-energy goods, respectively, with steady-state shares β_{ki} and $(1-\beta_{ki})$, and ϕ denotes the elasticity of substitution between energy and non-energy goods. These are given by:

$$X_{kiE,t} = \left[\sum_{j\in I_E} \widetilde{\nu}_{kij}^{\frac{1}{\chi}} X_{kij,t}^{\frac{\chi-1}{\chi}}\right]^{\frac{\chi}{\chi-1}}, \qquad X_{kiM,t} = \left[\sum_{j\in I_M} \widetilde{\upsilon}_{kij}^{\frac{1}{\xi}} X_{kij,t}^{\frac{\xi-1}{\xi}}\right]^{\frac{\xi}{\xi-1}}, \tag{15}$$

where $X_{kij,t}$ denotes the input purchase of goods from sector j, with steady-state shares ν_{kij} and υ_{kij}, and χ and ξ denote the elasticity of substitution between goods in energy and non-energy sectors, respectively. The aggregation over sectoral goods produced in different countries is given by CES aggregators:

$$X_{kij,t} = \left[\sum_{l=1}^{K} \widetilde{\zeta}_{kl\,ij}^{\frac{1}{\mu}} X_{kl\,ij,t}^{\frac{\mu-1}{\mu}}\right]^{\frac{\mu}{\mu-1}}. \tag{16}$$

where $X_{kl\,ij,t}$ denotes the input purchases of goods from sector j in country l, with steady-state shares $\zeta_{kl\,ij}$, and μ denotes the Armington trade elasticity of substitution between goods in sectors in different countries. Finally, each origin–sector input bundle $X_{kl\,ij,t}$ is itself a Dixit-Stiglitz aggregator over differentiated goods produced by firms in sector j in country l:

$$X_{kl\,ij,t} = \left(\int_0^1 X_{kl\,ijff',t}^{(\epsilon_{kj}^p-1)/\epsilon_{kj}^p} df'\right)^{\epsilon_{kj}^p/(\epsilon_{kj}^p-1)} \tag{17}$$

Cost minimization by firms delivers the following first-order conditions for labor and intermediate goods demands:

$$W_{k,t} = \mathrm{MC}_{ki,t}\frac{\partial F_{ki}}{\partial N_{kif,t}}, \quad P_{ij,t}^X = \mathrm{MC}_{ki,t}\frac{\partial F_{ki}}{\partial X_{kijf,t}} \tag{18}$$

and the allocation of intermediate goods demand across sectors and countries:

$$\frac{\partial \mathcal{X}_{ki}}{\partial X_{kij,t}} = \frac{P_{kij,t}^X}{P_{ki,t}^X}, \quad \frac{\partial \mathcal{X}_{kij}}{\partial X_{kl\,ij,t}} = \frac{(1+\tau_{kl\,j,t})P_{kl\,j,t}^k}{P_{kij,t}^X}, \quad X_{kl\,ijff',t} = \left(\frac{P_{kl\,ijff',t}^k}{P_{kl\,j,t}^k}\right)^{-\epsilon_{kj}^p} X_{kl\,ij,t}. \tag{19}$$

Above, $P_{i,t}^X$ denotes the price index of the intermediate input bundle $X_{kif,t}$ faced by firms in sector i in country k, and $P_{ij,t}^X$ is the price index of the sectoral intermediate input $X_{kijf,t}$ faced by firms in sector i in country k. As in the case of households, the prices faced by domestic industries are subject to sectoral tariffs $\tau_{kl\,j,t}$.

Nominal marginal costs in sector i of country k are denoted by $\mathrm{MC}_{ki,t}$. Note that, under CRS, all firms in a given sector choose the same combination of inputs. Hence, $\mathrm{MC}_{ki,t}$ is common across firms and given by:

$$\mathrm{MC}_{ki,t} = \min_{N_{ki,t}, X_{kl\,ij,t}} W_{k,t}N_{ki,t} + \sum_{j=1}^{I}\sum_{l=1}^{K}(1+\tau_{kl\,j,t})P_{kl\,j,t}^k X_{kl\,ij,t} \tag{20}$$

Price Setting. Firms set prices in a staggered manner (Calvo 1983). Specifically, firms in sector i of country k can reset their price with probability $1 - \theta^{p}_{ki}$ each period. We allow for a country– and sector–specific probability of price adjustment.

We consider three pricing regimes for exports: producer currency pricing (PCP), local currency pricing (LCP), and dominant currency pricing (DCP). Under PCP, the firm sets its export price in the domestic currency, and hence the selling price to the domestic and foreign market coincide, and the price of a good in sector i produced in country k and purchased by agents in country l, in currency units of country m, is given by

$$P^{m}_{l\,ki,t} = \mathcal{E}^{k}_{m,t} P_{ki,t}. \tag{21}$$

A firm that is resetting its price chooses a single producer-currency price $\overline{P}_{ki,t}$ for all destinations by solving the following problem:

$$\max_{\overline{P}_{ki,t}} \mathbb{E}_t \sum_{s=0}^{\infty} \mathrm{SDF}_{t,t+s} (\theta^{p}_{ki})^{s} \sum_{l=1}^{K} \left(\overline{P}_{ki,t} - \mathrm{MC}_{ki,t+s} \right) \mathcal{D}_{l\,ki,t+s}, \tag{22}$$

where $\mathrm{SDF}_{t,t+s} = \beta^{s} U'(C_{k,t+s})/U'(C_{k,t})$ is the stochastic discount factor between periods t and $t+s$, and $\mathcal{D}_{l\,ki,t+s}$ denotes demand from agents in destination country l for the firm's good produced in sector i and country k, as implied by equations (8) and (19).

Under LCP, the firm sets its export price to country l in the currency of country l, denoted by $\overline{P}_{l\,ki,t}$,

$$P^{m}_{l\,ki,t} = \mathcal{E}^{l}_{m,t} P_{l\,ki,t}, \tag{23}$$

solving the problem:

$$\max_{\overline{P}_{l\,ki,t}} \mathbb{E}_t \sum_{s=0}^{\infty} \mathrm{SDF}_{t,t+s} (\theta^{p}_{ki})^{s} \left(\mathcal{E}^{l}_{k,t+s} \overline{P}_{l\,ki,t} - \mathrm{MC}_{ki,t+s} \right) \mathcal{D}_{l\,ki,t+s}, \tag{24}$$

where $\mathcal{D}_{l\,ki,t+s}$ is the demand from agents from country l for the firm's production.

Finally, under DCP, the firm sets its export price to country l in US dollars, denoted by $\overline{P}^{\mathrm{US}}_{ki,t}$,

$$P^{m}_{l\,ki,t} = \mathcal{E}^{\mathrm{US}}_{m,t} P^{\mathrm{US}}_{ki,t}, \tag{25}$$

solving the following problem:

$$\max_{\overline{P}^{\text{US}}_{ki,t}} \mathbb{E}_t \sum_{s=0}^{\infty} \text{SDF}_{t,t+s}(\theta^p_{ki})^s \left(\mathcal{E}^{\text{US}}_{k,t+s}\overline{P}^{\text{US}}_{ki,t} - \text{MC}_{ki,t+s}\right)\mathcal{D}_{ki,t+s}. \tag{26}$$

where $\mathcal{D}_{ki,t+s}$ is the demand from agents from country l for the firm's production.

Profits in sector i of country k are given by export- and domestic-market revenues net of costs, $\Pi_{ki,t} = \sum_{l=1}^{K} P^m_{lki,t} D_{lki,t} - \text{MC}_{ki,t} Y_{ki,t}$, and aggregate profits rebated to households are $\Pi_{k,t} = \sum_{i=1}^{I} \Pi_{ki,t}$.

2.3. Government

The government is composed of a fiscal authority and a central bank. The fiscal authority issues domestic debt $B_{k,t}$, provides production subsidies to firms, and collects lump-sum taxes $T_{k,t}$ from the household in order to balance its budget constraint:

$$\frac{B_{k,t}}{1+i_{k,t}} + T_{k,t} + \sum_{l\neq k}^{K}\sum_{i=1}^{I}\left(\tau_{kli,t}P^k_{kli,t}C_{kli,t} + \sum_{j=1}^{I}\tau_{kli,t}P^k_{kli,t}X_{klji,t}\right) = B_{k,t-1} + \sum_{i=1}^{I}\tau_{k,i}\text{MC}_{k,i,t}Y_{k,i,t} \tag{27}$$

where $Y_{k,i,t}$ denotes the production of sector i in country k.

2.4. Monetary Authority

There is a monetary authority in each country $k \in K$, and the central bank sets the nominal interest rate, $i_{k,t}$. We assume that each central bank follows a Taylor rule:

$$i_{k,t} = \rho_{kr} i_{k,t-1} + (1-\rho_{kr})\left(\phi_{k\pi}\pi_{k\phi,t} + \phi_{ky}\widehat{y}_{k,t} + \phi_{k\Delta\pi}\Delta\pi_{k\phi,t} + \phi_{k\Delta y}\Delta y_{k,t}\right) + \varepsilon^r_{k,t} \tag{28}$$

where the coefficients are allowed to vary by country; ρ_{kr} denotes the degree of interest rate smoothing in the monetary instrument, coefficients $\{\phi_{k\pi}, \phi_{ky}\}$ modulate the elasticity of the policy rate with respect to changes in a given inflation index $\pi_{k\phi,t}$, and output $\widehat{y}_{k,t}$, measured as the log-deviation from its steady-state value; and coefficients $\{\phi_{k\Delta\pi}, \phi_{k\Delta y}\}$ modulate the elasticity of the policy rate with respect to changes in a given inflation index growth $\Delta\pi_{k\phi,t} = \pi_{k\phi,t} - \pi_{k\phi,t-1}$, and output $\Delta\widehat{y}_{k,t} = \widehat{y}_{k,t} - \widehat{y}_{k,t-1}$. The Taylor rule features a monetary policy shock, which follows $\varepsilon^r_{k,t} \sim \mathcal{N}\left(0, (\sigma_{kr})^2\right)$.

Furthermore, we allow the monetary authority to choose the particular inflation measure (CPI, PCE, PPI, GDP deflator, etc.) that they aim to stabilize, $\pi_{k\phi,t} = \boldsymbol{\Phi}^\top\boldsymbol{\pi}_{k,t} = \sum_{l=1}^{K}\sum_{i=1}^{I}\phi_{kli}\pi_{kli,t}$, where $\sum_{l=1}^{K}\sum_{i=1}^{I}\phi_{kli} = 1$, and $\pi_{kki,t} = \pi_{ki,t}$. For example, when ϕ_{kli} is equal to the consumption share of sector i in the country k, then the central bank targets headline inflation.

2.5. Market Clearing and GDP

Market clearing in the goods market requires that the quantity produced of each good matches the quantity demanded at home and abroad, either for direct consumption or intermediate use. That is,

$$Y_{ki,t} = \sum_{l=1}^{K} C_{lki,t} + \sum_{l=1}^{K}\sum_{j=1}^{I} X_{lkji,t}. \tag{29}$$

Market clearing in the labor market requires that the aggregate labor supplied matches the sum of labor demand across sectors, for each country. That is,

$$N_{k,t} = \sum_{i=1}^{I} N_{ki,t}. \tag{30}$$

Finally, the aggregate resource constraint of the economy requires that the net foreign asset position of country k evolves with the trade balance:

$$\mathrm{NFA}^{K}_{k,t} - (1 + i_{K,t-1})\frac{\mathcal{E}_{kK,t}}{\mathcal{E}_{kK,t-1}}\mathrm{NFA}^{K}_{k,t-1} = P_{k\mathrm{EXP},t}\mathrm{EXP}_{k,t} - P_{k\mathrm{IMP},t}\mathrm{IMP}_{k,t} \tag{31}$$

where $\mathrm{NFA}^{K}_{k,t}$ is denominated in country k's currency and reflects holdings of the internationally traded bond issued by country K. The first term in the right-hand side of (31) is the total nominal exports of country k,

$$P_{k\mathrm{EXP},t}\mathrm{EXP}_{k,t} = \sum_{l\neq k}^{K}\sum_{i=1}^{I}\left(P^{k}_{lki,t}C_{lki,t} + \sum_{j=1}^{I} P^{k}_{lki,t}X_{lkji,t}\right), \tag{32}$$

and the second term is the total nominal imports,

$$P_{k\mathrm{IMP},t}\mathrm{IMP}_{k,t} = \sum_{l\neq k}^{K}\sum_{i\in I}\left(P^{k}_{kli,t}C_{kli,t} + \sum_{j=1}^{I} P^{k}_{kli,t}X_{klji,t}\right). \tag{33}$$

Nominal GDP is defined as the sum of total household consumption and nominal net exports,

$$\mathcal{Y}_{k,t} = P_{kC,t}C_{k,t} + P_{k\mathrm{EXP},t}\mathrm{EXP}_{k,t} - P_{k\mathrm{IMP},t}\mathrm{IMP}_{k,t} \tag{34}$$

Defining the GDP deflator, $P_{kY,t}$, as the ratio between nominal GDP measured using time–t prices and nominal GDP measured using steady-state prices

$$P_{kY,t} = \frac{P_{kC,t}C_{k,t} + P_{k\mathrm{EXP},t}\mathrm{EXP}_{k,t} - P_{k\mathrm{IMP},t}\mathrm{IMP}_{k,t}}{P_{kC}C_{k,t} + P_{k\mathrm{EXP}}\mathrm{EXP}_{k,t} - P_{k\mathrm{IMP}}\mathrm{IMP}_{k,t}},$$

we have that real GDP is given by:

$$Y_{k,t} = \mathcal{Y}_{k,t}/P_{kY,t}. \tag{35}$$

2.6. Tariffs

We are interested in analyzing the propagation and consequences of tariffs in the economy. We assume that the (log-)tariff, introduced in (8), follows an AR(1) process:

$$\tau_{lkj,t} = \rho^{\tau}_{lkj}\tau_{lkj,t-1} + \varepsilon^{\tau}_{lkj,t} \tag{36}$$

where $\varepsilon^{\tau}_{lkj,t} \sim \mathcal{N}\left(0, (\sigma^{\tau}_{lkj})^2\right)$. The three subscripts allow for origin-, destination-, and sector-specific tariff shocks.

3. Calibration

We calibrate the model economy presented in Section 2 at the quarterly frequency to $K = 4$ countries: the euro area, the United States, China, and the Rest of the World (ROW). The production structure within each country contains $I = 44$ sectors.[5] We next discuss the calibration strategy and collect in Table 1 the main parameter values and the corresponding targets or sources.

Households. We set the household discount factor β to 0.99, a standard value implying an annualized real interest rate of approximately 4%. The intertemporal elasticity of substitution σ is set to 1, a common value in the literature. The inverse of the Frisch elasticity φ is set to 1, in line with the estimates presented in Chetty *et al.* (2011). The household borrowing

[5]The 44 sectors follow the NACE Rev. 2 two-digit classification, covering 3 energy, 21 services, and 20 tradeable manufacturing sectors. A complete list of the sectors included can be found in Appendix B, Table 1.

Parameter	Description	Value	Target / Source
Households			
β	Discount factor	0.99	$R \approx 4\%$ p.a.
σ	Inv. Intertemp. Elast. Subs.	1	Standard value
φ	Inv. Frisch Elasticity	1	Chetty *et al.* (2011)
γ	Elast. Subst. E and M	0.4	Böhringer and Rivers (2021)
η	Elast. Subst. E	0.9	Atalay (2017)
ι	Elast. Subst. M	0.9	Atalay (2017)
δ	Trade Elasticity	2	Boehm *et al.* (2023)
$\{\widetilde{\beta}_k, \widetilde{\nu}_{ki}, \widetilde{\upsilon}_{ki}, \widetilde{\zeta}_{kl\,i}\}$	Quasi-shares consumption		ICIO tables (OECD)
θ^w_k	Calvo wage prob.	0.75	Christoffel *et al.* (2008)
Firms			
ψ	Elast. Subst. N and X	0.5	Atalay (2017)
ϕ	Elast. Subst. E and M	0.4	Böhringer and Rivers (2021)
χ	Elast. Subst. E	0.2	Atalay (2017)
ξ	Elast. Subst. M	0.2	Atalay (2017)
μ	Trade Elasticity	2	Boehm *et al.* (2023)
$\{\widetilde{\alpha}_{ki}, \widetilde{\vartheta}_{ki}, \widetilde{\beta}_{ki}, \widetilde{\nu}_{kij}, \widetilde{\upsilon}_{kij}, \widetilde{\zeta}_{kij}\}$	Quasi-shares production		ICIO tables (OECD)
$\mathcal{M}_{ki}$	Markups		Labor shares (Eurostat)
θ^p_{ki}	Calvo price prob.		Gautier *et al.* (2024)
Monetary Policy			
$\rho_{k,r}$	Interest Rate Smoothing	0.7	Standard value
$\phi_{k,\pi}$	Reaction to Inflation	1.5	Galí (2015)
$\phi_{k,\Delta\pi}$	Reaction to Inflation	0	Galí (2015)
$\phi_{k,y}$	Reaction to GDP growth	0	Standard value
$\phi_{k,\Delta y}$	Reaction to GDP growth	0.125	Standard value
Exogenous Shock Process			
$\rho^\tau_{kl\,i}$	Persistence Tariff	0.96	(–)
$\sigma^\tau_{kl\,i}$	Std. Dev. tariff shock	1	Standard value
σ^r_k	Std. Dev. monetary shock	1	Standard value

Notes: List of calibrated parameters. See the main text for a discussion of targets, values, and data used.

TABLE 1. Calibration

premium γ_* is set to 0.001 so that the evolution of net foreign assets has only a small impact on the exchange rate and trade in the short run while guaranteeing that the net foreign asset position is stabilized at zero in the long run (Schmitt-Grohe and Uribe 2003).

The elasticity of substitution in consumption between energy and non-energy goods γ is set to 0.4 following Böhringer and Rivers (2021). The elasticity of substitution in consumption between energy sources η and between non-energy sectors ι is set to 0.9 following Atalay (2017). The household trade elasticity δ is set to 2 in the benchmark calibration.[6]

To calibrate the quasi-consumption shares $\{\widetilde{\beta}_k, \widetilde{\nu}_{ki}, \widetilde{\upsilon}_{ki}, \widetilde{\zeta}_{kl\,i}\}$, we rely on the linearized model to target the respective sectoral consumption shares in each country. More precisely, in Appendix A we show that once the model has been linearized, it is possible to read consumption shares directly from the data as long as we have as many quasi-consumption shares parameters as data targets. Implementing this strategy, we obtain consumption shares by country from Inter-Country Input-Output (ICIO) tables produced by the OECD, using 2019 as our baseline period.

Regarding wage rigidities, ECB (2009) report limited cross-sectoral heterogeneity in wage frequency adjustments for euro area countries. Therefore, we fix the Calvo frequency wage adjustment probability θ_k^w to 0.75 for all countries, in line with the evidence presented in Christoffel *et al.* (2008) for the EA.

Production. The elasticity of substitution in production between labor and intermediate inputs ψ is set to 0.5 (Atalay 2017). The elasticity of substitution in production between energy and non-energy goods ϕ is set to 0.4 (Böhringer and Rivers 2021). The elasticity of substitution in production between energy sectors χ and between non-energy sectors ξ is set to 0.2, following the estimates of Atalay (2017). Finally, as with households, we set the trade elasticity for firms, μ, equal to 2 in the benchmark.

We follow the same strategy as with households to calibrate the quasi-shares in production $\{\widetilde{\alpha}_{ki}, \widetilde{\vartheta}_{ki}, \widetilde{\beta}_{ki}, \widetilde{\nu}_{kij}, \widetilde{\upsilon}_{kij}, \widetilde{\zeta}_{kij}\}$. Namely, using the linearized model around the steady state, we read from the data the share of each intermediate good in production as well as the shares of labor and intermediate inputs in total costs. Our data source here again is the 2019 ICIO tables from the OECD.

We complement the ICIO tables with the FIGARO database by Eurostat to calibrate the labor share of each industry. Namely, once the quasi-shares in production have been used, we calibrate the sector-specific markups $\mathcal{M}_{ki}$ to target the wage-bill-over-sales observed in the data.

Sectoral price rigidities are obtained from the PRISMA project conducted by the ECB (Gautier *et al.* 2024). Using CPI micro-data from several EA countries, the authors report the frequency of price adjustment by COICOP category for each country separately and for the

[6] A growing body of literature has estimated the value of these elasticities for different time horizons, finding that the values of trade elasticities are significantly greater than one in the long term but not in the short term, with values around 1 for horizons of up to two years (Boehm *et al.* 2023). A sensitivity analysis is conducted in Section 4.4.

aggregate EA. Using the COICOP–to–NACE correspondence tables (Kouvavas *et al.* 2021), we compute the frequency of price adjustment by each NACE category in each country, and obtain the heterogeneous price rigidities θ^{p}_{ki} for the euro area and similarly for the United States. Finally, we assume that Chinese and ROW price rigidities coincide with those of the US. This is a data-driven constraint: sector-level price adjustment frequencies comparable to the Gautier *et al.* (2024) PRISMA data are not available for China or the rest of the world. To the extent that Chinese manufacturing sectors adjust prices more frequently than their US counterparts, our calibration may overstate the degree of nominal rigidity in China and thereby understate the speed of price adjustment to the tariff shock.

A drawback of the evidence presented in Gautier *et al.* (2024) is that it does not contain consistent price adjustment frequency data on energy goods. Therefore, we complement this with the evidence presented in Dhyne *et al.* (2006) on price adjustments for energy goods in euro area countries. In line with the data presented there, and not surprisingly, energy sectors in the model have the steepest price Phillips curves, with nearly fully flexible prices.

Invoicing. We consider three invoicing regimes. We classify each sector into one of three invoicing regimes: producer currency pricing (PCP), local currency pricing (LCP), or dominant currency pricing (DCP); based on observed invoicing patterns (Egorov and Mukhin 2023; Gopinath *et al.* 2020): 7 sectors invoice in producer currency (PCP), 20 in local currency (LCP), and 17 in dominant (dollar) currency (DCP).[7] Since the majority of tradeable manufacturing sectors fall under DCP, the benchmark economy is substantially tilted toward dollar invoicing, which is the regime in which exchange-rate adjustment provides the least insulation.

Monetary Policy. All Taylor rule parameters are set to standard values, and are homogeneous across countries.[8] The interest-rate smoothing coefficient $\rho_{k,r}$ is set to 0.7. The coefficients for inflation and output growth, $\phi_{k,\pi}$ and $\phi_{k,\Delta y}$, are set to their standard values of 1.5 and 0.125, respectively. Furthermore, we assume that central banks target headline inflation.

Exogenous Processes. The tariff shock follows an exogenous AR(1) process with persistence $\rho^{\tau}_{kl\,i} = 0.96$. As reported in Table 1, the innovation standard deviations of the tariff and monetary-policy shocks, $\sigma^{\tau}_{kl\,i}$ and σ^{r}_{k}, are normalized to 1.

[7]The invoicing classification is sector-specific: all trade in a given sector (both intermediate and final goods) follows the same invoicing regime. PCP sectors include utilities and public services; DCP sectors are concentrated in primary commodities, basic manufacturing, and transport; LCP sectors cover food processing, pharmaceuticals, machinery, and most services.

[8]Section 4.4 explores the case in which the central bank of China operates a managed float with capital controls, and the ECB's smoothing parameter likely differs from the Fed's.

4. Aggregate transmission of a US–China tariff war

This section studies the aggregate transmission of a US–China tariff war in the model. We focus on a reciprocal 10 percentage-point tariff increase between the US and China across the 20 tradeable manufacturing sectors, and use that experiment to organize the discussion around three questions. First, how are the aggregate effects distributed across the two countries directly involved in the trade war? Second, through which model mechanisms do those direct effects propagate and amplify? Third, how are those shocks transmitted to a third economy, namely the EA, which is not directly targeted by either tariff measure?

The benchmark simulation suggests three broad facts. The targeted country experiences the larger output contraction, while the tariff-imposing country bears more of the short-run inflationary burden. The EA response remains small in the aggregate, but only because several offsetting margins operate simultaneously. In the model, these outcomes are shaped by direct expenditure switching, propagation through production networks, and the invoicing structure.

The rest of the section is organized accordingly. Section 4.1 presents the benchmark aggregate dynamics. Section 4.2 then isolates the main mechanisms behind those dynamics, focusing on direct incidence and expenditure switching, production-network amplification, and currency invoicing. Section 4.3 turns to third-country transmission and interprets the EA response as a net object driven by offsetting bilateral and multilateral margins. Finally, Section 4.4 reports additional robustness exercises on the trade elasticity and the exchange-rate regime.

4.1. Benchmark aggregate dynamics

We begin with the benchmark experiment: a reciprocal 10 percentage-point tariff increase between the US and China across the 20 tradeable manufacturing sectors in the model. Figure 1 reports the responses of CPI inflation, real GDP, consumption, the trade balance, and the real effective exchange rate (REER) in the US, China, and the EA, separating the US tariff on Chinese imports (solid blue) from the Chinese retaliation on US imports (dashed dark red).[9] In the benchmark calibration, the tariff process is persistent and decays only gradually over the 12-quarter horizon.

Two aggregate asymmetries stand out. First, the targeted country experiences the larger output contraction, while the tariff-imposing country bears more of the short-run inflationary burden. Second, the EA response is small in the aggregate, but this small net effect reflects

[9] CPI inflation is reported as year-on-year inflation, $\pi^{C,\text{yoy}}_{k,t} \equiv \sum_{s=0}^{3} \pi^{C}_{k,t-s}$. The trade balance is reported in pp of GDP. Real GDP, consumption, and the REER are expressed as percentage deviations from steady state.

offsetting trade and production margins rather than insulation from the shock. These patterns are consistent with the broader literature on tariff incidence and international transmission. Empirically, recent evidence for the US tariff episode points to substantial pass-through of tariffs into US import prices and costs faced by US buyers, rather than to large offsetting cuts in exporters' pre-tariff prices (Amiti *et al.* 2019). In quantitative trade models, tariffs likewise reallocate expenditure across origins while compressing demand in the targeted economy, with the aggregate incidence depending on general-equilibrium feedbacks and cross-sector linkages (Baqaee and Farhi 2024).

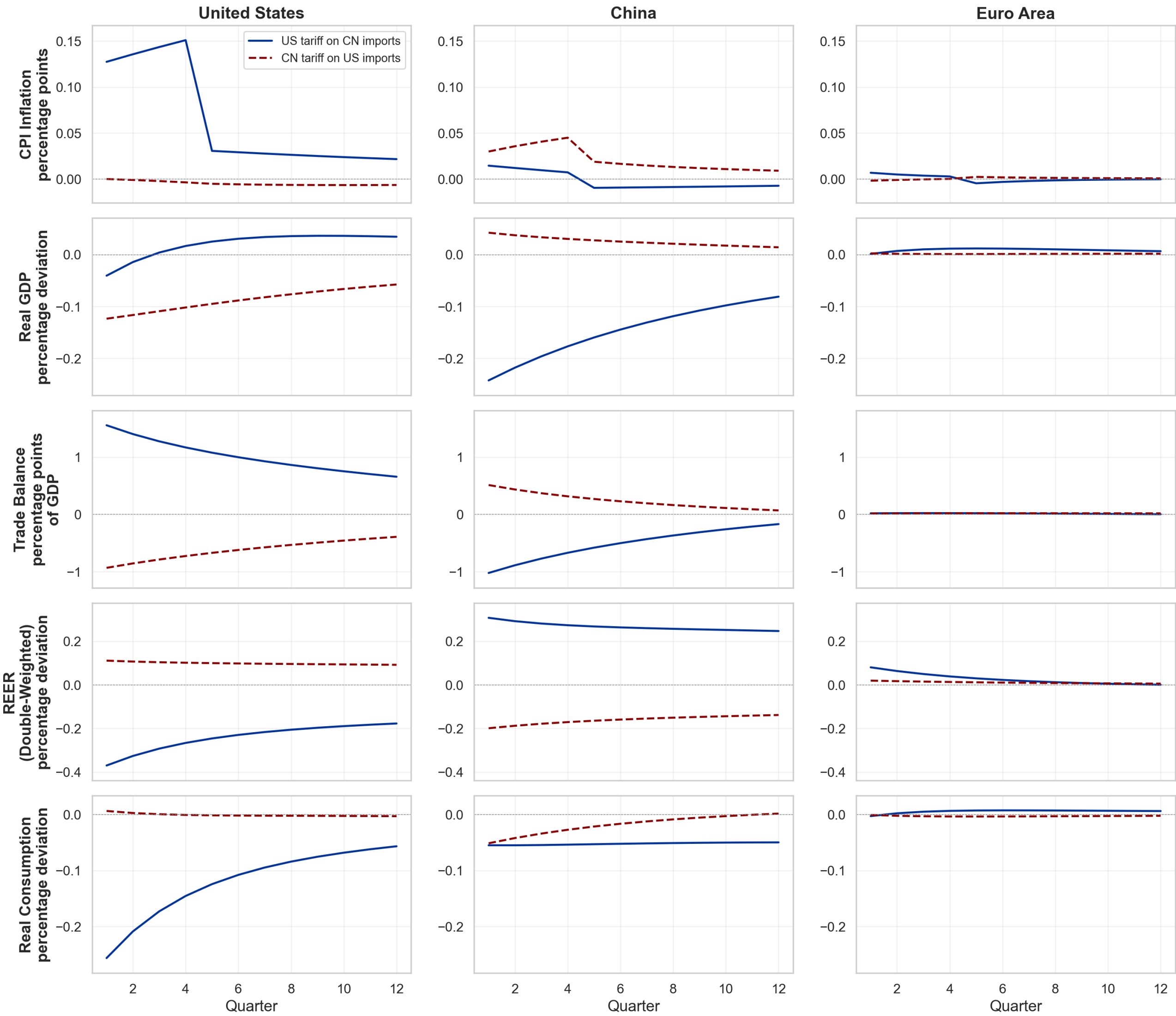
4.1.1. Direct incidence in the US and China

The benchmark responses of the US and China are best understood through the distinction between the *price incidence* of the tariff and the *quantity incidence* of the foreign-demand contraction. When the US imposes a tariff on Chinese goods, the tariff enters directly into the domestic price of imported final and intermediate goods purchased in the US. This raises US inflation on impact. China, by contrast, is hit through lower external demand. The reverse logic applies under the Chinese retaliation.

This distinction is visible in the CPI panel of Figure 1. Following the US tariff on China, US CPI inflation rises by about 0.13 pp on impact and remains positive for several quarters, whereas the Chinese inflation response is an order of magnitude smaller. Under the Chinese tariff on US imports, the sign pattern reverses: Chinese CPI rises on impact because Chinese buyers now face tariff-inclusive prices on US imports, while the US inflation response becomes close to zero and slightly negative later in the horizon. The basic mechanism is therefore straightforward: the country levying the tariff absorbs the immediate import-price shock, whereas the targeted country absorbs it only indirectly through exchange-rate movements and general-equilibrium feedback. This asymmetry is in line with the evidence in Amiti *et al.* (2019), who document that the burden of the recent US tariffs fell mainly on US importers and consumers rather than on foreign exporters, and with the mechanisms emphasized by Kalemli-Özcan *et al.* (2025): in a New Keynesian open economy with production networks, tariff shocks operate simultaneously through demand, cost, monetary-policy, exchange-rate, and input–output channels, so that their macroeconomic incidence is not pinned down by bilateral trade shares alone.

The GDP responses point to the complementary quantity margin. Under the US tariff on China, US GDP falls only modestly on impact, by about 0.04%, and then recovers, while Chinese GDP contracts much more sharply, by about 0.24%, and remains below steady state throughout the horizon shown in Figure 1. Under the Chinese retaliation, the same pattern applies in reverse. In both directions, the targeted country is the one that absorbs the larger

Figure 1: Benchmark Macroeconomic Dynamics



Notes: 12-quarter aggregate responses under reciprocal 10 pp US–China tariffs across the 20 tradeable manufacturing sectors. Solid blue lines are the US tariff on Chinese imports; dashed dark red lines are the Chinese tariff on US imports. The CPI panel reports the rolling four-quarter sum of CPI inflation in pp; the trade balance is in pp of GDP; real GDP, consumption, and the REER are percentage deviations from steady state.

output loss. In the model, this is the natural implication of market clearing and expenditure switching. The foreign tariff compresses demand for the targeted country's exports, while the tariff-imposing country partly offsets the decline in real activity through substitution toward domestic goods and toward imports from non-targeted origins. This logic is standard in Armington environments and in recent multi-sector trade models with tariffs (Armington 1969; Baqaee and Farhi 2024).

A second feature of the benchmark is that GDP and consumption need not move together in the tariff-imposing country. Under the US tariff on China, US consumption declines much more than US GDP (around 6 times larger fall on impact), reflecting the fact that household absorption is hit directly by higher tariff-inclusive import prices while GDP is partly cushioned by the improvement in net exports. The reason is the expenditure-side GDP decomposition (see equation A.97 in Appendix A), which follows from the open-economy national income identity in the model. Consumption is directly exposed to the increase in tariff-inclusive import prices through the consumption price index, whereas GDP is partly cushioned by the improvement in net exports as tariffed imports fall. Tariff revenue is also rebated lump-sum to households through the fiscal authority. This transfer relaxes the household budget constraint and mitigates the consumption loss. In the benchmark, the tariff revenue rebated to US households amounts to approximately 0.2–0.3 percent of GDP on impact, so the muted US GDP response reflects the combination of import compression, expenditure switching, and fiscal rebates that prevent household absorption from falling even further.

The trade-balance and real effective exchange rate (REER) panels complete the picture.[10] Under the US tariff on China, the US trade balance improves and the US REER appreciates on impact (-0.37 percent deviation from steady state), while China experiences a trade-balance deterioration and a REER depreciation. Under the Chinese retaliation, the pattern reverses. In the model, these relative-price movements support the expenditure-switching margin: the tariff-imposing country compresses imports and shifts demand away from tariffed goods, while the targeted country undergoes a deterioration in foreign demand. The appreciation of the imposing country's currency is also consistent with the standard open-economy prediction that import restrictions strengthen the relative price of domestic goods in the short run when nominal rigidities limit the speed of adjustment.

Taken together, the benchmark US–China responses reflect a clean division of incidence. The tariff-imposing country bears more of the short-run inflationary burden because the tariff directly raises domestic import prices. The targeted country bears more of the contractionary burden because the tariff destroys external demand. The quantitative importance of that asymmetry is then shaped by the two mechanisms we study below: production-network propagation and the invoicing structure.

4.1.2. Third-country transmission in the EA

The EA is not directly involved in the trade war: it neither imposes the tariff nor is it directly targeted by one. Its response is therefore entirely indirect. Yet the near-zero movement of EA GDP in Figure 1 should not be interpreted as evidence that the trade war has little effect on third countries. Rather, the EA response reflects several offsetting forces.

On the one hand, US demand is reallocated away from Chinese goods and toward alternative suppliers, including EA producers. On the other hand, weaker Chinese activity reduces

[10] We compute the REER as the double-weighted average of bilateral real exchange rates.

Chinese demand for EA exports, while changes in global relative prices and imported input costs feed back into EA production and inflation. These margins largely offset one another in the benchmark. Following the US tariff on China, EA inflation remains close to flat and EA GDP stays near zero on impact, before turning mildly positive at short horizons. Under the Chinese retaliation, the EA again exhibits only a muted aggregate response.

The relevant point is therefore not that the EA effect is small, but that it is small only after offsetting gross reallocations have been netted out in equilibrium. Section 4.3 returns to this third-country case and shows more directly how trade diversion, weaker Chinese demand, production networks, and the invoicing structure interact.

4.2. Mechanisms behind the benchmark

The benchmark responses in Figure 1 reflect three mechanisms in the model. The first is the direct incidence of the tariff on domestic import prices and the associated expenditure-switching margin. The second is the propagation of the shock through domestic and international input–output linkages. The third is the invoicing structure, which governs how exchange-rate movements map into border prices and therefore into expenditure switching, imported inflation, and third-country spillovers. We discuss each mechanism in turn.

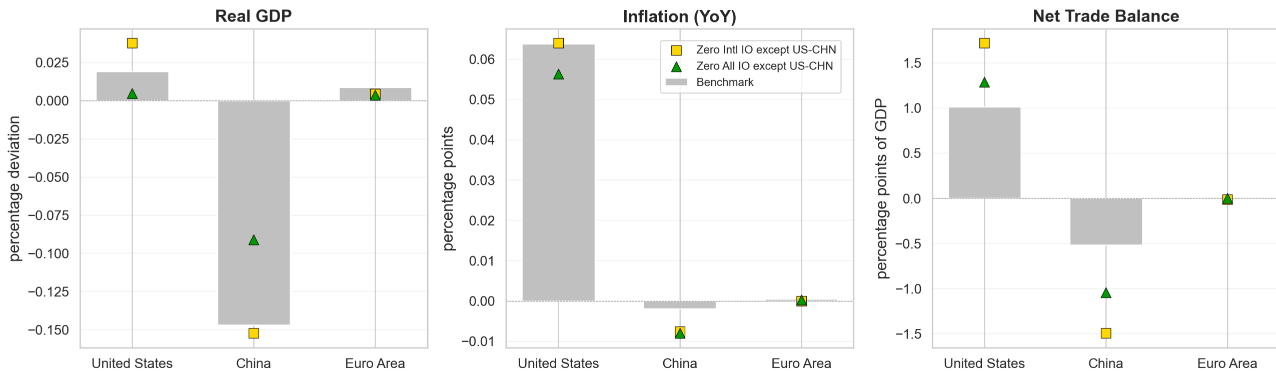
4.2.1. Direct incidence and expenditure switching

A tariff is a wedge between the border price received by the exporter and the domestic-currency price paid by the importer. In the model, that wedge enters both household consumption demand and firms' intermediate-input demand. The immediate consequence is that the country imposing the tariff faces a rise in the domestic relative price of imported goods, while the targeted country is hit primarily through weaker foreign demand.

Two margins then shape the aggregate allocation. First, households and firms substitute away from tariffed imports toward domestic goods and toward imports from alternative origins. Second, the targeted economy experiences a contraction in exports, production, and income. These two forces are enough to explain the benchmark asymmetry: the tariff-imposing country bears more of the short-run inflationary burden, while the targeted country bears the larger output loss. In that sense, the benchmark follows the standard incidence logic of recent empirical and quantitative work on tariffs (Amiti *et al.* 2019; Armington 1969; Baqaee and Farhi 2024).

What matters for the rest of the section is that expenditure switching operates not only in final demand, but also through intermediate-input demand. Once imported inputs become more expensive, the tariff changes domestic marginal costs as well as trade shares. That is the margin through which production networks amplify the benchmark responses.

Figure 2: Production-Network Counterfactuals



Notes: three-year average production-network counterfactual responses under a 10 pp US tariff on Chinese imports across the 20 tradeable manufacturing sectors. Real GDP and net trade balance are averages; inflation is the average of year-on-year CPI inflation. Gray bars: benchmark; dots: zero international IO except US–China and zero all IO except US–China, both preserving directly tariffed US–China intermediate-input links.

4.2.2. Production-network amplification

The benchmark output responses cannot be understood from bilateral trade reallocation alone. In the model, imported goods are used as intermediate inputs throughout the production structure, and sectors are connected through domestic and international input–output linkages. As a result, the tariff shock propagates beyond the directly exposed tradeable sectors. A tariff that raises the domestic price of imported intermediates increases marginal costs in downstream sectors; a contraction in export demand for a targeted sector reduces intermediate-input demand for its suppliers. The shock therefore travels both downstream through costs and upstream through demand.

This propagation mechanism is summarized by the open-economy marginal-cost equation in the log-linearized model (see equation A.73 in Appendix A), where sectoral marginal costs depend on domestic wages and the prices of intermediate goods sourced across countries and sectors. The economic implication is that the direct trade shock is transformed into a broader aggregate disturbance through the production network. This is closely related to the broader literature showing that network structure amplifies the aggregate effects of sectoral distortions and relative-price wedges (Acemoglu *et al.* 2012; Baqaee and Farhi 2024). In New Keynesian environments, the interaction of those networks with nominal rigidities is particularly important because part of the cost adjustment takes place through quantities rather than being absorbed immediately by prices (Pasten *et al.* 2020; Rubbo 2023).

Figure 2 isolates this mechanism while holding fixed the directly tariffed US–China bilateral input margin. We compare the benchmark with two counterfactuals. In the first, we remove all international IO linkages except the direct US–China links affected by the tariff. In the second, we additionally remove domestic IO linkages, again preserving the direct US–China bilateral links. The remaining responses therefore retain the direct incidence of the tariff on final demand and on directly tariffed US–China intermediates, while the attenuation relative to the benchmark measures the amplification generated by propagation through the wider production network.

Relative to the benchmark, shutting down international IO propagation outside the directly tariffed US–China bilateral links marginally strengthens the Chinese contraction, while removing domestic propagation as well attenuates it substantially. This implies that the large Chinese response is not driven only by the direct tariff incidence on US–China trade in intermediates. A sizable part arises because the shock propagates through the broader domestic and international production network. The same exercise also reduces the responses of the US and the EA, confirming that the wider network amplifies the tariff shock on both the cost and demand margins.

The inflation responses point in the same direction. Once the propagation margins outside the directly tariffed bilateral US–China links are removed, average US CPI inflation falls, the Chinese inflation response moves closer to zero, and the EA response becomes even more muted. Without those broader propagation channels, the tariff still changes bilateral relative prices, but the downstream cost cascade becomes weaker.

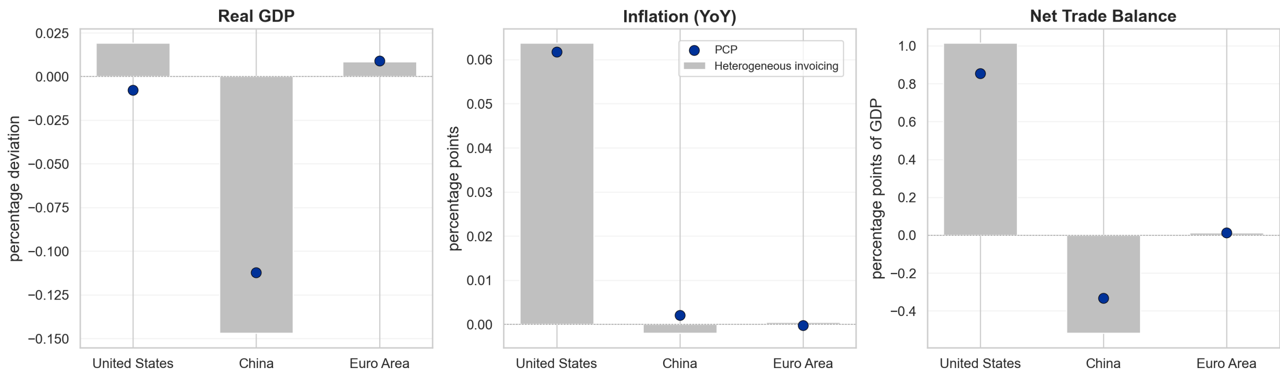
For the interpretation of the benchmark, production networks are not a peripheral robustness dimension. They are one of the main reasons the targeted economy contracts as much as it does. In particular, they show that the Chinese response is not simply the direct consequence of weaker exports to the US. It is that direct export-demand shock amplified by propagation through the wider production network.

4.2.3. Currency invoicing and exchange-rate pass-through

The third mechanism is the invoicing structure. The currency in which goods are invoiced determines the currency in which border prices are sticky, and therefore determines how much of any exchange-rate movement is relevant for expenditure switching at short horizons. This point is central in the dominant-currency literature. When prices are sticky in dollars, exchange-rate movements between non-US currencies need not pass through to border prices in the way implied by producer-currency pricing, and the international transmission of shocks changes accordingly (Gopinath *et al.* 2020; Egorov and Mukhin 2023).

In our model, the distinction is sharpest for Chinese exports under the US tariff. Under producer-currency pricing, a renminbi depreciation lowers Chinese export prices in foreign currency and cushions the decline in foreign demand. Under dominant-currency pricing, by contrast, dollar-invoiced prices remain sticky in dollars within the Calvo contract duration, so the depreciation does not generate the same expenditure-switching support. This mechanism is captured by the DCP Phillips curve in Appendix A; see equation (A.70). The practical

Figure 3: Invoicing Regime Average Responses



Notes: three-year average invoicing-regime counterfactual responses under a 10 pp US tariff on Chinese imports across the 20 tradeable manufacturing sectors. Real GDP and net trade balance are averages; inflation is the average of year-on-year CPI inflation. Gray bars: heterogeneous invoicing; blue dots: producer-currency pricing (PCP).

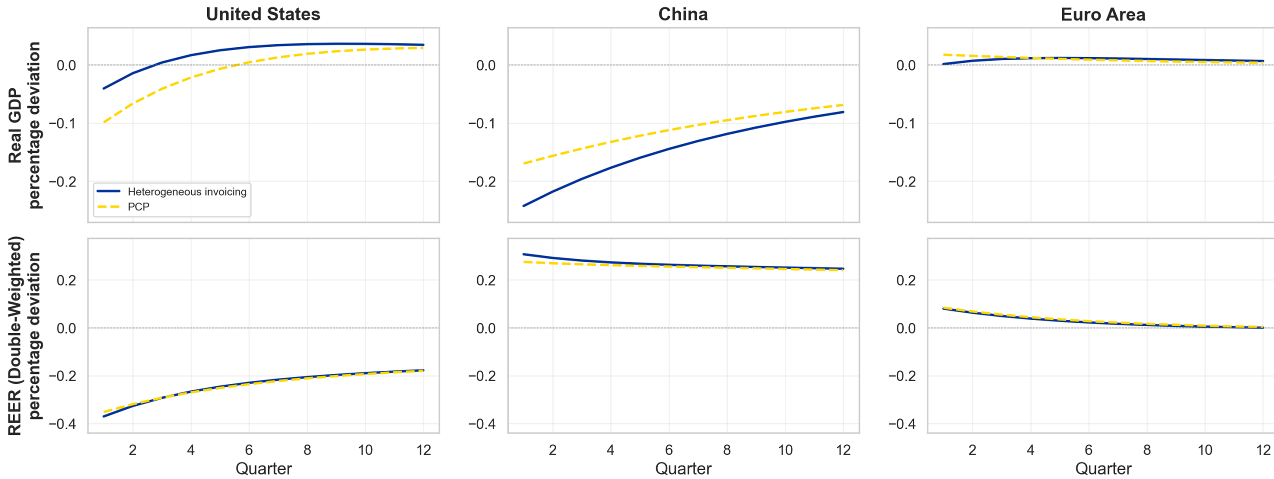
consequence is that exchange-rate adjustment provides less insulation for China under heterogeneous invoicing than under PCP.

Figure 3 quantifies this margin. We compare the three-year average response of real GDP, CPI inflation and trade balance in our benchmark framework with a counterfactual economy in which all trade is invoiced in the currency of the producer, PCP, represented by a blue dot in the figure.

Relative to PCP, the benchmark heterogeneous-invoicing regime delivers a markedly larger Chinese contraction—approximately 30 percent larger in three-year average GDP terms. The interpretation is immediate: under heterogeneous invoicing, the renminbi depreciation does less to restore Chinese competitiveness because a sizable share of border prices is preset in dollars. Under PCP, the depreciation mechanically lowers foreign-currency export prices and cushions export demand; under DCP, this channel is blocked within the Calvo contract duration, so the full force of the foreign-demand contraction is borne by quantities. This result is closely aligned with the dominant-currency paradigm of Gopinath *et al.* (2020) and Egorov and Mukhin (2023), both of which emphasize that dollar pricing weakens the expenditure-switching role of exchange-rate movements for non-US economies.

The invoicing structure also changes how the shock propagates through costs. In the model, invoicing is sector-specific, not use-specific: the same invoicing regime applies whether the good is used for final demand or as an intermediate input. When imported intermediates are invoiced in dollars, exchange-rate movements cannot buffer the local-currency cost increase in the same way as under PCP. The initial cost impulse entering the production network is therefore larger. This is why invoicing and input–output propagation interact: the network mechanism is the same across invoicing regimes, but the size of the cost impulse entering the production network depends on the currency in which border prices are sticky. The interaction is therefore multiplicative—dollar invoicing amplifies the initial

Figure 4: Real Effective Exchange Rates by Invoicing Regime



Notes: 12-quarter double-weighted real effective exchange rate responses under a 10 pp US tariff on Chinese imports across the 20 tradeable manufacturing sectors. Solid lines: heterogeneous invoicing; dashed lines: PCP.

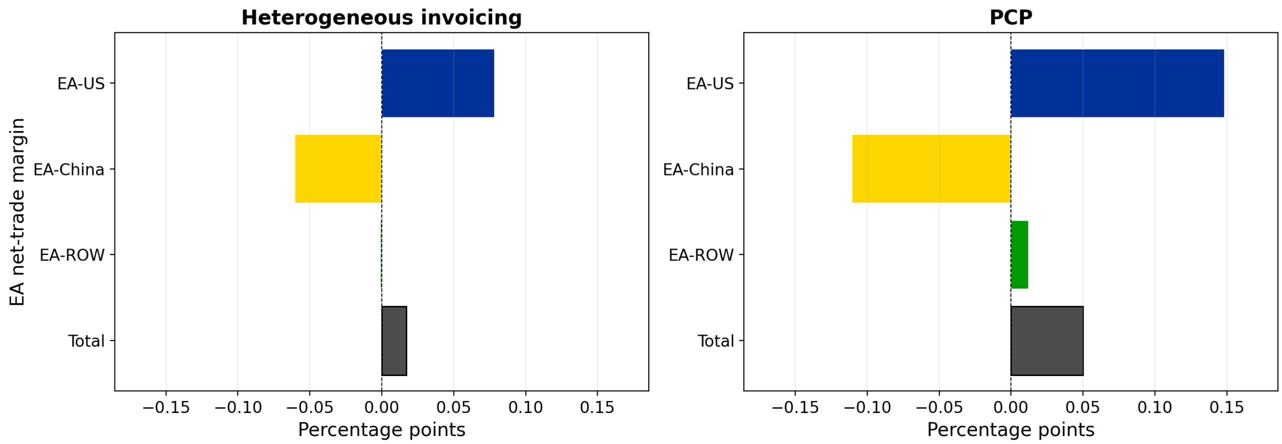
cost shock, and the network propagates that larger shock through the same input–output channels.

For the EA, the invoicing channel is most visible at short horizons. Figure 3 reports three-year-average responses, on which heterogeneous invoicing and PCP look similar for the EA. But the short-run dynamics differ more clearly: on impact, EA GDP is essentially flat under heterogeneous invoicing and positive under PCP, as shown in the top row of Figure 4. The reason is not a change in the sign of trade diversion vis-à-vis the US, but a change in the composition of the EA trade response across partners.

Figure 5 makes that composition explicit. Under PCP, the positive EA–US trade margin is larger, and the EA–ROW margin is also more supportive. The EA–China margin moves in the opposite direction: it is more negative under PCP, because the stronger pass-through of the renminbi depreciation improves Chinese relative prices vis-à-vis euro-area producers. The aggregate EA gain under PCP therefore does not come from the EA–China margin. It comes from the larger EA–US and EA–ROW margins, which more than offset the deterioration vis-à-vis China. Figure 4 is useful for interpreting this composition rather than a single bilateral margin: heterogeneous invoicing generates a larger Chinese depreciation because border-price pass-through is constrained, while PCP translates exchange-rate movements more directly into producer-currency price competitiveness. For the EA, that pass-through strengthens expenditure switching in third markets, especially the EA–ROW margin; under heterogeneous invoicing, dollar pricing dampens that response by reducing the sensitivity of ROW border prices to bilateral exchange-rate movements.

Taken together, the invoicing comparison shows that the benchmark is not simply a tariff story. It is a tariff shock filtered through nominal rigidities and the international role of the

Figure 5: Euro-area Net-trade Margins by Invoicing Regime



Notes: on-impact euro-area net-trade margin response under a 10 pp US tariff on Chinese imports across the 20 tradeable manufacturing sectors. Bars report the EA–US, EA–China, EA–ROW, and total margins under heterogeneous invoicing and PCP.

dollar. Under flexible prices, the invoicing regime would matter much less for the short-run allocation. With sticky border prices, however, invoicing becomes a first-order determinant of how much exchange-rate adjustment can cushion the shock, how much imported inflation is transmitted across countries, and how strongly third-country spillovers emerge.

4.3. Third-country transmission: the EA as a net object

The EA is useful precisely because it is not directly involved in the trade war. It neither imposes the tariff nor is it directly targeted by one. Its response therefore isolates the indirect transmission channels in the model: trade diversion, exchange-rate adjustment, and propagation through international input–output linkages. The small aggregate response of the EA in Figure 1 should thus not be interpreted as evidence that the trade war has little effect on third countries. Rather, it reflects the fact that several sizable margins operate in opposite directions and largely offset in equilibrium.

The first of those margins is trade diversion toward EA producers. When the US raises tariffs on Chinese goods, some US expenditure is reallocated toward alternative suppliers, including the EA. In a standard Armington environment, this substitution effect is the most immediate source of third-country gains. The second margin works in the opposite direction: as Chinese activity contracts, Chinese demand for EA goods weakens. The third margin is network-based. Because EA firms use imported intermediates and are embedded in international production chains, changes in foreign marginal costs and foreign demand affect EA production even when the initial tariff wedge does not apply to the EA directly. The fourth margin is financial and nominal: exchange-rate movements alter the EA's competitiveness vis-à-vis both the US and the ROW, and the strength of that adjustment depends on the invoicing regime.

This combination of margins explains why the EA is best viewed as a *net object*. The benchmark responses in Figure 1 are close to zero on impact and remain small throughout the horizon shown, but those small aggregates mask the interaction of positive and negative forces. Under the US tariff on China, the positive force is the diversion of US demand away from Chinese suppliers. The negative forces are the contraction in Chinese demand and the propagation of the shock through international production linkages. Under the Chinese retaliation, the same logic applies with the bilateral roles reversed. In both cases, the EA response is the residual of offsetting gross reallocations rather than the manifestation of a single dominant channel.

This interpretation is consistent with the recent literature on the international incidence of the US–China trade war. Empirical work has shown that the shock generated meaningful reallocation toward third countries, but that the aggregate gains of those third countries were far from mechanical because they depended on sectoral composition, foreign-demand spillovers, and general-equilibrium adjustment (Fajgelbaum *et al.* 2024). In our model, those same features are present in a transparent way. The EA benefits from trade diversion in some sectors and vis-à-vis some destinations, but that gain is partially offset by weaker Chinese demand and by the propagation of foreign cost and demand disturbances through the network.

The mechanism exercises from Section 4.2 sharpen this interpretation. Figure 2 shows that once propagation through the wider IO network is removed—while preserving the directly tariffed US–China bilateral intermediate-input links—the EA response becomes even smaller in absolute value. This indicates that a non-negligible part of third-country transmission operates through production networks rather than only through final-goods substitution. Figure 3 shows that the three-year-average EA response differs across invoicing regimes only modestly, while the short-run dynamics discussed above are more visibly separated in Figure 4. Figure 5 makes the trade accounting explicit: dollar pricing changes the balance between the positive EA–US diversion margin and the negative margins associated with China and the ROW.

The REER responses in Figure 4 are helpful in this regard. Under heterogeneous invoicing, exchange-rate adjustment is less effective at delivering expenditure switching through border prices than under PCP, so more of the adjustment must take place through quantities and through the path of the real exchange rate itself. This is especially visible for China, but it matters for the EA as well because the EA's competitiveness is determined not only by bilateral trade with the US, but also by how dollar movements affect its position relative to China and to other third markets. The EA therefore provides the clearest case in which invoicing and third-country transmission interact.

For exposition, the main lesson is simple. The EA response is small not because the model shuts down spillovers to third countries, but because the model generates several spillovers with different signs. That is, the EA is not a small-effect object; it is an offsetting-margins object. This is why aggregate IRFs alone are not enough to understand third-country transmission. Figure 5 shows directly that the small EA aggregate is the net result of larger gross movements vis-à-vis the US, China, and the ROW. In that sense, the EA is not a small-effect object; it is an offsetting-margins object.

This perspective also helps connect the aggregate analysis in this section with the sectoral analysis that follows in Section 5. Once the EA is understood as the outcome of offsetting bilateral and multilateral margins, it becomes natural to ask which sectors gain from trade diversion, which sectors lose from weaker Chinese demand, and how those sectoral responses add up to an aggregate that remains close to zero.

4.4. Additional robustness

The previous subsections isolate the two main structural mechanisms behind the benchmark responses: production-network propagation and the invoicing structure. We now consider two additional dimensions of model structure that are best interpreted as robustness exercises rather than as separate mechanisms in their own right: the trade elasticity and the exchange-rate regime. These experiments change the quantitative magnitude of the benchmark responses, sometimes substantially, but do not overturn the qualitative ordering of the main results.

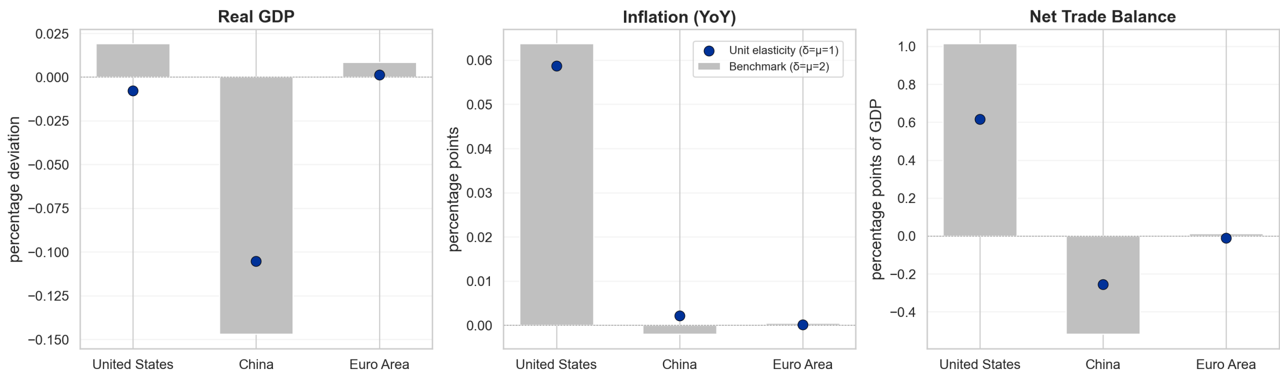
4.4.1. Trade elasticity

We begin with the trade elasticity. In the benchmark calibration, both the household and intermediate-input Armington elasticities are set to two. To assess how much the benchmark depends on expenditure switching, Figure 6 compares that calibration to a unit-elasticity counterfactual with $\delta = \mu = 1$.

Lower trade elasticities weaken expenditure switching. Households substitute less aggressively away from tariffed imports in final demand, and firms do the same for tariffed foreign inputs in production. As a result, tariff-induced relative-price movements generate a smaller reallocation of quantities across origins. This is the standard role of trade elasticities in quantitative trade models (Armington 1969; Boehm *et al.* 2023; Baqaee and Farhi 2024).

Figure 6 shows that lowering the trade elasticity attenuates the benchmark responses. The Chinese contraction becomes smaller, the small positive average US GDP response disappears, and the EA gain becomes even closer to zero. This pattern is exactly what one would expect

Figure 6: Trade Elasticity Average Responses



Notes: three-year average unit-elasticity counterfactual responses under a 10 pp US tariff on Chinese imports across the 20 tradeable manufacturing sectors. Real GDP and net trade balance are averages; inflation is the average of year-on-year CPI inflation. Gray bars: benchmark ($\delta = \mu = 2$); blue dot: unit elasticity ($\delta = \mu = 1$).

when substitution across suppliers is weaker. With less expenditure switching, the tariff-imposing country receives less support from demand reallocation, the targeted country is hit less strongly through trade diversion, and third-country gains become smaller because there is less reallocation toward alternative exporters.

The inflation responses change more modestly, but in the same direction. Average US CPI inflation falls somewhat when the trade elasticity is reduced, consistent with a weaker expenditure-switching response and a smaller induced reallocation of demand and costs across sectors and origins. The EA remains close to zero, and China moves only slightly. The main message of the elasticity exercise is therefore quantitative rather than qualitative: the strength of the benchmark responses depends on how easily trade flows are reallocated across countries, but the ranking of the cross-country effects remains unchanged.

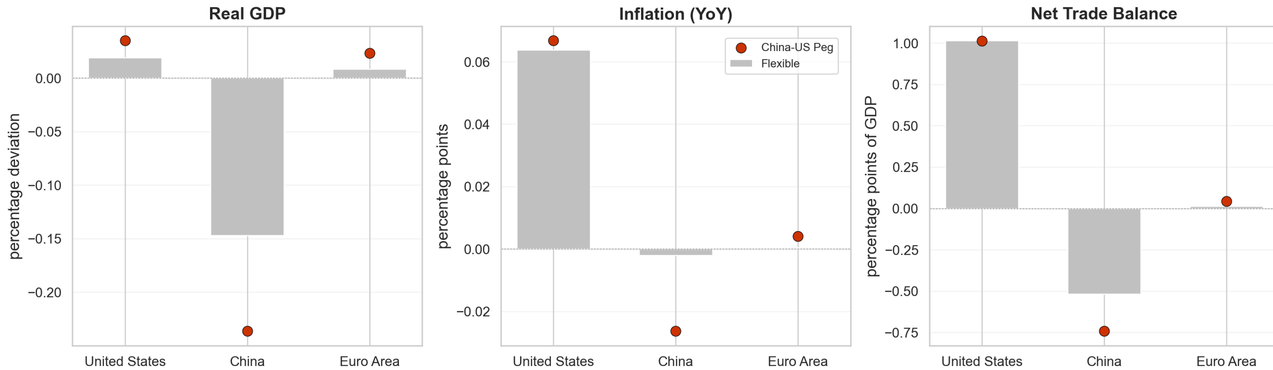
4.4.2. Exchange-rate regime

We next consider the exchange-rate regime. In the benchmark, China adjusts through the exchange rate, so a US tariff on Chinese goods induces a renminbi depreciation. Figure 7 instead considers a counterfactual in which China stabilizes its exchange rate tightly against the dollar through a foreign-exchange intervention rule. Operationally, the peg is implemented through an effective interest rate that enters only the UIP condition, so the exchange rate is stabilized without changing the domestic policy rate relevant for intertemporal decisions.[11]

A tighter exchange-rate regime shuts down that margin. This is the classic open-economy role of exchange-rate flexibility in the presence of nominal rigidities, and it becomes especially relevant when border-price pass-through is already limited by the invoicing structure

[11]Formally, $i^{\text{eff}}_{k,t} = i_{k,t} + \phi_{k,e}\Delta e_{k,\text{US},t}$ enters the exchange-rate determination equation, while the domestic policy rate $i_{k,t}$ continues to govern all other intertemporal decisions. We set $\phi_{k,e} = 15$, which keeps the nominal exchange rate close to its steady state.

Figure 7: Exchange-rate Regime Average Responses



Notes: three-year average China–US peg counterfactual responses under a 10 pp US tariff on Chinese imports across the 20 tradeable manufacturing sectors. Real GDP and net trade balance are averages; inflation is the average of year-on-year CPI inflation. Gray bars: flexible rates; red dot: China–US peg ($\phi_{k,e} = 15$)*.*

(Devereux and Engel 2003; Egorov and Mukhin 2023).

Figure 7 shows that this adjustment margin is quantitatively important. Under the peg, the Chinese contraction becomes much larger than in the benchmark. The intuition is immediate: once the renminbi is prevented from depreciating, China loses an important buffer against the fall in US demand. At the same time, the US and the EA benefit somewhat more from trade diversion, because Chinese producers are less able to restore competitiveness through the exchange rate. The peg therefore amplifies the benchmark asymmetry between the targeted and non-targeted economies.

This result is particularly relevant for interpretation. In the benchmark, the Chinese contraction already exceeds the US contraction by a wide margin. The exchange-rate-regime exercise shows that this asymmetry would be even stronger under tighter exchange-rate management. In that sense, the benchmark can be read as conservative with respect to the output cost borne by the targeted economy. More broadly, the exercise confirms that the depreciation channel remains economically important even in a model with dominant-currency pricing. Dollar invoicing weakens the mapping from exchange rates into border prices, but it does not eliminate the exchange rate as an adjustment margin altogether.

Taken together, the robustness exercises reinforce the interpretation of the benchmark. Lower trade elasticities weaken expenditure switching and attenuate the responses; a tighter exchange-rate regime shuts down a key adjustment margin and amplifies the contraction in the targeted economy. Neither exercise overturns the central conclusions of the section. A natural further robustness check—not pursued here—would vary the degree of price rigidity in China, where our benchmark assigns US-based Calvo frequencies from the Gautier et al. (2024) PRISMA data. To the extent that Chinese manufacturing sectors adjust prices more frequently, the Chinese contraction would moderate, though the qualitative ranking of cross-country effects would likely survive. Across specifications, the targeted country still bears the larger output loss, the tariff-imposing country still bears more of the short-run inflationary burden, and the EA still emerges as a net object shaped by offsetting third-country margins.

5. Sectoral channels of US–China tariff transmission

We now turn from aggregate transmission to sectoral incidence. The exercise in this section is simple: we impose a 10 percentage-point US tariff on one Chinese tradeable sector at a time and ask how much that sector contributes to aggregate GDP and CPI in the US, China, and the EA over the subsequent three years. This decomposition lets us address three questions. Which Chinese sectors matter most for aggregate incidence? To what extent can those aggregate responses be inferred from what happens inside the directly affected sector? And why does the EA remain close to zero in the aggregate even when several sectors move materially?

Three results emerge. First, the cross-sectional ranking is organized mainly by direct bilateral exposure: the sectors that matter most are those in which US expenditure is most concentrated on Chinese supply. Second, aggregate incidence is not a sector-local object in the directly exposed economies, because domestic propagation materially reshapes the own-sector response. Third, the EA remains a net object at the sectoral level: stronger exports to the US are offset by weaker exports to China and by smaller multilateral adjustments.

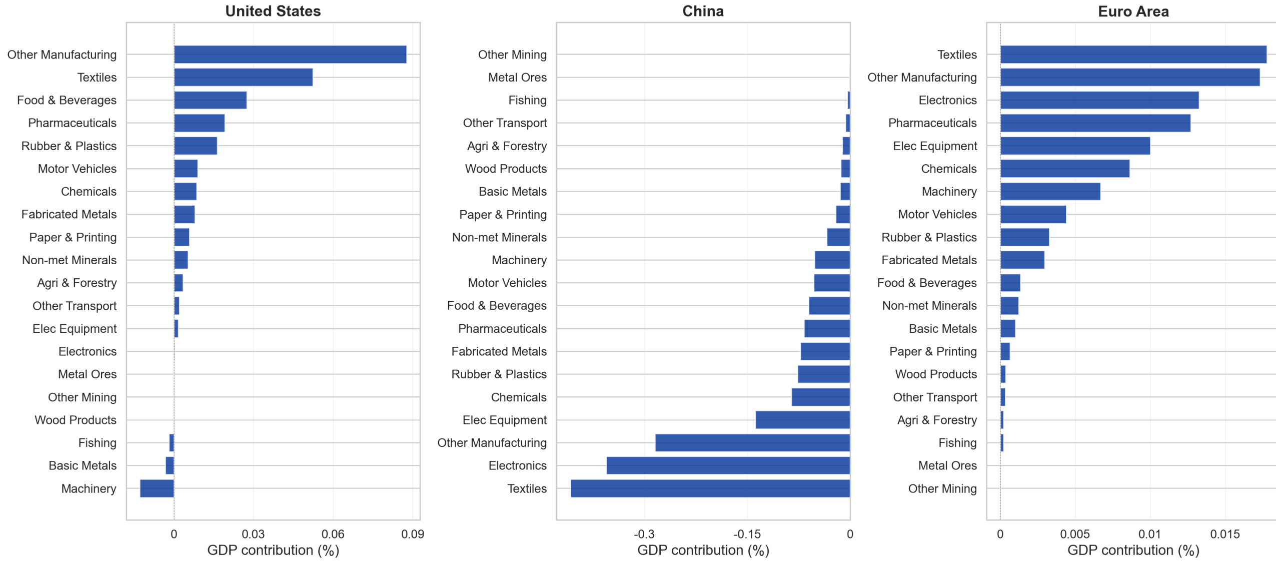
These findings connect the sectoral decomposition to three related literatures. The role of direct bilateral exposure is consistent with the empirical literature on tariff incidence and trade reallocation (Amiti *et al.* 2019; Fajgelbaum *et al.* 2024). The wedge between own-sector and aggregate responses reflects the propagation logic emphasized in the production-network literature (Acemoglu *et al.* 2012; Pasten *et al.* 2020; Rubbo 2023; Baqaee and Farhi 2024). And the euro-area results are closely related to recent work showing that third-country transmission depends on the interaction between trade diversion, exchange-rate adjustment, and the invoicing structure (Gopinath *et al.* 2020; Egorov and Mukhin 2023).

The section proceeds in three steps. Section 5.1 establishes the sectoral ranking of aggregate GDP and CPI contributions. Section 5.2 then shows why those aggregate contributions cannot be read off from the tariffed sector alone. Section 5.3 finally turns to the EA and shows why, even at the sectoral level, its response remains an offsetting-margins object.

5.1. Which Chinese sectors matter most for aggregate incidence?

We begin by documenting the sectoral ranking of aggregate incidence. We first identify which Chinese sectors matter most for cumulative GDP and CPI in each economy, and then relate the ranking to a small set of sectoral characteristics in the model.

Figure 8: Sectoral Contributions to GDP



Notes: three-year cumulative aggregate GDP responses under single-sector 10 pp US tariffs on Chinese tradeable manufacturing sectors. Each bar is the cumulative country-level response to tariffing the sector shown on the horizontal axis. In each country panel, the 20 bars sum to the aggregate GDP effect of a 10 pp tariff on all tradeable manufacturing sectors.

5.1.1. Benchmark sectoral contributions

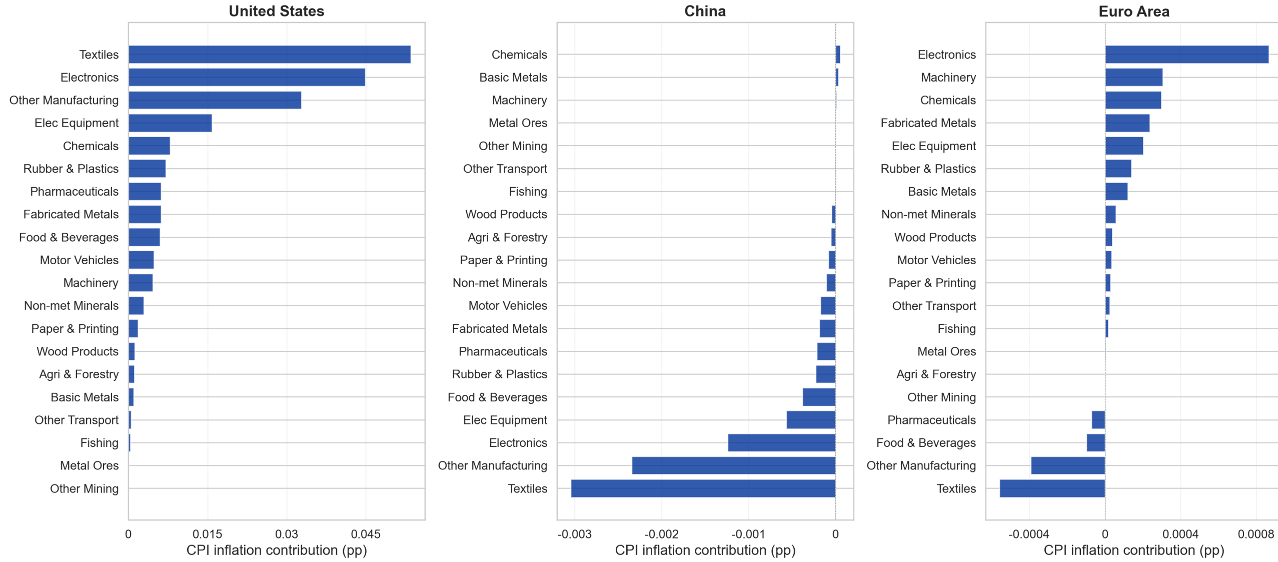
Throughout this section, the US raises the tariff by 10 percentage points in one Chinese tradeable sector at a time. Each bar reports the three-year cumulative aggregate contribution of that sector to aggregate GDP or CPI inflation, computed from a single-sector experiment in which only sector i is tariffed and the rest of the economy adjusts. Under the first-order solution, these 20 sectoral contributions add up to the benchmark cumulative response over the same horizon.

Figures 8 and 9 show that sectoral incidence is concentrated rather than diffuse. A small set of tariffed Chinese sectors accounts for most of the three-year movement in the directly exposed economies. In cumulative GDP space, the largest US gains come from *other manufacturing*, *textiles*, and *food and beverages*, while China's largest losses come from *textiles*, *electronics*, and *other manufacturing*. The ranking is therefore far from uniform across the 20 tariffed sectors.

The same concentration appears in prices, but with a different geography. On the cumulative inflation side, the largest positive contributions accrue to the US, especially in *textiles*, *electronics*, and *other manufacturing*, whereas China's cumulative inflation contributions are mostly negative and the EA remains small in absolute terms. The sectoral ranking therefore reproduces the aggregate asymmetry from Section 4.1: quantity incidence is concentrated in the targeted economy, while price incidence is concentrated in the tariff-imposing economy.

The natural interpretation is that the initial sorting is done by the bilateral margin the

Figure 9: Sectoral Contributions to Inflation



Notes: three-year cumulative aggregate CPI inflation responses under single-sector 10 pp US tariffs on Chinese tradeable manufacturing sectors. Each bar is the cumulative country-level response to tariffing the sector shown on the horizontal axis. In each country panel, the 20 bars sum to the aggregate inflation effect of a 10 pp tariff on all tradeable manufacturing sectors.

tariff hits first. This is consistent with the recent empirical literature on the US–China trade war, which shows that tariff incidence and trade reallocation were concentrated in the sectors directly exposed to the tariff wedge (Amiti *et al.* 2019; Fajgelbaum *et al.* 2024). In our model, the sectors that matter most are the sectors in which the US is most exposed to Chinese supply before the shock arrives.

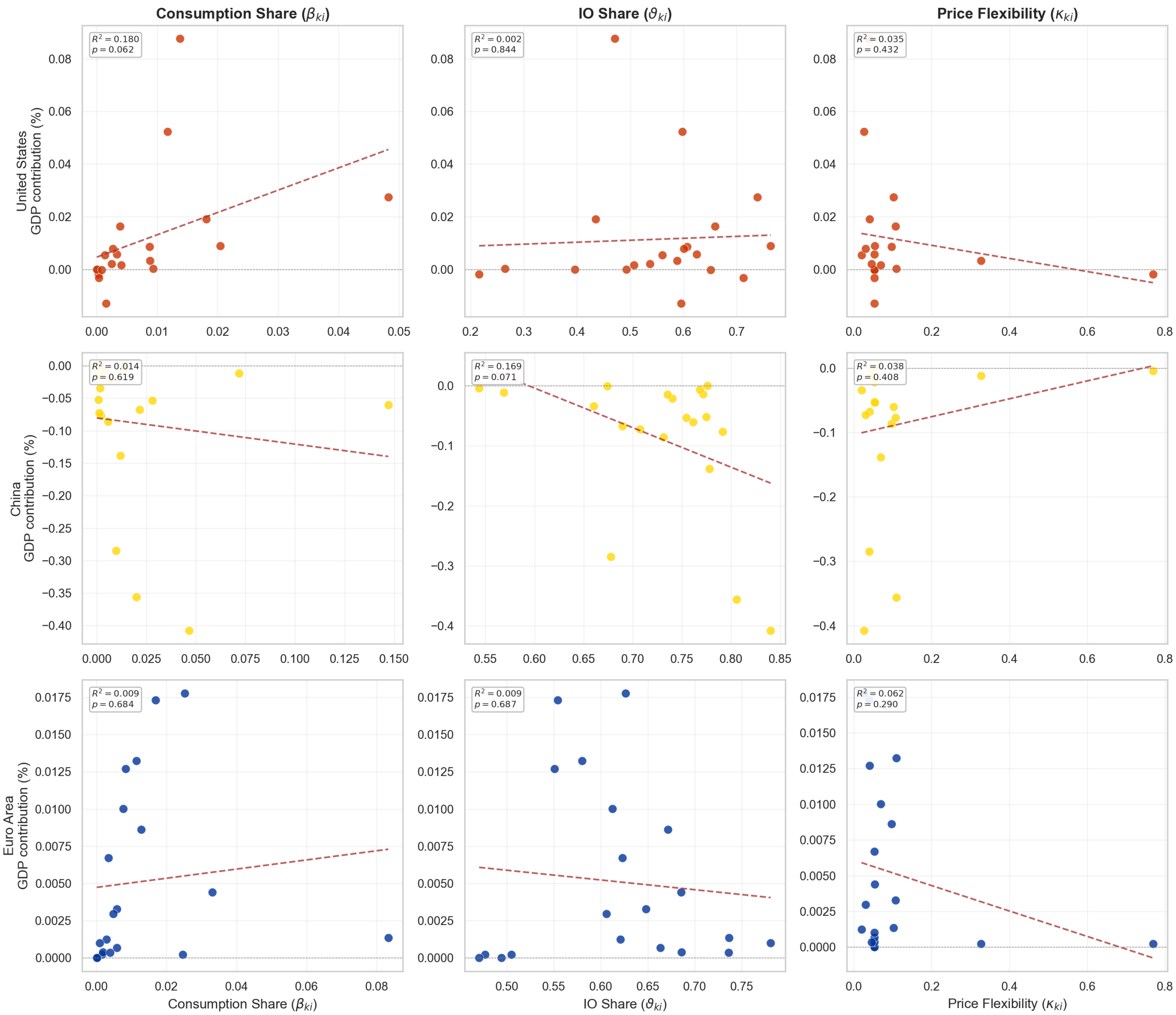
5.1.2. Sectoral characteristics and amplification

Three sectoral characteristics are natural candidates to help organize the ranking in Figures 8 and 9: the domestic consumption share, the IO share, and price flexibility. These objects do not measure where the tariff lands initially; rather, they help describe how a sectoral disturbance maps into aggregate quantities and prices once general-equilibrium adjustment is underway.

A sector with a larger domestic consumption share matters more for aggregate incidence because it carries more weight in domestic absorption. A sector with a higher IO share has more scope to transmit the shock through upstream and downstream production linkages. And a sector with more flexible prices should, in principle, accommodate more of the adjustment through prices rather than quantities.

Figures 10 and 11 show that these sectoral characteristics do help organize the cross section, but only imperfectly. In the US, the clearest pattern is that sectors with larger consumption shares tend to be associated with larger cumulative GDP contributions. In China, the IO share

Figure 10: Structural Determinants of Sectoral GDP

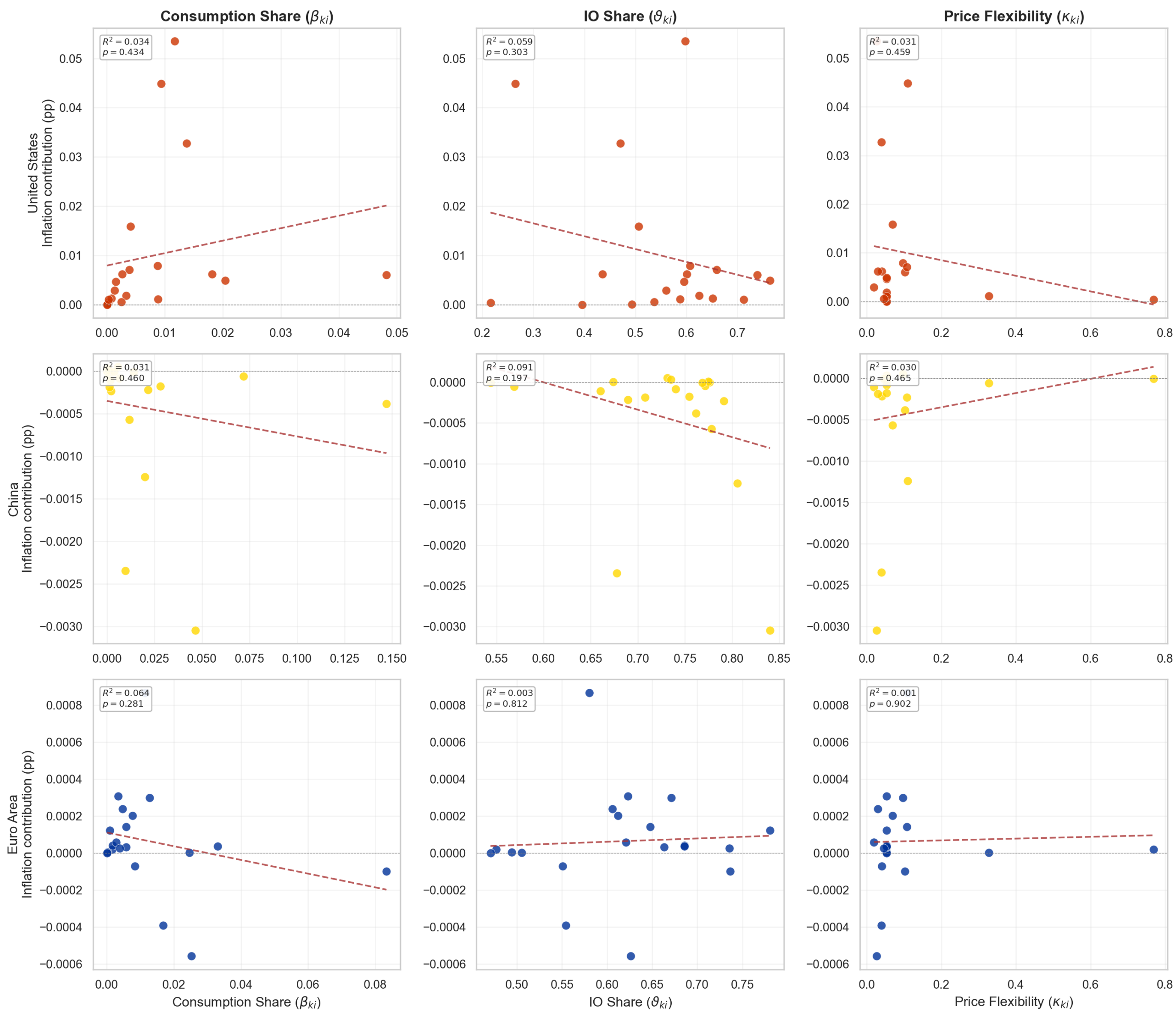


Notes: three-year cumulative sectoral GDP effects and sector characteristics under single-sector 10 pp US tariffs on Chinese tradeable manufacturing sectors. Rows show the US, China, and the EA; columns show consumption share ($\sum_{l=1}^{K} \beta_{kl\,i}$), IO share ($\vartheta_{ki}$), and price flexibility (κ_{ki}). Each point is one tariffed sector; the dashed line is the univariate linear trend; each panel reports R^2 and p-value.

displays the strongest relationship with cumulative GDP, consistent with a larger role for domestic propagation in sectors that are more tightly embedded in production networks. By contrast, price flexibility is a weak stand-alone predictor in most panels, and none of the three characteristics organizes the EA cross section tightly on its own.

These sectoral characteristics shape the amplification and composition of the response, rather than determine the ranking by themselves. The main message of Figures 10 and 11 is thus modest but useful: the sectoral ranking is partly systematic, and the clearest part of that systematic variation comes from sector size and network position rather than from price flexibility alone.

Figure 11: Structural Determinants of Sectoral Inflation



Notes: three-year cumulative sectoral inflation effects and sector characteristics under single-sector 10 pp US tariffs on Chinese tradeable manufacturing sectors. Rows show the US, China, and the EA; columns show consumption share ($\sum_{l=1}^{K} \beta_{kl\,i}$), IO share ($\vartheta_{ki}$), and price flexibility (κ_{ki}). Each point is one tariffed sector; the dashed line is the univariate linear trend; each panel reports R^2 and p-value.

5.2. Why aggregate incidence is not a sector-local object

The aggregate ranking in Figures 8 and 9 cannot be read off from what happens inside the tariffed sector alone. A country-level GDP contribution is the sum of what happens inside the sector hit directly by the tariff and what happens in the rest of the domestic economy after the shock propagates. This distinction between the own-sector term and the cross-sector term is the key to understanding why sectoral protection in the US and sectoral contraction in China do not map one-for-one into aggregate incidence.

Using the aggregate relation (A.110) from Appendix A, we can decompose the aggregate response after a US tariff on Chinese sector s as the weighted aggregation of sectoral value added

$$\widehat{y}_{k,t|\tau_{\text{US,CHN},s}} = \lambda_{ks}(1-\vartheta_{ks})\widehat{\text{va}}_{ks,t|\tau_{\text{US,CHN},s}} + \sum_{i\neq s}^{I} \lambda_{ki}(1-\vartheta_{ki})\widehat{\text{va}}_{ki,t|\tau_{\text{US,CHN},s}}, \tag{37}$$

where $\widehat{y}_{k,t|\tau_{\text{US,CHN},s}}$ is the aggregate GDP response, $\widehat{\text{va}}_{ki,t|\tau_{\text{US,CHN},s}}$ denotes the response of sector i's value added when sector s in China is tariffed by the US, and $\lambda_{ki}(1-\vartheta_{ki})$ is the steady-state value-added share of sector i in country k. Real value added is defined formally in Appendix A.

The broader idea is familiar from the production-network literature: aggregate effects need not be read off from the directly shocked sector because input–output linkages propagate shocks across the rest of the economy (Acemoglu *et al.* 2012; Pasten *et al.* 2020; Rubbo 2023; Baqaee and Farhi 2024). What is specific here is that the same logic operates inside a multicountry tariff environment.

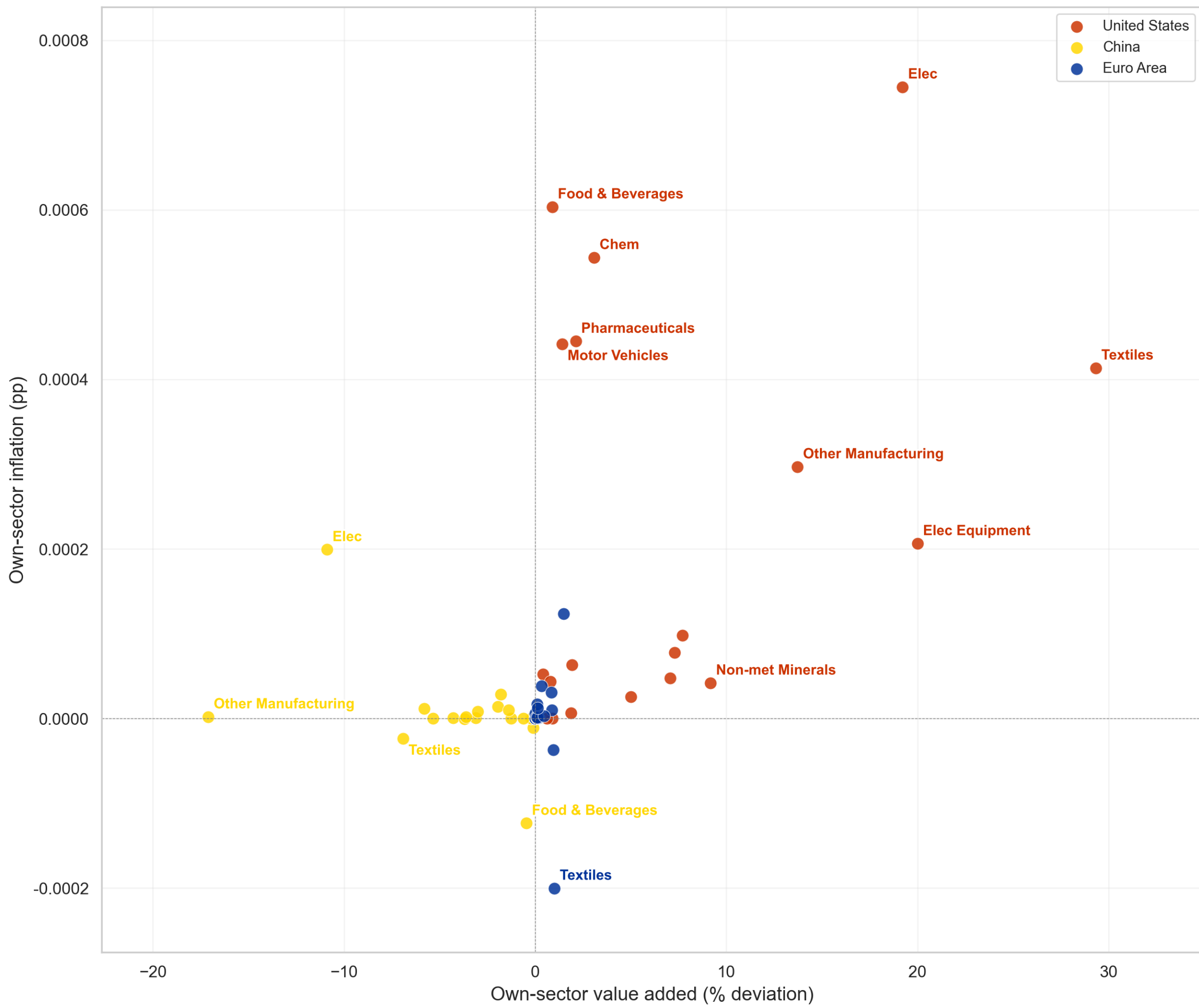
5.2.1. Own-sector incidence

Figure 12 isolates the own-sector term. Each point corresponds to one of the 20 tariffed Chinese sectors in a single-sector experiment, with the horizontal axis measuring the 12-quarter cumulative response of value added in the corresponding domestic sector and the vertical axis the associated cumulative sectoral inflation.

In the US, the tariffed domestic sector tends to expand because demand is diverted away from Chinese suppliers and toward US producers. The US observations lie mostly in the region of positive cumulative own-sector value added and positive own-sector inflation, with especially large gains in *textiles*, *electrical equipment*, and *electronics*. In China, the tariffed sector contracts because it loses US demand. The Chinese observations lie on the negative side of cumulative own-sector value added, led by *other manufacturing*, *electronics*, and *textiles*. In the EA, which is not on the tariff margin itself, the own-sector response remains close to zero. EA observations remain much closer to the origin, although a few tradeable sectors still post visible positive own-sector value-added responses.

A sector can move sharply on its own and still matter little for aggregate GDP if it is small in steady state, and a large diagonal response can be offset once the rest of the domestic economy is added back. *Electronics* in the US is the clearest example: the tariffed sector posts a very large cumulative own gain, yet its aggregate GDP contribution is close to zero once the cross-sector term is included.

Figure 12: Own-sector Value Added and Inflation



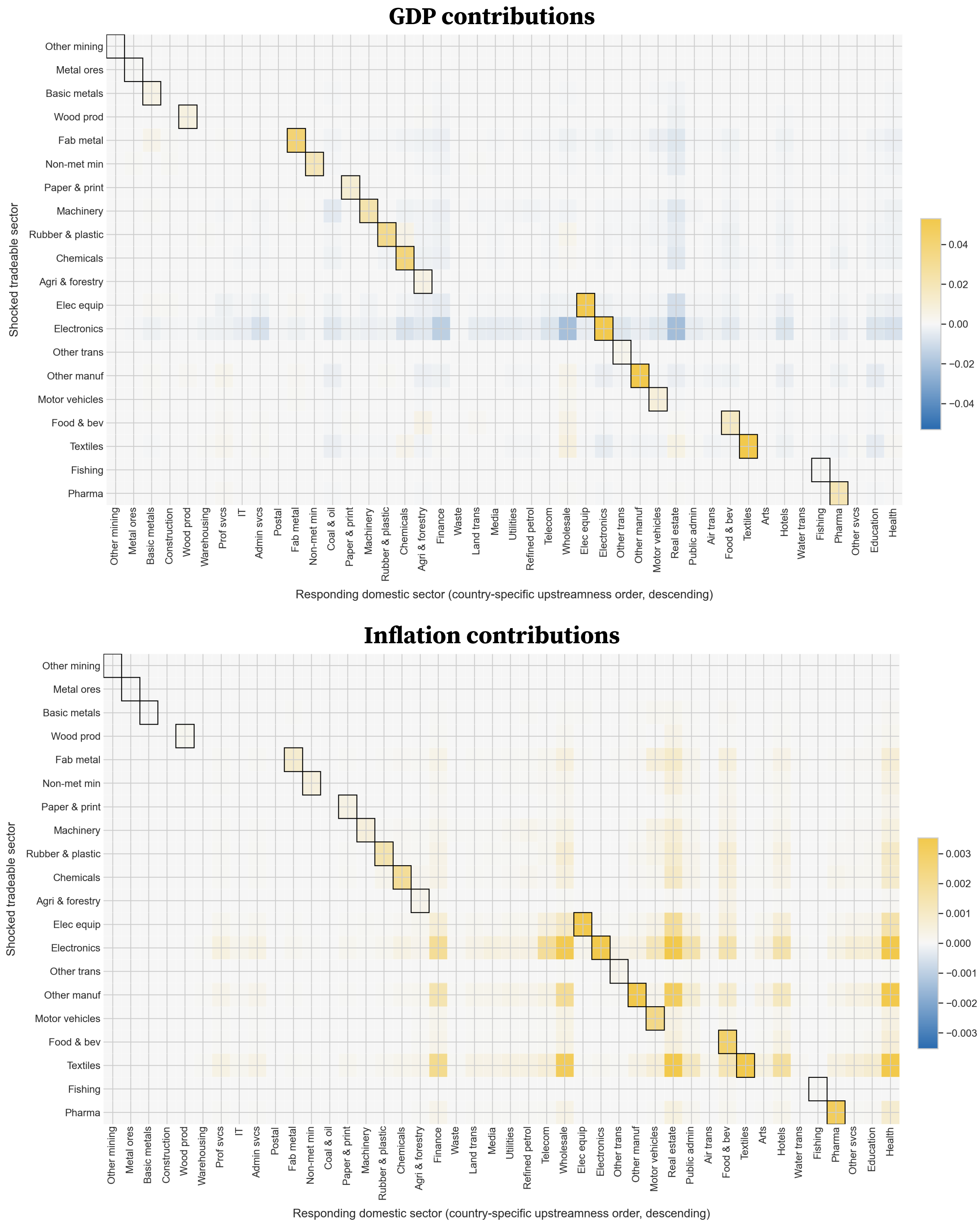
Notes: three-year cumulative own-sector value added and own-sector inflation under single-sector 10 pp US tariffs on Chinese tradeable manufacturing sectors. Each point is one tariffed sector. US points are red, China points are gold, and EA points are blue; key outliers are labeled.

5.2.2. Cross-sector propagation

Figures 13 and 14 visualize that decomposition directly. Each row corresponds to the Chinese sector hit by the tariff; each column corresponds to a domestic sector in the responding economy. The black-outlined diagonal cell is the own-sector term, while the off-diagonal cells capture cross-sector spillovers. The first matrix reports GDP contributions and the second reports the corresponding CPI contributions.

These matrices show that aggregate incidence is not a small correction to the directly affected sector. In the US, the rest-of-economy term often reshapes the ranking and sometimes overturns it. *Machinery, basic metals, wood products*, and *fishing* are the clearest cases in which an initial sectoral gain is more than offset by losses elsewhere in the economy: the cross-

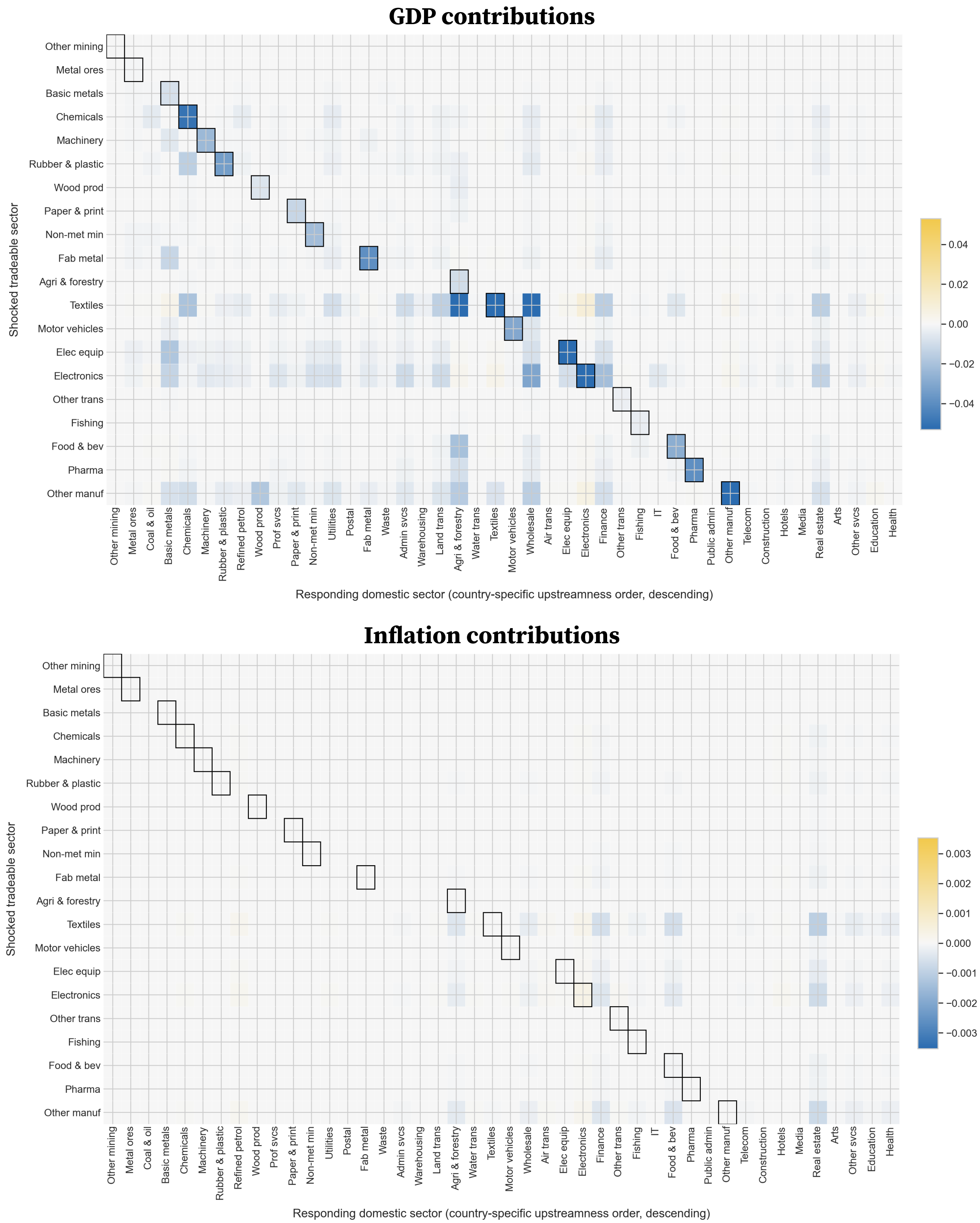
Figure 13: US Sectoral Transmission Matrix



Notes: three-year cumulative US sectoral transmission under single-sector 10 pp US tariffs on Chinese tradeable manufacturing sectors. Rows are tariffed sectors; columns are US sectors ordered by upstreamness. In each matrix, a row sums to the cumulative aggregate US GDP or inflation effect of tariffing that row's Chinese sector; black outlines mark own-sector terms.

sector block (off-diagonal cells) is negative and large enough to flip the sign of the aggregate contribution. By contrast, *textiles* and *other manufacturing* remain net positive because the

Figure 14: China Sectoral Transmission Matrix



Notes: three-year cumulative Chinese sectoral transmission under single-sector 10 pp US tariffs on Chinese tradeable manufacturing sectors. Rows are tariffed sectors; columns are Chinese sectors ordered by upstreamness. In each matrix, a row sums to the cumulative aggregate Chinese GDP or inflation effect of tariffing that row's Chinese sector; black outlines mark own-sector terms.

own-sector gain (diagonal cell) is large enough to survive the cross-sector offsets. *Electronics* is the knife-edge case: the directly protected sector expands strongly, but that gain is almost fully offset once downstream and upstream losses are added back.

In China, the same accounting points even more clearly to domestic amplification. The tariffed sector is already contracting because it loses US demand, and the rest of the Chinese production network usually reinforces that contraction rather than buffering it. The strongest rows—especially *textiles*, *electronics*, *other manufacturing*, and *electrical equipment*—therefore show negative mass both on the diagonal and across the rest of the matrix. The relevant Chinese mechanism is not just a direct export-loss margin. It is an export-demand shock that is transmitted further through domestic production linkages.

The inflation matrices show the corresponding cumulative CPI contributions. In the US, positive price pressure is concentrated in the manufacturing rows that matter most for cumulative incidence, especially *textiles*, *electronics*, and *other manufacturing*. In China, cumulative CPI contributions are mostly near zero or negative and remain much more localized than the GDP decomposition. Taken together, the figures support the same reading as the aggregate section: own-sector incidence identifies where the tariff hits first, and cross-sector propagation determines how much of that local disturbance survives in country-level quantities and prices.

5.3. Why the EA remains an offsetting-margins object

The EA should not be read as a mild version of either the US or China. Because the EA neither imposes the tariff nor is directly targeted by it, its sectoral ranking is not organized around a domestic tariff wedge. The right question is instead whether the own-sector trade-diversion gain within the indirectly exposed sector is large enough to survive the negative China margin and the smaller multilateral offsets that accompany it.

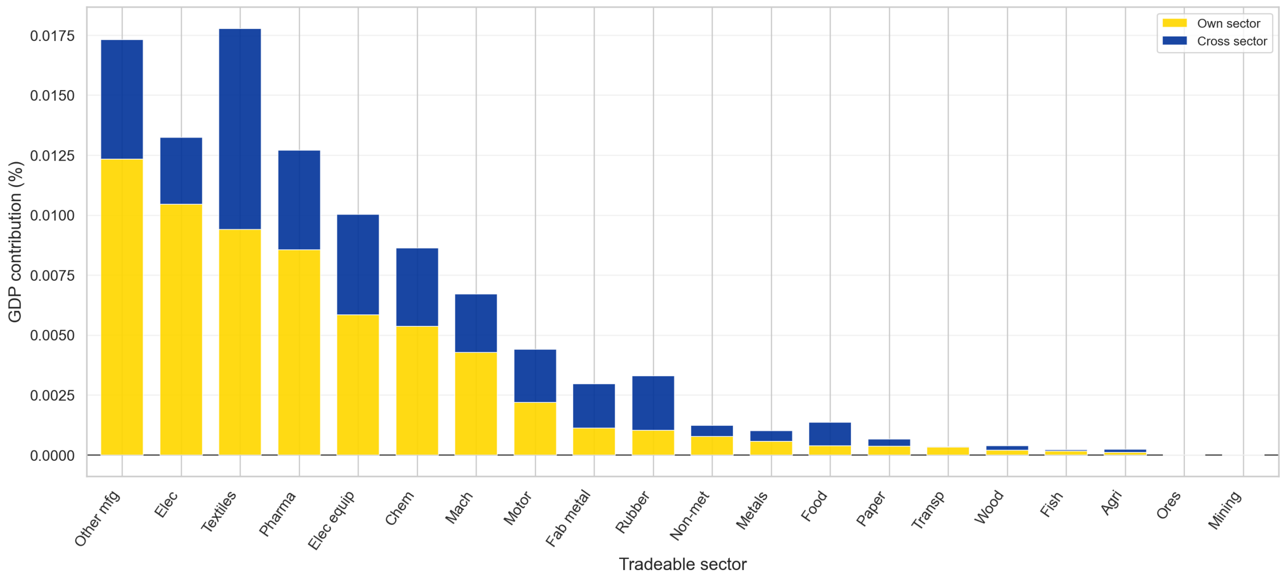
This is also consistent with the recent literature on third-country transmission, which emphasizes that trade diversion is only one margin among several and that the net effect depends on broader equilibrium adjustments (Fajgelbaum *et al.* 2024; Gopinath *et al.* 2020; Egorov and Mukhin 2023).

5.3.1. Own-sector and cross-sector terms in the EA

Figure 15 decomposes each EA aggregate GDP contribution into an own-sector term and a cross-sector term. Each column corresponds to one of the 20 tariffed Chinese sectors in the single-sector decomposition, ordered by the size of the own-sector contribution.

The main contrast with the US and China is that the cross-sector block has the opposite sign. In the US, cross-sector terms mostly offset the protected sector's own gains; in China, they reinforce the own-sector contraction. In the EA, by contrast, the cross-sector term is

Figure 15: Euro-area Sectoral GDP Contributions

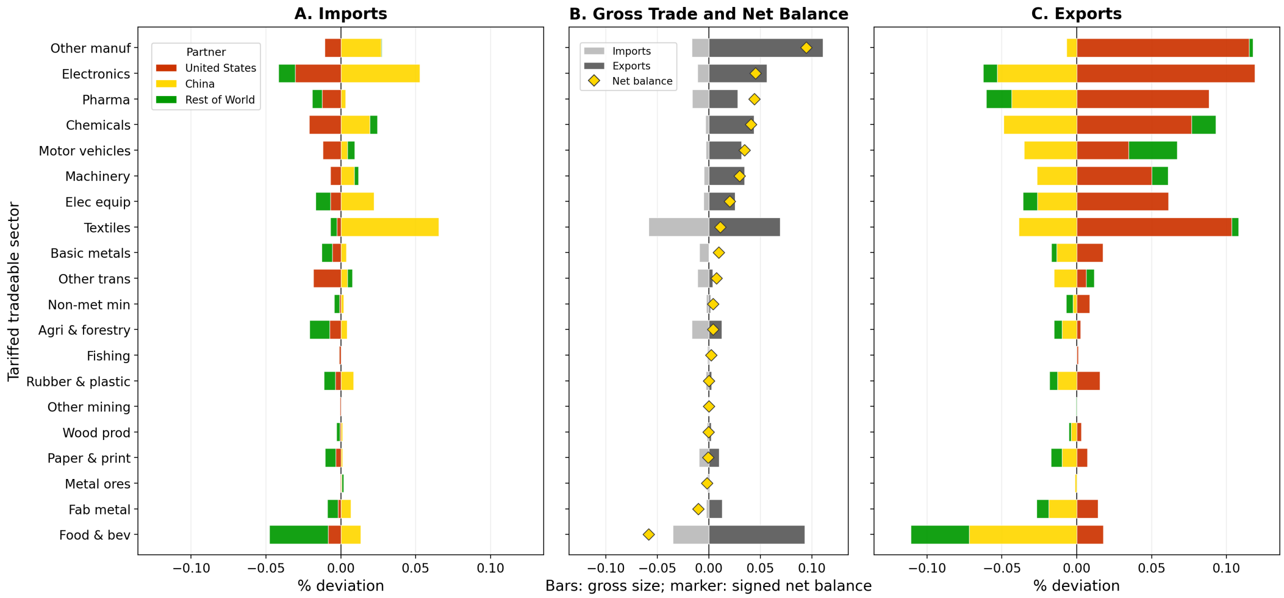


Notes: three-year cumulative EA GDP responses under single-sector 10 pp US tariffs on Chinese tradeable manufacturing sectors. Each column is one tariffed sector, ordered by the own-sector GDP contribution. Gold segments show the shocked sector's own contribution to EA GDP; blue segments show the contribution from all other EA sectors.

positive in all economically relevant sectors, but it lacks the quantitative incidence to drive the ranking. The sectors that matter most for EA GDP remain anchored by the own-sector term: *textiles*, *other manufacturing*, *electronics*, *pharmaceuticals*, *electrical equipment*, *chemicals*, and *machinery* all fit that pattern. The cases in which the cross-sector block dominates the own-sector block are concentrated in quantitatively minor rows. In that precise sense, the EA is the opposite of the US case and milder than China: the ranking is driven mainly by the indirectly exposed sector itself, not by a large domestic propagation block.

That distinction is economically important. In the directly exposed economies, the aggregate row is often determined by what happens after the tariff shock propagates through the domestic production network: the cross-sector block can dominate the own-sector term. In the EA, the most important object is the sector-specific own-sector contribution generated before domestic propagation becomes important. The own-sector contribution, driven mainly by trade reallocation, accounts for most of the variation across sectors. For a third country, that contribution is not created by domestic protection or by a direct collapse in export demand. It reflects mainly trade reallocation: sectors that capture enough demand diverted from China toward the US deliver positive own contributions, while sectors for which the China drag remains too strong contribute little or even slightly negatively.

Figure 16: Euro-area Sectoral Trade Margins



Notes: three-year cumulative sectoral EA trade margins under the full benchmark shock: a 10 pp US tariff on Chinese imports across all 20 tradeable manufacturing sectors. Left: imports by partner. Middle: gross imports, gross exports, and the sectoral net trade balance. Right: exports by partner. Sectors are ordered by the total net trade balance.

5.3.2. Large trade effects cancel in net

Figure 16 complements, rather than mirrors, the single-sector decomposition. Unlike Figures 8–15, it reports sectoral trade margins under the full benchmark tariff shock across all 20 tradeable sectors. Its role is therefore not to rank the single-sector contributions, but to show more directly which bilateral trade margins underpin the EA's sectoral gains and losses in the benchmark equilibrium.

Figure 16 shows that the gross trade effects are large, but they do not point in the same direction. *Other manufacturing*, *electronics*, *pharmaceuticals*, *chemicals*, and *motor vehicles* account for most of the positive sectoral net-trade gains, while *food and beverages* is the clearest negative outlier. The EA is therefore not a small-effect object. It is a net object built from several large sectoral trade margins with different signs.

Four margins organize this accounting. First, trade diversion toward the US is the clearest positive force in the data. The EA–US margin is positive in all 20 tariffed sectors, and especially large in *other manufacturing*, *electronics*, *textiles*, *pharmaceuticals*, and *chemicals*. Second, weaker Chinese demand is the clearest negative force. The EA–China margin is negative in all 20 sectors, so every positive US margin is partially offset by weaker exposure to China. Third, broader multilateral and network adjustment is present, but smaller: the ROW margin is more mixed across sectors and does not organize the ranking the way the US and China margins do. Fourth, exchange-rate and relative-price adjustment matters through the bilateral reallocations it supports, even if it is not isolated directly by Figure 16.

The right interpretation is therefore not that the EA is insulated. It is that the EA gains only where the positive US-diversion margin is large enough and concentrated enough to survive the negative China margin and the smaller multilateral offsets. This is why the EA remains an offsetting-margins object even at the sectoral level: there are large gross gains, but they are concentrated in a few sectors and they do not generalize into broad-based aggregate expansion.

Taken together, the sectoral evidence leaves three conclusions. Direct bilateral exposure sorts the sectors that matter most. Domestic propagation determines whether that local disturbance survives as a country-level aggregate outcome. And the EA remains a third-country net object in which large gross reallocations offset in aggregate. With that sectoral foundation in place, the conclusion can return to the paper's aggregate claims with a cleaner account of the margins that support them.

6. Conclusions

This paper studies the short-run macroeconomic consequences of a US–China tariff war in a four-block, multi-sector New Keynesian model with input–output linkages, sectoral nominal rigidities, and heterogeneous invoicing regimes. The framework is sufficiently rich to speak to aggregate inflation and output responses in a tariff conflict, but also sufficiently granular to trace those responses to the sectors and margins through which they propagate.

Three conclusions emerge. First, the aggregate incidence of tariffs depends crucially on production networks. In the benchmark reciprocal tariff experiment, the targeted economy bears the larger output loss, but that loss is not the mechanical consequence of weaker bilateral exports alone. It is amplified by the propagation of the shock through domestic and international input–output linkages. In that sense, tariffs are transmitted not only through expenditure switching, but through the wider production structure in which trade flows are embedded.

Second, the invoicing structure is a first-order determinant of short-run transmission. Under dominant-currency pricing, exchange-rate adjustment provides less insulation to the targeted economy because border prices do not move in the same way as under producer-currency pricing. This changes both the magnitude and the geography of the adjustment. The main role of invoicing is therefore not to alter the sign of the benchmark results, but to govern how much of the adjustment takes place through quantities, through prices, and through third-country spillovers at policy-relevant horizons.

Third, sectoral incidence is a net object rather than a direct-exposure object. The sectors that matter most are not identified solely by the bilateral margin on which the tariff first lands. In the directly exposed economies, what happens inside the tariffed sector must be combined with the response of the rest of the domestic economy. In the US, protected sectors often expand on their own, but those gains can be materially offset by losses elsewhere. In China, domestic propagation typically reinforces an already negative own-sector shock. In the EA, by contrast, domestic cross-sector propagation is smaller, and the relevant margin is instead the balance between stronger exports to the US, weaker exports to China, and smaller multilateral adjustments. The sectoral results therefore reinforce the aggregate message of the paper: bilateral trade exposure is informative, but not sufficient, for understanding tariff incidence.

Two broader implications follow. The first is substantive. In a networked world economy, the burden of a tariff is not confined to the bilateral trade relationship at which the policy is aimed. It depends on how the shock is transmitted through intermediate-input linkages, on how border prices are invoiced, and on how third countries absorb the induced reallocation of trade. The second is methodological. For short-run policy analysis, a framework that combines production networks, nominal rigidities, and heterogeneous invoicing can deliver a materially different assessment from one built on bilateral trade shares alone.

The analysis also has clear limitations. The model is solved using a first-order perturbation around a zero-tariff steady state and therefore abstracts from nonlinearities, endogenous supply-chain restructuring, and strategic tariff dynamics. The first-order solution also holds input–output shares fixed at their steady-state values, so the production-network amplification documented in Section 4.2 should be interpreted as the propagation implied by existing trade and production structures rather than a prediction of how those structures would evolve under persistent tariff regimes. Firms do not re-optimize their supplier network over time, so the propagation documented here should be interpreted as the short-run transmission implied by existing trade and production structures. A natural extension would be to combine the present framework with endogenous re-sourcing, tariff persistence uncertainty, or strategic interactions in trade policy. Another would be to bring the sectoral predictions more directly to the data, especially on third-country reallocation and the own-sector versus cross-sector decomposition.

These limitations notwithstanding, the central message is clear. The effects of tariffs cannot be read from direct bilateral exposure alone. What matters is where the tariff lands, how it propagates through production networks, how invoicing governs border-price adjustment, and how sectoral gains and losses offset in aggregate. Those margins jointly determine whether a tariff is mainly inflationary or mainly contractionary, and for which economies.

References

Acemoglu, D., Carvalho, V. M., Ozdaglar, A. and Tahbaz-Salehi, A. (2012). The network origins of aggregate fluctuations. *Econometrica*, **80** (5), 1977–2016.

Aguilar, P., Domínguez-Díaz, R., Gallegos, J.-E. and Quintana, J. (2026). The Transmission of Foreign Shocks in a Networked Economy. *Working Papers 2607, Banco de España*.

Amiti, M., Redding, S. J. and Weinstein, D. E. (2019). The impact of the 2018 tariffs on prices and welfare. *Journal of Economic Perspectives*, **33** (4), 187–210.

Armington, P. S. (1969). A theory of demand for products distinguished by place of production. *Staff Papers (International Monetary Fund)*, **16** (1), 159–178.

Atalay, E. (2017). How important are sectoral shocks? *American Economic Journal: Macroeconomics*, **9** (4), 254–280.

Baqaee, D. R. and Farhi, E. (2019). The macroeconomic impact of microeconomic shocks: Beyond hulten's theorem. *Econometrica*, **87** (4), 1155–1203.

— and — (2022). Supply and demand in disaggregated keynesian economies with an application to the covid-19 crisis. *American Economic Review*, **112** (5), 1397–1436.

— and — (2024). Networks, barriers, and trade. *Econometrica*, **92** (2), 505–541.

Boehm, C. E., Levchenko, A. A. and Pandalai-Nayar, N. (2023). The long and short (run) of trade elasticities. *American Economic Review*, **113** (4), 861–905.

Böhringer, C. and Rivers, N. (2021). The energy efficiency rebound effect in general equilibrium. *Journal of Environmental Economics and Management*, **109**, 102508.

Cakmakli, C., Demiralp, S., Kalemli-Özcan, Ş., Yesiltas, S. and Yildirim, M. A. (2021). *The Economic Case for Global Vaccinations: An Epidemiological Model with International Production Networks*. NBER Working Paper 28395, National Bureau of Economic Research, Cambridge, MA.

Calvo, G. (1983). Staggered Prices in a Utility Maximizing Framework. *Journal of Monetary Economics*, **12** (1978), 383–398.

Chetty, R., Guren, A., Manoli, D. and Weber, A. (2011). Are micro and macro labor supply elasticities consistent? a review of evidence on the intensive and extensive margins. *American Economic Review*, **101** (3), 471–75.

Christoffel, K. P., Coenen, G. and Warne, A. (2008). *The new area-wide model of the euro area: a micro-founded open-economy model for forecasting and policy analysis*. Tech. rep., ECB working paper.

Devereux, M. B. and Engel, C. (2003). Monetary policy in the open economy revisited: Price setting and exchange-rate flexibility. *Review of Economic Studies*, **70** (4), 765–783.

Dhyne, E., Alvarez, L. J., Le Bihan, H., Veronese, G., Dias, D., Hoffmann, J., Jonker, N., Lunnemann, P., Rumler, F. and Vilmunen, J. (2006). Price changes in the euro area and the united states: Some facts from individual consumer price data. *Journal of Economic Perspectives*, **20** (2), 171–192.

di Giovanni, J., Kalemli-Özcan, Ş., Silva, A. and Yildirim, M. A. (2022). *Global Supply Chain Pressures, International Trade, and Inflation*. NBER Working Paper 30240, National Bureau of Economic Research, Cambridge, MA.

—, —, — and Yildirim, M. A. (2023). *Pandemic-Era Inflation Drivers and Global Spillovers*. NBER Working Paper 31887, National Bureau of Economic Research, Cambridge, MA.

Dixit, A. K. and Stiglitz, J. E. (1977). Monopolistic competition and optimum product diversity. *The American Economic Review*, **67** (3), 297–308.

ECB (2009). Wage dynamics in europe: Final report of the wage dynamics network (wdn). *European Central Bank Report*.

EGOROV, K. and MUKHIN, D. (2023). Optimal policy under dollar pricing. *American Economic Review*, **113** (7), 1783–1824.

ERCEG, C. J., HENDERSON, D. W. and LEVIN, A. T. (2000). Optimal monetary policy with staggered wage and price contracts. *Journal of Monetary Economics*, **46** (2), 281–313.

FAJGELBAUM, P. D., GOLDBERG, P. K., KENNEDY, P. J. and KHANDELWAL, A. K. (2024). The return to protectionism. *American Economic Review: Insights*, **6** (1), 1–18.

GABAIX, X. (2011). The granular origins of aggregate fluctuations. *Econometrica*, **79** (3), 733–772.

GALÍ, J. (2015). *Monetary policy, inflation, and the business cycle: An introduction to the new Keynesian framework and its applications: Second edition*. Princeton University Press.

GAUTIER, E., CONFLITTI, C., FABER, R. P., FABO, B., FADEJEVA, L., JOUVANCEAU, V., MENZ, J.-O., MESSNER, T., PETROULAS, P., ROLDAN-BLANCO, P., RUMLER, F., SANTORO, S., WIELAND, E. and ZIMMER, H. (2024). New facts on consumer price rigidity in the euro area. *American Economic Journal: Macroeconomics*, **16** (4), 386–431.

GOPINATH, G., BOZ, E., CASAS, C., DÍEZ, F. J., GOURINCHAS, P.-O. and PLAGBORG-MOELLER, M. (2020). Dominant currency paradigm. *American Economic Review*, **110** (3), 677–719.

KALEMLI-ÖZCAN, Ş., SOYLU, C. and YILDIRIM, M. A. (2025). *Global Networks, Monetary Policy and Trade*. NBER Working Paper 33686, National Bureau of Economic Research, Cambridge, MA.

KOUVAVAS, O., OSBAT, C., REINELT, T. and VANSTEENKISTE, I. (2021). *Markups and inflation cyclicality in the euro area*. Tech. rep., ECB Working Paper.

PASTEN, E., SCHOENLE, R. and WEBER, M. (2020). The propagation of monetary policy shocks in a heterogeneous production economy. *Journal of Monetary Economics*, **116**, 1–22.

RUBBO, E. (2023). Networks, phillips curves, and monetary policy. *Econometrica*, **91** (4), 1417–1455.

SCHMITT-GROHE, S. and URIBE, M. (2003). Closing small open economy models. *Journal of International Economics*, **61** (1), 163–185.

A. Model Derivation and Log-linearization

In this section, we derive model equilibrium conditions, outlining the final set of log-linearized equations. We further enlarge the theoretical model presented in the main text—which only contains exogenous perturbations to the foreign price wedges and the monetary policy shock—to accommodate the standard shocks in the literature: an internal demand shock, sectoral TFP, sectoral price cost-push shocks, and wage cost-push shocks.

A.1. Households

Each household is made up of a continuum of members, each specialized in a different labor service, indexed by $g \in [0, 1]$. Income is pooled within each household, acting as a risk-sharing mechanism. The per-period utility function (1) is modified to accommodate a discount factor disturbance. Namely,

$$U_t = \left(\frac{C_{k,t}^{1-\sigma}}{1-\sigma} - \int_0^1 \frac{\mathcal{N}_{gk,t}^{1+\varphi}}{1+\varphi}\, dg \right) Z_{k,t}, \tag{A.1}$$

where $\mathcal{N}_{gk,t}$ denotes the labor supply of labor service g, $Z_{k,t}$ is an exogenous preference shifter, and σ denotes the inverse of the intertemporal elasticity of substitution.[12]

Consumption Demand Curves. The allocation optimal allocation between energy and non-energy goods is the result of a cost minimization programme min $P_{k,t}^{C,E} C_{k,t}^{E} + P_{k,t}^{C,M} C_{k,t}^{M}$ subject to (14). Similarly, the optimal allocation between energy (non-energy) consumption is the result of a cost minimization programme min $\sum_{i \in I_E} P_{ki,t}^{C} C_{ki,t}$ (min $\sum_{i \in I_M} P_{ki,t}^{C} C_{ki,t}$) subject to (4). Finally, the optimal allocation between the different consumption goods is the result of a cost minimization programme min $\sum_{l=1}^{K} (1+\tau_{kl\,i,t}) P_{kl\,i,t}^{k} C_{kl\,i,t}$ subject to (5). The implied demand curves are given by

$$P_{k,t}^{C,E} = P_{k,t}^{C} \left(\frac{\widetilde{\rho}_k^{C} C_{k,t}}{C_{k,t}^{E}} \right)^{\frac{1}{\gamma_C}} \quad \text{and} \quad P_{k,t}^{C,M} = P_{k,t}^{C} \left(\frac{(1-\widetilde{\rho}_k^{C}) C_{k,t}}{C_{k,t}^{M}} \right)^{\frac{1}{\gamma_C}} \tag{A.2}$$

$$P_{ki,t}^{C} = P_{k,t}^{C,E} \left(\frac{\widetilde{\nu}_{ki}^{C} C_{k,t}^{E}}{C_{ki,t}} \right)^{\frac{1}{\eta_C}} \quad \forall \quad i \in I_M \quad \text{and} \quad P_{ki,t}^{C} = P_{k,t}^{C,M} \left(\frac{\widetilde{\upsilon}_{ki}^{C} C_{k,t}^{M}}{C_{ki,t}} \right)^{\frac{1}{\iota_C}} \quad \forall \quad i \in I_E \tag{A.3}$$

$$(1+\tau_{kl\,i,t}) P_{kl\,i,t}^{k} = P_{ki,t}^{C} \left(\frac{\widetilde{\zeta}_{kl\,i}^{C} C_{ki,t}}{C_{kl\,i,t}} \right)^{\frac{1}{\delta_C}} \quad \forall \quad l \in K. \tag{A.4}$$

The log-linearized versions of the consumption demand curves (A.2)-(A.4) are given by

$$\widehat{p}_{k,t}^{C,E} = \frac{1}{\gamma_C} (\widehat{c}_{k,t} - \widehat{c}_{k,t}^{E}) \quad \text{and} \quad \widehat{p}_{k,t}^{C,M} = \frac{1}{\gamma_C} (\widehat{c}_{k,t} - \widehat{c}_{k,t}^{M}) \tag{A.5}$$

$$\widehat{p}_{ki,t}^{C} - \widehat{p}_{k,t}^{C,E} = \frac{1}{\eta_C} (\widehat{c}_{k,t}^{E} - \widehat{c}_{ki,t}) \quad \text{and} \quad \widehat{p}_{ki,t}^{C} - \widehat{p}_{k,t}^{C,M} = \frac{1}{\iota_C} (\widehat{c}_{k,t}^{M} - \widehat{c}_{ki,t}) \tag{A.6}$$

$$\tau_{kl\,i,t} + \widehat{p}_{kl\,i,t}^{k} - \widehat{p}_{ki,t}^{C} = \frac{1}{\delta_C} \left(\widehat{c}_{ki,t} - \widehat{c}_{kl\,i,t} \right) \tag{A.7}$$

where $\widehat{p}_{k,t}^{C,E} = p_{k,t}^{C,E} - p_{k,t}^{C}$, $\widehat{p}_{k,t}^{C,M} = P_{k,t}^{C,M} - p_{k,t}^{C}$, $\widehat{p}_{ki,t}^{C} = P_{ki,t}^{C} - p_{k,t}^{C}$, and $\widehat{p}_{kl\,i,t}^{k} = p_{kl\,i,t}^{k} - p_{k,t}^{C}$ are

[12] Each household takes as given labor income since wages are set by labor unions and employment is decided by firms. Thus, the only decisions made by the household are the optimal allocation of consumption expenditures among different good varieties across different countries, and the optimal intertemporal allocation of consumption.

well-defined as a ratio of prices.[13]

Currency Pricing Paradigm. Under PCP, using the log-linearized version of (21), we obtain

(A.8)
$$\widehat{p}^{k}_{kl\,i,t} = p^{k}_{kl\,i,t} - p^{C}_{k,t} = e^{l}_{k,t} + p_{kl\,i,t} - p^{C}_{k,t} = e^{l}_{k,t} + (p_{l\,i,t} - p^{C}_{l,t}) + (p^{C}_{l,t} - p^{C}_{k,t}) = \widehat{p}_{l\,i,t} + q^{l}_{k,t},$$

(A.9)
$$\widehat{p}^{k}_{l\,ki,t} = p^{k}_{l\,ki,t} - p^{C}_{k,t} = p_{l\,ki,t} - p^{C}_{k,t} = p_{ki,t} - p^{C}_{k,t} = \widehat{p}_{ki,t},$$

where we have used that, under PCP, $p_{kl\,i,t} = p_{l\,i,t}$ since the law of one price (LOP) is satisfied; the transformation $\widehat{p}_{l\,i,t} = p_{l\,i,t} - p^{C}_{l,t}$; and the definition of the bilateral real exchange rate between country k and country l: $\mathcal{Q}_{kl,t} = P_{l\,C,t}\mathcal{E}_{kl,t}/P^{C}_{k,t}$, which log-linearized takes the form

(A.10)
$$q^{m}_{k,t} = e^{m}_{k,t} + p^{C}_{m,t} - p^{C}_{k,t}.$$

Under LCP, using the log-linearized version of (23), we can write

(A.11)
$$\widehat{p}^{k}_{kl\,i,t} = p^{k}_{kl\,i,t} - p^{C}_{k,t} = \widehat{p}_{kl\,i,t},$$

(A.12)
$$\widehat{p}^{k}_{l\,ki,t} = p^{k}_{l\,ki,t} - p^{C}_{k,t} = e^{l}_{k,t} + p_{l\,ki,t} - p^{C}_{k,t} = (p_{l\,ki,t} - p^{C}_{l,t}) + e^{l}_{k,t} + p^{C}_{l,t} - p^{C}_{k,t} = q^{l}_{k,t} + \widehat{p}_{l\,ki,t},$$

where we have made use of (A.10), and of the definition $\widehat{p}_{kl\,i,t} = p_{kl\,i,t} - p^{C}_{k,t}$.

Under DCP, using the log-linearized version of (25), we obtain

(A.13)
$$\widehat{p}^{k}_{kl\,i,t} = p^{k}_{kl\,i,t} - p^{C}_{k,t} = e^{\mathrm{US}}_{k,t} + p_{l\,i,t} - p^{C}_{k,t} = e^{\mathrm{US}}_{k,t} + (p_{l\,i,t} - p^{C}_{\mathrm{US},t}) + p^{C}_{\mathrm{US},t} - p^{C}_{k,t} = q^{\mathrm{US}}_{k,t} + \widehat{p}_{l\,i,t},$$

(A.14)
$$\widehat{p}^{k}_{l\,ki,t} = p^{k}_{l\,ki,t} - p^{C}_{k,t} = e^{\mathrm{US}}_{k,t} + p_{ki,t} - p^{C}_{k,t} = e^{\mathrm{US}}_{k,t} + (p_{ki,t} - p^{C}_{\mathrm{US},t}) + p^{C}_{\mathrm{US},t} - p^{C}_{k,t} = q^{\mathrm{US}}_{k,t} + \widehat{p}_{ki,t}$$

where we have used that, under DCP, $p_{kl\,i,t} = p_{l\,i,t}$ although it should be noted that the LOP is not satisfied, the transformation $\widehat{p}_{l\,i,t} = p_{l\,i,t} - p^{C}_{\mathrm{US},t}$, and the definition of the bilateral real exchange rate (A.10).

[13] The individual price levels are not well-defined in steady state, but their ratio is.

To summarize pricing, using (A.8), (A.11), (A.13), (A.9), (A.12), and (A.14), we have that

$$\text{(A.15)} \qquad \widehat{p}^{k}_{kl\,i,t} = \begin{cases} q^{l}_{k,t} + \widehat{p}_{l\,i,t}, & \text{if } i \in I_{\text{PCP}}, \\ \widehat{p}_{kl\,i,t}, & \text{if } i \in I_{\text{LCP}}, \\ q^{\text{US}}_{k,t} + \widehat{p}_{l\,i,t}, & \text{if } i \in I_{\text{DCP}}. \end{cases}, \qquad \widehat{p}^{k}_{l\,ki,t} = \begin{cases} \widehat{p}_{ki,t}, & \text{if } i \in I_{\text{PCP}}, \\ q^{l}_{k,t} + \widehat{p}_{l\,ki,t}, & \text{if } i \in I_{\text{LCP}}, \\ q^{\text{US}}_{k,t} + \widehat{p}_{ki,t}, & \text{if } i \in I_{\text{DCP}}. \end{cases}$$

Consumption Baskets. The log-linearized consumption aggregator (14) is given by

$$\text{(A.16)} \qquad \widehat{c}_{k,t} = \rho^{C}_{k}\widehat{c}^{E}_{k,t} + (1-\rho^{C}_{k})\widehat{c}^{M}_{k,t}$$

where $\rho^{C}_{k} = \frac{P^{C,E}_{k}C^{E}_{k}}{P^{C}_{k}C_{k}} = (\widetilde{\rho}^{C}_{k})^{\frac{1}{\gamma_C}}\left(\frac{C^{E}_{k}}{C_{k}}\right)^{\frac{\gamma_C-1}{\gamma_C}}$ and $1-\rho^{C}_{k} = \frac{P^{C,M}_{k}C^{M}_{k}}{P^{C}_{k}C_{k}} = (1-\widetilde{\rho}^{C}_{k})^{\frac{1}{\gamma_C}}\left(\frac{C^{M}_{k}}{C_{k}}\right)^{\frac{\gamma_C-1}{\gamma_C}}$ can be verified using the steady-state consumption aggregator (14) and the demand curves (A.2).

The log-linearized versions of the energy and non-energy consumption aggregators (4) are given by

$$\text{(A.17)} \qquad \widehat{c}^{E}_{k,t} = \sum_{i \in I_E} \nu_{ki}\widehat{c}_{ki,t} \qquad \text{and} \qquad \widehat{c}^{M}_{k,t} = \sum_{i \in I_M} \upsilon_{ki}\widehat{c}_{ki,t}$$

where $\nu_{ki} = \frac{P^{C}_{ki}C_{ki}}{P^{C,E}_{k}C^{E}_{k}} = (\widetilde{\nu}^{C}_{ki})^{\frac{1}{\eta_C}}\left(\frac{C_{ki}}{C^{E}_{k}}\right)^{\frac{\eta_C-1}{\eta_C}}$ and $\upsilon_{ki} = \frac{P^{C}_{ki}C_{ki}}{P^{C,M}_{k}C^{M}_{k}} = (\widetilde{\upsilon}^{C}_{ki})^{\frac{1}{\iota_C}}\left(\frac{C_{ki}}{C^{M}_{k}}\right)^{\frac{\iota_C-1}{\iota_C}}$ can be verified using the steady-state energy and non-energy consumption aggregators (4) and the demand curves (A.3).

The log-linearized version of the final layer of the consumption aggregator, (5), is given by

$$\text{(A.18)} \qquad \widehat{c}_{ki,t} = \sum_{l=1}^{K} \zeta_{kl\,i}\widehat{c}_{kl\,i,t}$$

where $\zeta_{kl\,i} = \frac{P_{kl\,i}C_{kl\,i}}{P^{C}_{ki}C_{ki}} = (\widetilde{\zeta}^{C}_{kl\,i})^{\frac{1}{\delta_C}}\left(\frac{C_{kl\,i}}{C_{ki}}\right)^{\frac{\delta_C-1}{\delta_C}}$ can be verified using the steady-state international consumption aggregator (5) and the consumption demand curves (A.4).

Price Indices. The different price indices can be derived by combining the consumption demand curves previously derived with the different consumption aggregators. The consumption price index, the energy and non-energy price index, and the consumption import price index are given by $P^{C}_{k,t} = \left[\widetilde{\rho}^{C}_{k}(P^{C,E}_{k,t})^{1-\gamma_C} + (1-\widetilde{\rho}^{C}_{k})(P^{C,M}_{k,t})^{1-\gamma_C}\right]^{\frac{1}{1-\gamma_C}}$, $P^{C,E}_{k,t} = \left[\sum_{i \in I_E}\widetilde{\nu}^{C}_{ki}(P^{C}_{ki,t})^{1-\eta_C}\right]^{\frac{1}{1-\eta_C}}$, $P^{C,M}_{k,t} = \left[\sum_{i \in I_M}\widetilde{\upsilon}^{C}_{ki}(P^{C}_{ki,t})^{1-\iota_C}\right]^{\frac{1}{1-\iota_C}}$, and $P^{C}_{ki,t} = \left\{\sum_{l=1}^{K}\widetilde{\zeta}^{C}_{kl\,i}[(1+\tau_{kl\,i,t})P^{k}_{kl\,i,t}]^{1-\delta_C}\right\}^{\frac{1}{1-\delta_C}}$. Their log-linearized counterparts are given by

$$0 = \rho_k^C \widehat{p}_{k,t}^{C,E} + (1-\rho_k^C)\widehat{p}_{k,t}^{C,M} \tag{A.19}$$

$$\widehat{p}_{k,t}^{C,E} = \sum_{i\in I_E} \nu_{ki}\widehat{p}_{ki,t}^{C} \quad \text{and} \quad \widehat{p}_{k,t}^{C,M} = \sum_{i\in I_M} \upsilon_{ki}\widehat{p}_{ki,t}^{C} \tag{A.20}$$

$$\widehat{p}_{ki,t}^{C} = \sum_{l=1}^{K} \zeta_{kl\,i}(\widehat{p}_{kl\,i,t}^{k} + \tau_{kl\,i,t}) \tag{A.21}$$

Intertemporal Household Problem. International financial markets are incomplete, with households in each country only having access to two risk-free bonds. More precisely, the household in country k has access to a domestic bond, and an internationally traded bond, $B_{k,t}^K$, issued, without loss of generality, by country K and denominated in country K's currency. The agent maximizes the present discounted value of per-period utility flows (A.1), with discount factor β, subject to her budget constraint,

$$\begin{aligned} &P_{k,t}^C C_{k,t} + B_{k,t} + B_{k,t}^K \left[1-\Gamma(\text{NFA}_{k,t}^K)\right]^{-1} \mathcal{E}_{k,t}^K + \Xi_{k,t} \leq \\ &B_{k,t-1}(1+i_{k,t-1}) + B_{k,t-1}^K \mathcal{E}_{k,t}^K (1+i_{K,t-1}) + \int_0^1 W_{gk,t}\mathcal{N}_{gk,t}dg + \Pi_{k,t} - T_{k,t} \end{aligned} \tag{A.22}$$

where $\int_0^1 W_{gk,t}\mathcal{N}_{gk,t}dg$ is the nominal labor income received by the representative household, $\text{NFA}_{k,t}^K = B_{k,t}^K \mathcal{E}_{k,t}^K$ is the net foreign asset position of households in the country k, and where $\Gamma(x) = \gamma_* \left(\exp\left\{x/\mathcal{Y}_{k,t}\right\} - 1\right)$ is an external financial intermediary premium that depends on the economy-wide net holdings of internationally traded foreign bonds as a ratio to the national nominal GDP $\mathcal{Y}_{k,t}$, with $\gamma_* > 0$.[14] The incurred intermediation premium is rebated to households in a lump-sum manner through the fiscal instrument $\Xi_{k,t}$. Finally, $T_{k,t}$ denotes government transfers, also rebated to households in lump sum.

The above program delivers two sets of different Euler conditions,

$$C_{k,t}^{-\sigma} = \mathbb{E}_t \beta C_{k,t+1}^{-\sigma} \frac{1+i_{k,t}}{1+\pi_{k,t+1}^C} \frac{Z_{k,t+1}}{Z_{k,t}} \tag{A.23}$$

$$C_{k,t}^{-\sigma} = \mathbb{E}_t \beta C_{k,t+1}^{-\sigma} \frac{1+i_{K,t}}{1+\pi_{k,t+1}^C} \left[1-\Gamma(\text{NFA}_{k,t}^K)\right] \frac{\mathcal{E}_{k,t+1}^K}{\mathcal{E}_{k,t}^K} \frac{Z_{k,t+1}}{Z_{k,t}} \quad \forall k \neq K \tag{A.24}$$

[14] The role of this intermediation premium is to stabilize the net foreign asset position in response to transitory shocks, a common practice in open economies with incomplete financial markets (Schmitt-Grohe and Uribe 2003). Furthermore, this specification guarantees that, in the non-stochastic steady state, households have no incentive to hold foreign bonds and the economy's net foreign asset position is zero.

where we assume that the (log-)demand shock follows an AR(1) process:

$$z_{k,t} = \rho_k^z z_{k,t-1} + \varepsilon_{k,t}^z \tag{A.25}$$

where $z_{k,t} := \log Z_{k,t}$, and $\varepsilon_{k,t}^z \sim \mathcal{N}\left(0, \sigma_{kz}^2\right)$.

The log-linearized version of the household's first-order conditions (A.23)-(A.24) are given by

$$\widehat{c}_{k,t} = -\frac{1}{\sigma}(i_{k,t} - \mathbb{E}_t\pi_{k,t+1}^C) + \mathbb{E}_t\widehat{c}_{k,t+1} + \frac{1}{\sigma}(1-\rho_k^z)z_{k,t} \tag{A.26}$$

$$\widehat{c}_{k,t} = -\frac{1}{\sigma}(i_{K,t} - \mathbb{E}_t\pi_{k,t+1}^C) + \mathbb{E}_t\widehat{c}_{k,t+1} + \frac{1}{\sigma}(1-\rho_k^z)z_{k,t} - \frac{1}{\sigma}\mathbb{E}_t\Delta e_{k,t+1}^K + \frac{1}{\sigma}\gamma_*\mathrm{nfa}_{k,t}^K \quad \forall k \neq K \tag{A.27}$$

where we define the different log-linear NFA positions as $\mathrm{nfa}_{k,t}^K = B_{k,t}^K \mathcal{E}_{k,t}^K/\mathcal{Y}_k$ and $\mathrm{nfa}_{k,t}^{\mathrm{MU}} = B_{k,t}^{\mathrm{MU}}\mathcal{E}_{k\mathrm{MU},t}/\mathcal{Y}_k$ since $B_{k,t}^K = 0$ and $B_{k,t}^{\mathrm{MU}} = 0$ in the steady state, and

$$\Delta e_{k,t}^l = e_{k,t}^l - e_{k,t-1}^l. \tag{A.28}$$

Combining the log-linearized first-order conditions for the holdings of domestic and internationally traded bonds (A.26)-(A.27), yields a risk-adjusted Uncovered Interest Parity (UIP) condition

$$i_{k,t} - i_{K,t} = \mathbb{E}_t\Delta e_{k,t+1}^K - \gamma_*\mathrm{nfa}_{k,t}^K. \tag{A.29}$$

A.2. Firms

We augment the production function (12) to include a sectoral TFP shock $A_{ki,t}$,

$$Y_{kif,t} = A_{ki,t}\left[\widetilde{\alpha}_{ki}^{\frac{1}{\psi}} N_{kif,t}^{\frac{\psi-1}{\psi}} + \widetilde{\vartheta}_{ki}^{\frac{1}{\psi}} X_{kif,t}^{\frac{\psi-1}{\psi}}\right]^{\frac{\psi}{\psi-1}\psi_{ki}}, \tag{A.30}$$

where the (log-)TFP shock follows an AR(1) process:

$$a_{ki,t} = \rho_{ki}^a a_{ki,t-1} + \varepsilon_{ki,t}^a, \tag{A.31}$$

where $a_{ki,t} := \log A_{ki,t}$, and $\varepsilon_{ki,t}^a \sim \mathcal{N}\left(0, \sigma_{kia}^2\right)$.

Intermediate Input Demand Curves. The optimal allocation between labor and intermediate inputs is the result of a cost minimization programme min $W_{k,t}N_{kif,t} + P_{ki,t}^X X_{kif,t}$ subject to

(A.30). The optimal allocation between energy and non-energy intermediate goods is the result of a cost minimization programme min $P^{X,E}_{ki,t}X^E_{ki,t} + P^{X,M}_{ki,t}X^M_{ki,t}$ subject to (14). Similarly, the optimal allocation between energy (non-energy) intermediate inputs is the result of a cost minization programme min $\sum_{j\in I_E} P^X_{kij,t}X_{kij,t}$ (min $\sum_{j\in I_M} P^X_{kij,t}X_{kij,t}$) subject to (15). Finally, the optimal allocation between the different consumption goods is the result of a cost minimization programme min $\sum_{l=1}^{K}(1+\tau_{klj,t})P^k_{klj,t}X_{kl\,ij,t}$ subject to (5). The implied intermediate input demand curves are given by

$$W_{k,t} = \mathrm{MC}_{kif,t}A_{ki,t}^{\frac{\psi-1}{\psi\psi_{ki}}}\psi_{ki}Y_{kif,t}^{\frac{(1-\psi_{ki})(1-\psi)}{\psi\psi_{ki}}}\left(\frac{\widetilde{\alpha}_{ki}Y_{kif,t}}{N_{kif,t}}\right)^{\frac{1}{\psi}} \tag{A.32}$$

$$P^X_{ki,t} = \mathrm{MC}_{ki,t}A_{ki,t}^{\frac{\psi-1}{\psi\psi_{ki}}}\psi_{ki}Y_{kif,t}^{\frac{(1-\psi_{ki})(1-\psi)}{\psi\psi_{ki}}}\left(\frac{\widetilde{\vartheta}_{ki}Y_{kif,t}}{X_{kif,t}}\right)^{\frac{1}{\psi}} \tag{A.33}$$

$$P^{X,E}_{ki,t} = P^X_{ki,t}\left(\frac{\widetilde{\rho}^X_{ki}X_{ki,t}}{X^E_{ki,t}}\right)^{\frac{1}{\gamma_X}} \quad \text{and} \quad P^{X,M}_{ki,t} = P^X_{ki,t}\left(\frac{(1-\widetilde{\rho}^X_{ki})X_{ki,t}}{X^M_{ki,t}}\right)^{\frac{1}{\gamma_X}} \tag{A.34}$$

(A.35)

$$P^X_{kij,t} = P^{X,E}_{ki,t}\left(\frac{\widetilde{\nu}^X_{kij}X^E_{ki,t}}{X_{kij,t}}\right)^{\frac{1}{\eta_X}} \quad \forall \quad j\in I_E \quad \text{and} \quad P^X_{kij,t} = P^{X,M}_{ki,t}\left(\frac{\widetilde{\upsilon}^X_{kij}X^M_{ki,t}}{X_{kij,t}}\right)^{\frac{1}{\iota_X}} \quad \forall \quad j\in I_M$$

$$(1+\tau_{klj,t})P^k_{klj,t} = P^X_{kij,t}\left(\frac{\zeta^X_{kl\,ij}X_{kij,t}}{X_{kl\,ij,t}}\right)^{\frac{1}{\delta_X}} \quad \forall \quad l\in K \tag{A.36}$$

The log-linearized versions of the labor and intermediate inputs demand curves (A.32)-(A.36) are given by

$$\widehat{w}_{k,t} - \widehat{\mathrm{mc}}_{ki,t} = \frac{\psi-1}{\psi\psi_{ki}}a_{ki,t} + \frac{(1-\psi_{ki})(1-\psi)}{\psi\psi_{ki}}\widehat{y}_{kif,t} + \frac{1}{\psi}\left(\widehat{y}_{kif,t} - \widehat{n}_{kif,t}\right) \tag{A.37}$$

$$\widehat{p}^X_{ki,t} - \widehat{\mathrm{mc}}_{ki,t} = \frac{\psi-1}{\psi\psi_{ki}}a_{ki,t} + \frac{(1-\psi_{ki})(1-\psi)}{\psi\psi_{ki}}\widehat{y}_{kif,t} + \frac{1}{\psi}\left(\widehat{y}_{kif,t} - \widehat{x}_{kif,t}\right) \tag{A.38}$$

$$\widehat{p}^{X,E}_{ki,t} - \widehat{p}^X_{ki,t} = \frac{1}{\gamma_X}\left(\widehat{x}_{ki,t} - \widehat{x}^E_{ki,t}\right) \quad \text{and} \quad \widehat{p}^{X,M}_{ki,t} - \widehat{p}^X_{ki,t} = \frac{1}{\gamma_X}\left(\widehat{x}_{ki,t} - \widehat{x}^M_{ki,t}\right) \tag{A.39}$$

(A.40)

$$\widehat{p}^X_{kij,t} - \widehat{p}^{X,E}_{ki,t} = \frac{1}{\eta_X}\left(\widehat{x}^E_{ki,t} - \widehat{x}_{kij,t}\right) \quad \forall \quad j\in I_E \quad \text{and} \quad \widehat{p}^X_{kij,t} - \widehat{p}^{X,M}_{ki,t} = \frac{1}{\iota_X}\left(\widehat{x}^M_{ki,t} - \widehat{x}_{kij,t}\right) \quad \forall \quad j\in I_M$$

$$\tau_{klj,t} + \widehat{p}^k_{klj,t} - \widehat{p}^X_{kij,t} = \frac{1}{\delta_X}\left(\widehat{x}_{kij,t} - \widehat{x}_{kl\,ij,t}\right) \quad \forall \quad l\in K \tag{A.41}$$

where $\widehat{w}_{k,t} = w_{k,t} - p^C_{k,t}$, $\widehat{\mathrm{mc}}_{ki,t} = \mathrm{mc}_{ki,t} - p^C_{k,t}$, $\widehat{p}^X_{ki,t} = p^X_{ki,t} - p^C_{k,t}$, $\widehat{p}^{X,E}_{ki,t} = p^{X,E}_{ki,t} - p^C_{k,t}$, $\widehat{p}^{X,M}_{ki,t} =$

$p^{X,M}_{ki,t} - p^{C}_{k,t}$, and $\widehat{p}^{X}_{kij,t} = p^{X}_{kij,t} - p^{C}_{k,t}$.

Intermediate Inputs Baskets. The log-linearized intermediary input aggregator (14) is given by

$$\widehat{x}_{ki,t} = \rho^{X}_{ki}\widehat{x}^{E}_{ki,t} + (1-\rho^{X}_{ki})\widehat{x}^{M}_{ki,t} \tag{A.42}$$

where $\rho^{X}_{ki} = \frac{P^{X,E}_{ki}X^{E}_{ki}}{P^{X}_{ki}X_{ki}} = (\widetilde{\rho}^{X}_{ki})^{\frac{1}{\gamma_X}}\left(\frac{X^{E}_{ki}}{X_{ki}}\right)^{\frac{\gamma_X-1}{\gamma_X}}$ and $1-\rho^{X}_{ki} = \frac{P^{X,M}_{ki}X^{M}_{ki}}{P^{M}_{ki}X_{ki}} = (1-\widetilde{\rho}^{X}_{ki})^{\frac{1}{\gamma_X}}\left(\frac{X^{M}_{ki}}{X_{ki}}\right)^{\frac{\gamma_X-1}{\gamma_X}}$ can be verified using the steady-state intermediate input aggregator (14) and the input demand curves (A.34).

The log-linearized versions of the energy and non-energy intermediate input aggregators (15) are given by

$$\widehat{x}^{E}_{ki,t} = \sum_{j\in I_E} \nu^{X}_{kij}\widehat{x}_{kij,t} \qquad \text{and} \qquad \widehat{x}^{M}_{ki,t} = \sum_{j\in I_M} \upsilon^{X}_{kij}\widehat{x}_{kij,t} \tag{A.43}$$

where $\nu^{X}_{kij} = \frac{P^{X}_{kij}X_{kij}}{P^{X,E}_{ki}X^{E}_{ki}} = (\widetilde{\nu}^{X}_{kij})^{\frac{1}{\eta_X}}\left(\frac{X_{kij}}{X^{E}_{ki}}\right)^{\frac{\eta_X-1}{\eta_X}}$ and $\upsilon^{X}_{kij} = \frac{P^{X}_{kij}X_{kij}}{P^{X,M}_{ki}X^{M}_{ki}} = (\widetilde{\upsilon}^{X}_{kij})^{\frac{1}{\iota_X}}\left(\frac{X_{kij}}{X^{M}_{ki}}\right)^{\frac{\iota_X-1}{\iota_X}}$ can be verified using the steady-state energy and non-energy intermediate input aggregators (15) and the demand curves (A.35).

The log-linearized version of the final layer of the intermediate input aggregator, (5), is given by

$$\widehat{x}_{kij,t} = \sum_{l=1}^{K} \zeta^{X}_{kl\,ij}\widehat{x}_{kl\,ij,t} \tag{A.44}$$

where $\zeta^{X}_{kl\,ij} = \frac{P_{klj}X_{kl\,ij}}{P^{X}_{kij}X_{kij}} = (\widetilde{\zeta}^{X}_{kl\,ij})^{\frac{1}{\delta_X}}\left(\frac{X_{kl\,ij}}{X_{kij}}\right)^{\frac{\delta_X-1}{\delta_X}}$ can be verified using the steady-state international intermediate input aggregators (5) and the demand curve (A.36).

Price Indices. The different price indices can be derived by combining the intermediate input demand curves previously derived with the other intermediate input aggregators. The marginal cost of production, the intermediate input price index, the energy and non-energy input price index, and the input import price index are given by $MC_{ki,t} = A^{-\psi_{ki}}_{ki,t}\psi^{-1}_{ki}Y^{(1-\psi_{ki})/\psi_{ki}}_{ki,t}\left[\widetilde{\alpha}_{ki}W^{1-\psi}_{k,t} + \widetilde{\vartheta}_{ki}(P^{X}_{ki,t})^{1-\psi}\right]^{\frac{1}{1-\psi}}$, $P^{X}_{ki,t} = \left[\widetilde{\rho}^{X}_{ki}(P^{X,E}_{ki,t})^{1-\gamma_X} + (1-\widetilde{\rho}^{X}_{ki})(P^{X,M}_{ki,t})^{1-\gamma_X}\right]^{\frac{1}{1-\gamma_X}}$, $P^{X,E}_{ki,t} = \left[\sum_{j\in I_E}\widetilde{\nu}^{X}_{kij}(P^{X}_{kij,t})^{1-\eta_X}\right]^{\frac{1}{1-\eta_X}}$, $P^{X,M}_{ki,t} =$

$\left[\sum_{j\in I_M}\tilde{\upsilon}^X_{kij}(P^X_{kij,t})^{1-\iota_X}\right]^{\frac{1}{1-\eta_X}}$, and $P^X_{kij,t} = \left\{\sum_{l=1}^K \tilde{\zeta}^X_{kl\,ij}[(1+\tau_{klj,t})P^k_{klj,t}]^{1-\delta_X}\right\}^{\frac{1}{1-\delta_X}}$. Their log-linearized counterparts are given by

$$\widehat{mc}_{ki,t} = \frac{1-\psi_{ki}}{\psi_{ki}}\widehat{y}_{ki,t} - \psi_{ki}a_{ki,t} + \frac{\mathcal{M}_{ki}}{\psi_{ki}}\frac{W_kN_{ki}}{P_{ki}Y_{ki}}\widehat{w}_{k,t} + \frac{\mathcal{M}_{ki}}{\psi_{ki}}\frac{P^X_{ki}X_{ki}}{P_{ki}Y_{ki}}\widehat{p}^X_{ki,t} \tag{A.45}$$

$$\widehat{p}^X_{ki,t} = \rho^X_{ki}\widehat{p}^{X,E}_{ki,t} + (1-\rho^X_{ki})\,\widehat{p}^{X,M}_{ki,t} \tag{A.46}$$

$$\widehat{p}^{X,E}_{ki,t} = \sum_{j\in I_E}\nu^X_{kij}\widehat{p}^X_{kij,t} \quad\text{and}\quad \widehat{p}^{X,M}_{ki,t} = \sum_{j\in I_M}\upsilon^X_{kij}\widehat{p}^X_{kij,t} \tag{A.47}$$

$$\widehat{p}^X_{kij,t} = \sum_{l=1}^K \zeta^X_{kl\,ij}(\widehat{p}^k_{klj,t} + \tau_{klj,t}) \tag{A.48}$$

where $\frac{\mathcal{M}_{ki}}{\psi_{ki}}\frac{W_kN_{ki}}{P_{ki}Y_{ki}} = \left(\frac{W_kY_{ki}^{\frac{1-\psi_{ki}}{\psi_{ki}}}}{\mathrm{MC}_{ki}\psi_{ki}}\right)^{1-\psi}\tilde{\alpha}_{ki}$ and $\frac{\mathcal{M}_{ki}}{\psi_{ki}}\frac{P^X_{ki}X_{ki}}{P_{ki}Y_{ki}} = \left(\frac{P^X_{ki}Y_{ki}^{\frac{1-\psi_{ki}}{\psi_{ki}}}}{\mathrm{MC}_{ki}\psi_{ki}}\right)^{1-\psi}\tilde{\vartheta}_{ki}$ can be derived using (A.32)-(A.33) in steady-state.

Production Structure. The log-linearized version of the production function (A.30) is given by

$$\widehat{y}_{kif,t} = a_{ki,t} + \mathcal{M}_{ki}\alpha_{ki}\widehat{n}_{kif,t} + \mathcal{M}_{ki}\vartheta_{ki}\widehat{x}_{kif,t} \tag{A.49}$$

where the identities $\mathcal{M}_{ki}\alpha_{ki} = \mathcal{M}_{ki}\frac{W_kN_{ki}}{P_{ki}Y_{ki}} = \frac{N_{ki}}{Y_{ki}}\psi_{ki}Y_{ki}^{\frac{(1-\psi_{ki})(1-\psi)}{\psi\psi_{ki}}}\left(\frac{\tilde{\alpha}_{ki}Y_{ki}}{N_{ki}}\right)^{\frac{1}{\psi}}$ and $\mathcal{M}_{ki}\vartheta_{ki} = \mathcal{M}_{ki}\frac{P^X_{ki}X_{ki}}{P_{ki}Y_{ki}} = \frac{X_{ki}}{Y_{ki}}\psi_{ki}Y_{ki}^{\frac{(1-\psi_{ki})(1-\psi)}{\psi\psi_{ki}}}\left(\frac{\tilde{\vartheta}_{ki}Y_{ki}}{X_{ki}}\right)^{\frac{1}{\psi}}$ can be verified using the first-order conditions from the firms' problem (A.32) and (A.33) in steady-state and the standard monopolistic competition pricing condition in steady-state, $P_{ki} = \mathcal{M}_{ki}\mathrm{MC}_{ki}$. Using the previous identities, together with the production function (A.30) in steady-state, one can verify that

$$\mathcal{M}_{ki}\frac{W_kN_{ki}}{P_{ki}Y_{ki}} + \mathcal{M}_{ki}\frac{P^X_{ki}X_{ki}}{P_{ki}Y_{ki}} = \frac{W_kN_{ki}}{\mathrm{MC}_{ki}Y_{ki}} + \frac{P^X_{ki}X_{ki}}{\mathrm{MC}_{ki}Y_{ki}} = \psi_{ki}. \tag{A.50}$$

A.3. Price–Setting

We extend our framework to allow for a time-varying elasticity of substitution between different good varieties, in order to micro-found price cost-push shocks. We extend (6) to

$$Y^m_{kl\,i,t} = \left(\int_0^1 (Y^m_{fkl\,i,t})^{(\epsilon^p_{ki,t}-1)/\epsilon^p_{ki,t}} df\right)^{\epsilon^p_{ki,t}/(\epsilon^p_{ki,t}-1)}, \tag{A.51}$$

where $Y^m_{kl\,i,t}$ denotes the demand for domestic sector i from country k invoiced in currency m in country l. The implied sectoral price index is

$$P^m_{kl\,i,t} = \left(\int_0^1 (P^m_{fkl\,i,t})^{1-\epsilon^p_{ki,t}} df\right)^{\frac{1}{1-\epsilon^p_{ki,t}}}. \tag{A.52}$$

Producers of each differentiated variety face the demand function

$$Y^m_{kl\,i,t+s|t} = \left(P^m_{fkl\,i,t}/P^m_{kl\,i,t+s}\right)^{-\epsilon^p_{ki,t}} Y^m_{kl\,i,t+s} \tag{A.53}$$

Firms set prices à la Calvo, which implies that the aggregate price dynamics are described by the equation

$$(\Pi^m_{l\,ki,t})^{1-\epsilon^p_{ki,t}} = \theta^p_{ki} + (1-\theta^p_{ki})\left(\overline{P}^m_{l\,ki,t}/P^m_{l\,ki,t-1}\right)^{1-\epsilon^p_{ki,t}}, \tag{A.54}$$

which can be written in log-linearized terms as

$$\pi^m_{l\,ki,t} = (1-\theta^p_{ki})(\overline{p}^m_{l\,ki,t} - p^m_{l\,ki,t-1}). \tag{A.55}$$

Denote by $P^m_{l\,ki,t}$ the price of a good from sector i, originating in country k, sold in country l, and invoiced in the currency of country m. The per-period nominal profits of the domestic firm producing good from sector i are then given by

$$\Pi_{ki,t} = \sum_{l=1}^{K}\sum_{m=1}^{K} \mathcal{E}^m_{k,t} P^m_{l\,ki,t} Y^m_{l\,ki,t} - \mathcal{C}_{ki,t}(Y_{ki,t}) \tag{A.56}$$

where $Y_{ki,t} = \sum_{l=1}^{K}\sum_{m=1}^{K} Y^m_{l\,ki,t}$ denotes the total demand across destination markets l and invoicing currencies k. $\mathcal{C}_{ki,t}(Y_{ki,t})$ denotes total costs of country k firms in their home currency.

Consider the pricing problem of a firm from country k selling in country l and invoicing in currency m, and denote $\overline{P}^m_{lki,t}$ its optimally-chosen reset price, resulting from the problem

$$\text{(A.57)}\quad \max_{\overline{P}^m_{lki,t}} \sum_{s=0}^{\infty}(\theta^p_{ki})^s \mathbb{E}_t \left\{ \frac{\Lambda_{t,t+s}}{P_{ki,t+s}} \left[\sum_{l=1}^{K}\sum_{m=1}^{K} \mathcal{E}^m_{k,t+s}\overline{P}^m_{lki,t} Y^m_{lki,t+s|t} - \mathcal{C}_{ki,t+s}\left(\sum_{l=1}^{K}\sum_{m=1}^{K} Y^m_{lki,t+s|t} \right) \right] \right\}$$

subject to the sequence of demand constraints (A.53), with $P_{ki,t} = P^k_{kki,t}$. The optimality condition associated with the problem takes the form

$$\text{(A.58)}\qquad \sum_{s=0}^{\infty}(\theta^p_{ki})^s \mathbb{E}_t \left\{ \frac{\Lambda_{t,t+s} Y^m_{lki,t+s|t}}{P_{ki,t+s}} [\mathcal{E}^m_{k,t+s}\overline{P}^m_{lki,t} - \mathcal{M}_{ki,t+s}\text{MC}_{ki,t+s|t}] \right\} = 0.$$

A first-order Taylor expansion of (A.58) around the zero inflation steady state yields

$$\text{(A.59)}\qquad \overline{p}^m_{lki,t} = (1-\beta\theta^p_{ki}) \sum_{s=0}^{\infty}(\beta\theta^p_{ki})^s \mathbb{E}_t \left(\text{mc}_{ki,t+s|t} - e^m_{k,t+s} + \mu_{ki,t+s} \right).$$

The log marginal cost for an individual firm that last set its price in period t is given by (A.37),

$$\text{mc}_{ki,t+s|t} = w_{k,t+s} - \frac{\psi-1}{\psi\psi_{ki}} a_{ki,t+s} - \log\psi_{ki} - \frac{(1-\psi_{ki})(1-\psi)}{\psi\psi_{ki}} y_{ki,t+s|t} - \frac{1}{\psi}\left[\log\tilde{\alpha}_{ki} + y_{ki,t+s|t} - n_{ki,t+s|t} \right],$$

where $n_{ki,t+s|t}$ denotes the log employment in period $t+s$ for a firm that last reset its price in period t, and where we have made use of (A.37). Letting $\text{mc}_{ki,t} = \int_0^1 \text{mc}_{kif,t}\, df = (1-\theta^p_{ki})\sum_{s=0}^{\infty}(\theta^p_{ki})^s \text{mc}_{ki,t|t-s}$ represent the log average marginal cost, it follows that

$$\text{mc}_{ki,t} = w_{k,t} - \frac{\psi-1}{\psi\psi_{ki}} a_{ki,t} - \log\psi_{ki} - \frac{(1-\psi_{ki})(1-\psi)}{\psi\psi_{ki}} y_{ki,t} - \frac{1}{\psi}\left[\log\tilde{\alpha}_{ki} + y_{ki,t} - n_{ki,t} \right].$$

Thus, the following relation holds between firm-specific and economy-wide marginal costs

$$\text{mc}_{ki,t+s|t} = \text{mc}_{ki,t+s} - \frac{(1-\psi_{ki})(1-\psi)}{\psi\psi_{ki}} \left(y_{ki,t+s|t} - y_{ki,t+s} \right) - \frac{1}{\psi}\left[\left(y_{ki,t+s|t} - y_{ki,t+s} \right) - \left(n_{ki,t+s|t} - n_{ki,t+s} \right) \right].$$

Notice that, making use of both marginal cost expressions (A.32)-(A.33), the identity $x_{ki,t+s|t} - x_{ki,t+s} = n_{ki,t+s|t} - n_{ki,t+s}$ is satisfied. Hence, we can write

$$y_{ki,t+s|t} - y_{ki,t+s} = \left[\mathcal{M}_{ki}\frac{W_k N_{ki}}{P_{ki}Y_{ki}} + \mathcal{M}_{ki}\frac{P^X_{ki}X_{ki}}{P_{ki}Y_{ki}} \right](n_{ki,t+s|t} - n_{ki,t+s}) = \psi_{ki}(n_{ki,t+s|t} - n_{ki,t+s}),$$

where we have used the identity (A.50), and where we have used the linearized production function (A.49). We can finally write the relation between marginal costs as

$$\text{mc}_{ki,t+s|t} = \text{mc}_{ki,t+s} + \frac{1-\psi_{ki}}{\psi_{ki}}\left(y_{ki,t+s|t} - y_{ki,t+s}\right) = \text{mc}_{ki,t+s} - \frac{(1-\psi_{ki})\epsilon^p_{ki}}{\psi_{ki}}\left(\overline{p}^m_{lki,t} - p^m_{lki,t+s}\right).$$

Introducing this last expression into the log-linearized firms' FOC (A.59), we can write

$$\overline{p}^m_{lki,t} = (1-\beta\theta^p_{ki})\sum_{s=0}^{\infty}(\beta\theta^p_{ki})^s\mathbb{E}_t\left[\text{mc}_{ki,t+s} - \frac{(1-\psi_{ki})\epsilon^p_{ki}}{\psi_{ki}}\left(\overline{p}^m_{lki,t} - p^m_{lki,t+s}\right) - e^m_{k,t+s} + \mu_{ki,t+s}\right] \tag{A.60}$$

$$= (1-\beta\theta^p_{ki})\sum_{s=0}^{\infty}(\beta\theta^p_{ki})^s\mathbb{E}_t\left[\Theta_{ki}\text{mc}_{ki,t+s} + (1-\Theta_{ki})\,p^m_{lki,t+s} - \Theta_{ki}e^m_{k,t+s} + \Theta_{ki}\mu_{ki,t+s}\right] \tag{A.61}$$

Rewriting the (linearized) inflation dynamics (A.62) as

$$\overline{p}^m_{lki,t} - p^m_{lki,t} = \frac{\theta^p_{ki}}{1-\theta^p_{ki}}\pi^m_{lki,t}. \tag{A.62}$$

Subtracting $p^m_{lki,t}$ on both sides of (A.59), and inserting (A.62) on the LHS,

$$\begin{aligned}\frac{\theta^p_{ki}}{1-\theta^p_{ki}}\pi^m_{lki,t} &= (1-\beta\theta^p_{ki})\sum_{s=0}^{\infty}(\beta\theta^p_{ki})^s\mathbb{E}_t\left[\Theta_{ki}(\text{mc}_{ki,t+s} - p^m_{lki,t+s} - e^m_{k,t+s}) + p^m_{lki,t+s} - p^m_{lki,t} + \Theta_{ki}\mu_{ki,t+s}\right]\\ &= \Theta_{ki}(1-\beta\theta^p_{ki})\sum_{s=0}^{\infty}(\beta\theta^p_{ki})^s\mathbb{E}_t\left(\text{mc}_{ki,t+s} - p^m_{lki,t+s} - e^m_{k,t+s} + \mu_{ki,t+s}\right)\\ &+ (1-\beta\theta^p_{ki})\sum_{s=1}^{\infty}(\beta\theta^p_{ki})^s\mathbb{E}_t\pi^m_{lki,t+s}.\end{aligned} \tag{A.63}$$

Taking conditional expectations at time $t+1$, multiplying by $\beta\theta^p_{ki}$, and rearranging terms using (A.55), we can write

$$\pi^m_{lki,t} = \kappa_{ki}\left(\text{mc}_{ki,t} - p^m_{lki,t} - e^m_{k,t}\right) + \beta\mathbb{E}_t\pi^m_{lki,t+1} + u^p_{ki,t}, \tag{A.64}$$

where $\kappa_{ki} = \Theta_{ki}(1-\theta^p_{ki})(1-\beta\theta^p_{ki})/\theta^p_{ki}$.

We assume that the sectoral price cost-push shocks $u^p_{ki,t} = \kappa_{ki}\widehat{\mu}_{ki,t} = \kappa_{ki}(\mu_{ki,t} - \mu_{ki})$, micro-founded through a time-varying elasticity of substitution $\epsilon^p_{ki,t}$ in (6), follow independent AR(1) processes:

$$u^p_{ki,t} = \rho^p_{ki}u^p_{ki,t-1} + \varepsilon^p_{ki,t}, \tag{A.65}$$

where $u^{p}_{ki,t} \sim \mathcal{N}\left(0, \sigma^2_{ki\,p}\right)$.

Producer Currency Pricing. Under PCP ($m = k$), we have $\pi^{l}_{l\,ki,t} = \pi_{ki,t}$, and we can write (A.64) as

$$
\begin{aligned}
\pi_{ki,t} &= \kappa_{ki}\left(\mathrm{mc}_{ki,t} - p_{ki,t}\right) + \beta\mathbb{E}_t\pi_{ki,t+1} + u^{p}_{ki,t} \\
&= \kappa_{ki}\left[(\mathrm{mc}_{ki,t} - p^{C}_{k,t}) - (p_{ki,t} - p^{C}_{k,t})\right] + \beta\mathbb{E}_t\pi_{ki,t+1} + u^{p}_{ki,t} \\
&= \kappa_{ki}\left(\widehat{\mathrm{mc}}_{ki,t} - \widehat{p}_{ki,t}\right) + \beta\mathbb{E}_t\pi_{ki,t+1} + u^{p}_{ki,t}, \qquad \text{(A.66)}
\end{aligned}
$$

where the sectoral-inflation rates and sectoral-level real prices ($\widehat{p}_{ki,t}$) are related through the identity

$$
\pi_{ki,t} = p_{ki,t} - p_{ki,t-1} = \widehat{p}_{ki,t} - \widehat{p}_{ki,t-1} + \pi^{C}_{k,t}, \qquad \text{(A.67)}
$$

where $\widehat{p}_{ki,t} = p_{ki,t} - p^{C}_{k,t}$.

Local Currency Pricing. Under LCP ($m = l$), we have $\pi^{l}_{l\,ki,t} = \pi_{l\,ki,t}$, and we can write (A.64) as

$$
\begin{aligned}
\pi_{l\,ki,t} &= \kappa_{ki}\left(\mathrm{mc}_{ki,t} - p_{l\,ki,t} - e^{l}_{k,t}\right) + \beta\mathbb{E}_t\pi_{l\,ki,t+1} + u^{p}_{ki,t} \\
&= \kappa_{ki}\left[(\mathrm{mc}_{ki,t} - p^{C}_{k,t}) - (p_{l\,ki,t} - p^{C}_{k,t}) - e^{l}_{k,t}\right] + \beta\mathbb{E}_t\pi_{l\,ki,t+1} + u^{p}_{ki,t} \\
&= \kappa_{ki}\left[(\mathrm{mc}_{ki,t} - p^{C}_{k,t}) - (p_{l\,ki,t} - p^{C}_{l,t}) + p^{C}_{k,t} - p^{C}_{l,t} - e^{l}_{k,t}\right] + \beta\mathbb{E}_t\pi_{l\,ki,t+1} + u^{p}_{ki,t} \\
&= \kappa_{ki}\left(\widehat{\mathrm{mc}}_{ki,t} - \widehat{p}_{l\,ki,t} - q^{l}_{k,t}\right) + \beta\mathbb{E}_t\pi_{l\,ki,t+1} + u^{p}_{ki,t}, \qquad \text{(A.68)}
\end{aligned}
$$

which is the export Phillips curve, where we have used (A.10), and where

$$
\pi_{l\,ki,t} = p_{l\,ki,t} - p_{l\,ki,t-1} = \widehat{p}_{l\,ki,t} - \widehat{p}_{l\,ki,t-1} + \pi^{C}_{l,t} \qquad \text{(A.69)}
$$

denotes price inflation in sector i exported to country l from country k, where $\widehat{p}_{l\,ki,t} = p_{l\,ki,t} - p^{C}_{l,t}$.

Dominant Currency Pricing. Under DCP ($m = \mathrm{US}$), we have $\pi^{\mathrm{US}}_{l\,ki,t} = \pi_{ki,t}$, and we can write (A.64) as

$$
\begin{aligned}
\pi_{ki,t} &= \kappa_{ki}\left(\mathrm{mc}_{ki,t} - p_{ki,t} - e^{\mathrm{US}}_{k,t}\right) + \beta\mathbb{E}_t\pi_{ki,t+1} + u^{p}_{ki,t} \\
&= \kappa_{ki}\left[(\mathrm{mc}_{ki,t} - p^{C}_{k,t}) - (p_{ki,t} - p^{C}_{\mathrm{US},t}) + p^{C}_{k,t} - p^{C}_{\mathrm{US},t} - e^{\mathrm{US}}_{k,t}\right] + \beta\mathbb{E}_t\pi_{ki,t+1} + u^{p}_{ki,t}
\end{aligned}
$$

(A.70) $$= \kappa_{ki}\left(\widehat{\mathrm{mc}}_{ki,t} - \widehat{p}_{ki,t} - q^{\mathrm{US}}_{k,t}\right) + \beta\mathbb{E}_t\pi_{ki,t+1} + u^p_{ki,t}.$$

where the sectoral-inflation rates and sectoral-level real prices ($\widehat{p}_{ki,t}$) are related through the identity

(A.71) $$\pi_{ki,t} = p_{ki,t} - p_{ki,t-1} = \widehat{p}_{ki,t} - \widehat{p}_{ki,t-1} + \pi^C_{\mathrm{US},t},$$

where $\widehat{p}_{ki,t} = p_{ki,t} - p^C_{\mathrm{US},t}$. For domestic sales, $k = l$, we have that

(A.72) $$\pi_{kki,t} = \pi_{ki,t} \quad \text{and} \quad \widehat{p}_{kki,t} = \widehat{p}_{ki,t},$$

and both the Phillips curve and the sectoral inflation rates are given by (A.66) and (A.67), respectively.

Marginal costs and aggregate inflation. In this open input-output (IO) economy, the price Phillips curves depends on the international supply network through the real marginal costs faced by firm i in country k, $\widehat{\mathrm{mc}}_{ki,t}$. Combining the log-linearized intermediate input prices indices (A.45)-(A.48), we obtain the marginal cost equation,

(A.73) $$\widehat{\mathrm{mc}}_{ki,t} = \frac{1-\psi_{ki}}{\psi_{ki}}\widehat{y}_{ki,t} - \psi_{ki}a_{ki,t} + \mathcal{M}_{ki}\alpha_{ki}\widehat{w}_{k,t} + \sum_{l=1}^{K}\sum_{j=1}^{I}\mathcal{M}_{ki}\omega_{klij}\left(\widehat{p}^k_{klj,t} + \tau_{klj,t}\right)$$

where, in the absence of a production subsidy, $\alpha_{ki} = \frac{W_k N_{ki}}{P_{ki}Y_{ki}} = \frac{W_k N_{ki}}{\mathcal{M}_{ki}\mathrm{MC}_{ki}Y_{ki}}$ denotes the (steady-state) labor income share of total sales of firm i, $\omega_{klij} = \frac{P_{klj}X_{klij}}{P_{ki}Y_{ki}} = \frac{P_{klj}X_{klij}}{\mathcal{M}_{ki}\mathrm{MC}_{ki}Y_{ki}}$ denotes the (steady-state) IO expenditure share of total sales of firm i, and $\mathcal{M}_{ki} = \epsilon^p_{ki}/(\epsilon^p_{ki} - 1)$ denotes the steady-state markup charged by firm i.

Writing (A.19)-(A.21) in first-differences, we can obtain consumer price inflation,

(A.74) $$\pi^C_{k,t} = \sum_{i=1}^{I}\sum_{l=1}^{K}\beta_{kli}\pi^k_{kli,t}$$

where $\beta_{kli} = \frac{P_{kli}C_{kli}}{P^C_k C_k} = \zeta_{kli}\left[\nu_{ki}\rho^C_k\mathbb{1}_{\{i\in I_E\}} + \upsilon_{ki}(1-\rho^C_k)\left(1 - \mathbb{1}_{\{i\in I_E\}}\right)\right]$, and

(A.75) $$\pi^k_{kli,t} = p^k_{kli,t} - p^k_{kli,t-1} = \widehat{p}^k_{kli,t} - \widehat{p}^k_{kli,t-1} + \pi^C_{k,t}.$$

A.4. Wage–Setting

Following Erceg *et al.* (2000), wage stickiness is introduced in a way analogous to price stickiness. Labor unions specialized in any given labor type can reset their nominal wage only with probability $1-\theta_k^w$ each period, independently of the time elapsed since they last adjusted their wage. We assume that firms employ a continuum of differentiated labor services. In particular, $N_{kif,t}$ is an index of labor input used by firm f, and defined by

$$N_{kif,t} = \left(\int_0^1 N_{fgki,t}^{\frac{\epsilon_k^w - 1}{\epsilon_k^w}} dg \right)^{\frac{\epsilon_k^w}{\epsilon_k^w - 1}}, \tag{A.76}$$

where $N_{fgki,t}$ denotes the quantity of type-g labor employed by firm f in period t. Note that $\epsilon_{k,t}^w$ represents the elasticity of substitution among labor varieties. Note also the assumption of a continuum of labor types, indexed by $g \in [0,1]$.

Let $W_{gk,t}$ denote the nominal wage for type-g labor prevailing in period t. Nominal wages are set by workers of each type (or a union representing them) and taken as given by firms. Given the wages effective at any point in time for the different types of labor services, cost minimization yields a corresponding set of demand schedules for each firm f and labor type g, given the firm's total employment $N_{fk,t}$,

$$N_{fgki,t} = \left(\frac{W_{gk,t}}{W_{k,t}} \right)^{-\epsilon_{k,t}^w} N_{kif,t}, \tag{A.77}$$

where

$$W_{k,t} \equiv \left(\int_0^1 W_{gk,t}^{1-\epsilon_{k,t}^w} dg \right)^{\frac{1}{1-\epsilon_{k,t}^w}} \tag{A.78}$$

is an aggregate wage index. Combining the previous conditions, one can obtain a convenient aggregation result, $\int_0^1 W_{gk,t} N_{fgki,t} dg = W_{k,t} N_{kif,t}$. That is, the wage bill of any given firm can be expressed as the product of the wage index and the firm's employment index.

Consider a union resetting its members' wage in period t, and let $W_{k,t}^*$ denote the newly set wage. The union chooses $W_{k,t}^*$ in a way consistent with utility maximization of its members' households, taking as given the decisions of other unions as well as the paths of aggregate consumption and prices. Specifically, the union seeks to maximize (9) subject to the sequence of labor demand schedules (10), where $N_{k,t+s|t}$ denotes the level of employment in period

$t + s$ among workers that last reset their wage in period t. The first-order condition is given by

$$\sum_{s=0}^{\infty}(\beta\theta_k^w)^l\,\mathbb{E}_t\left[N_{k,t+s|t}C_{k,t+s}^{-\sigma}\left(\frac{W_{k,t}^*}{P_{k,t+s}^C}-\mathcal{M}_{k,t}^w\mathrm{MRS}_{k,t+s|t}^{\varphi}\right)\right]=0,$$

where $\mathcal{M}_{k,t}^w = \frac{\epsilon_{k,t}^w}{\epsilon_{k,t}^w-1}$, and $\mathrm{MRS}_{k,t+s|t} = C_{t+s}^{-\sigma}N_{k,t+s|t}^{\varphi}$ denotes the marginal rate of substitution between household consumption and employment in period $t+s$ relevant to the workers resetting their wage in period t. Log-linearizing the above expression around a zero inflation steady-state yields the wage setting rule

$$w_{k,t}^* = (1-\beta\theta_k^w)\sum_{s=0}^{\infty}(\beta\theta_k^w)^l\,\mathbb{E}_t\left(\mathrm{mrs}_{t+s|t}+\mu_{k,t}^w+p_{k,t+s}^C\right) \tag{A.79}$$

where $\mu_{k,t}^w = \log\mathcal{M}_{k,t}^w$ and $\mathrm{mrs}_{t+s|t} = \sigma c_{k,t+s}+\varphi n_{k,t+s|t}$.

Letting $\mathrm{mrs}_{t+s} = \sigma c_{k,t+s}+\varphi n_{k,t+s}$ define the economy's average marginal rate of substitution, where $n_{k,t+s} = \log\int_0^1\int_0^1 N_{fgk}df\,dg$ denotes the log aggregate employment. Up to a first-order approximation,

$$\mathrm{mrs}_{t+s|t} = \mathrm{mrs}_{t+s}+\varphi(n_{k,t+s}-n_{k,t+s|t}) = \mathrm{mrs}_{t+s}-\epsilon_k^w\varphi(w_{k,t}^*-w_{k,t+s}).$$

Hence, we can write (A.79) as

$$w_{k,t}^* = \frac{1-\beta\theta_k^w}{1+\epsilon_k^w\varphi}\sum_{s=0}^{\infty}(\beta\theta_k^w)^l\,\mathbb{E}_t\left[(1+\epsilon_k^w\varphi)w_{k,t+s}-\left(w_{k,t+k}-p_{k,t+s}^C-\mathrm{mrs}_{k,t+s}-\mu_{k,t+s}^w\right)\right] \tag{A.80}$$

where $\mu_{k,t+s}^w$ denotes the deviations of the economy's log average wage markup.

Given the assumed wage setting structure, the evolution of the aggregate wage index is given by

$$W_{k,t} = \left(\theta_k^w W_{k,t-1}^{1-\epsilon_k^w}+(1-\theta_k^w)(W_{k,t}^*)^{1-\epsilon_k^w}\right)^{\frac{1}{1-\epsilon_k^w}}.$$

Log-linearized,

$$w_{k,t} = \theta_k^w w_{k,t-1}+(1-\theta_k^w)w_{k,t}^*.$$

Combining the last expression with (A.80), and letting $\pi_{k,t}^w = w_{k,t}-w_{k,t-1}$, we obtain the wage inflation equation:

$$\pi_{k,t}^w = \kappa_k^w\left(\sigma\widehat{c}_{k,t}+\varphi\widehat{n}_{k,t}-\widehat{w}_{k,t}\right)+\beta\mathbb{E}_t\pi_{k,t+1}^w+u_{ki,t}^w, \tag{A.81}$$

where

$$\pi^w_{k,t} = w_{k,t} - w_{k,t-1} = \widehat{w}_{k,t} - \widehat{w}_{k,t-1} + \pi^C_{k,t} \tag{A.82}$$

denotes wage inflation, where both aggregate consumption $\widehat{c}_{k,t}$ and employment $\widehat{n}_{k,t}$ appear in log-deviations from their steady-state values, and $u^w_{k,t} = \kappa^w_k \widehat{\mu}^w_{k,t} = \kappa^w_k(\widehat{\mu}^w_{k,t} - \widehat{\mu}^w_k)$.

We assume that the wage cost-push shock, micro-founded through a time-varying elasticity of substitution in the labor demand aggregator, follows an AR(1) processes:

$$u^w_{k,t} = \rho^w_k u^w_{k,t-1} + \varepsilon^w_{k,t} \tag{A.83}$$

where $u^w_{k,t} \sim \mathcal{N}\left(0, \sigma^2_{kw}\right)$.

A.5. Monetary Authority

The log-linearized bilateral nominal exchange rate is given by $e^{k^{\text{MU}}}_{k,t} = e^{k^{\text{MU}}}_{k,}$, $\forall k \in K^{MU}$. In stationary terms, taking first differences, this can be written as

$$\Delta e^{k^{\text{MU}}}_{k,t} = 0 \quad \forall k \in K^{MU} \tag{A.84}$$

Log-linearizing the expression for the real exchange rate (A.10) and first-differencing, we obtain

$$\Delta q^l_{k,t} = \Delta e^l_{k,t} + \pi_{l,t} - \pi_{k,t}. \tag{A.85}$$

The nominal exchange rates are cross-related by

$$e^l_{k,t} = e^m_{k,t} - e^m_{l,t}, \tag{A.86}$$

and symmetry between nominal exchange rates is satisfied,

$$e^l_{k,t} = -e^k_{l,t}. \tag{A.87}$$

Log-linearizing and first-differencing the symmetry of nominal exchange rates condition $\mathcal{E}^l_{k,t} = (\mathcal{E}^k_{l,t})^{-1}$ yields

$$\Delta e^l_{k,t} = -\Delta e^k_{l,t}. \tag{A.88}$$

A.6. Market Clearing, GDP, and Trade Balance

Market Clearing. We first consider the goods market clearing condition (29). Pre-multiplying by $\frac{P_{ki,t}}{P_{k,t}C_{k,t}} = \frac{P_{ki,t}}{E_{k,t}}$, and making use of (A.17),

$$\begin{aligned}
\frac{P_{ki,t}Y_{ki,t}}{E_{k,t}} &= \sum_{l=1}^{K} \frac{P_{ki,t}C_{lki,t}}{E_{k,t}} + \sum_{l=1}^{K}\sum_{j=1}^{I} \frac{P_{ki,t}X_{lkji,t}}{E_{k,t}} \\
&= \sum_{l=1}^{K} \frac{P_{ki,t}}{P_{lki,t}} \frac{P_{lki,t}C_{lki,t}}{E_{k,t}} + \sum_{l=1}^{K}\sum_{j=1}^{I} \frac{P_{ki,t}}{P_{lki,t}} \frac{P_{lj,t}Y_{lj,t}}{E_{k,t}} \frac{P_{lki,t}X_{lkji,t}}{P_{lj,t}Y_{lj,t}} \\
&= \sum_{l=1}^{K} \frac{P_{ki,t}}{P_{lki,t}} \frac{E_{l,t}}{E_{k,t}} \frac{P_{lki,t}C_{lki,t}}{E_{l,t}} + \sum_{l=1}^{K}\sum_{j=1}^{I} \frac{P_{ki,t}}{P_{lki,t}} \frac{E_{l,t}}{E_{k,t}} \frac{P_{lj,t}Y_{lj,t}}{E_{l,t}} \frac{P_{lki,t}X_{lkji,t}}{P_{lj,t}Y_{lj,t}} \quad \forall i \in I
\end{aligned} \tag{A.89}$$

which we can write in steady-state as

$$\lambda_{ki} = \sum_{l=1}^{K} \mathcal{Y}_{lk}\beta_{lki} + \sum_{l=1}^{K}\sum_{j=1}^{I} \mathcal{Y}_{lk}\lambda_{lj}\omega_{lkji} \quad \forall i \in I$$

where the Domar weight for sector i in country k is $\lambda_{ki} = \frac{P_{ki}Y_{ki}}{\mathcal{Y}_k}$, the nominal GDP ratio between countries l and k is defined as $\mathcal{Y}_{lk} = \frac{P_{lC}C_l}{P_k^C C_k}$, and the IO share is given by $\omega_{lkji} = \frac{P_{lki}X_{lkji}}{P_{lj}Y_{lj}} = \frac{P_{ki}X_{lkji}}{P_{lj}Y_{lj}} = \zeta_{lkji}\vartheta_{lj}\left[\nu_{lji}\beta_{lj}\mathbb{1}_{\{i\in I_E\}} + \upsilon_{lji}(1-\beta_{lj})\left(1-\mathbb{1}_{\{i\in I_E\}}\right)\right]$, where we have made use of the law of one price in steady-state, $P_{klj} = P_{lj}$. Notice that β_{lki} and ω_{lkji} can be extracted directly from the data.

Hence, we can write the log-linearized version of the goods market clearing condition (29), $Y_{ki}\widehat{y}_{ki,t} = \sum_{l=1}^{K}\left(C_{lki}\widehat{c}_{lki,t} + \sum_{j=1}^{I} X_{lkji}\widehat{x}_{lkji}\right)$. Pre-multiplying the expression by P_{ki}/E_k, we can write

$$\lambda_{ki}\widehat{y}_{ki,t} = \sum_{l=1}^{K} \mathcal{Y}_{lk}\left(\beta_{lki}\widehat{c}_{lki,t} + \sum_{j=1}^{I}\lambda_{lj}\omega_{lkji}\widehat{x}_{lkji}\right) \tag{A.90}$$

where we have pre-multiplied the first expression by $\frac{P_{ki}}{E_k}$.

The log-linearized version of the labor market clearing condition (30) is given by

$$\widehat{n}_{k,t} = \sum_{i=1}^{I} \delta_{ki}\widehat{n}_{ki,t} \tag{A.91}$$

where $\delta_{ki} = \frac{N_{ki}}{N_k} = \frac{W_k N_{ki}}{P_{ki}Y_{ki}} \frac{P_{ki}Y_{ki}}{P_k^C C_k} \frac{P_k^C C_k}{W_k N_k} = \frac{\alpha_{ki}\lambda_{ki}}{1-\sum_{j=1}^{I}\left(1-\frac{\psi_{kj}}{\mathcal{M}_{kj}}\right)\lambda_{kj}}$ can be derived using (A.96).

The NFA from the "global" country K (31) can be log-linearized to

$$\text{(A.92)} \qquad \frac{1}{\beta}\sum_{k=1}^{K-1} \mathrm{nfa}_{k,t-1}^{K} - \sum_{k=1}^{K-1} \mathrm{nfa}_{k,t}^{K} = \Upsilon_K \left(\widehat{\mathrm{exp}}_{K,t} - \widehat{\mathrm{imp}}_{K,t} + \widehat{p}_{K,t}^{\mathrm{EXP}} - \widehat{p}_{K,t}^{\mathrm{IMP}}\right),$$

the NFA from country $k \neq K$ $k \notin$ MU, (31) can be log-linearized to

$$\text{(A.93)} \qquad \mathrm{nfa}_{k,t}^{K} - \frac{1}{\beta}\mathrm{nfa}_{k,t-1}^{K} = \Upsilon_k \left(\widehat{\mathrm{exp}}_{k,t} - \widehat{\mathrm{imp}}_{k,t} + \widehat{p}_{k,t}^{\mathrm{EXP}} - \widehat{p}_{k,t}^{\mathrm{IMP}}\right)$$

where $\widehat{p}_{k,t}^{\mathrm{IMP}} = p_{k,t}^{\mathrm{IMP}} - p_{k,t}^{C}$ and $\widehat{p}_{k,t}^{\mathrm{EXP}} = p_{k,t}^{\mathrm{EXP}} - p_{k,t}^{C}$. The linearized export and import price deflators are given by:

(A.94)

$$\widehat{p}_{k,t}^{\mathrm{IMP}} = \sum_{l \neq k}\sum_{i=1}^{I} \frac{P_{kl\,i}C_{kl\,i} + \sum_{j=1}^{I} P_{kl\,i}X_{kl\,ji}}{P_k^{\mathrm{IMP}}\mathrm{IMP}_k}\widehat{p}_{kl\,i,t}^{\,k} = \sum_{l \neq k}\sum_{i=1}^{I} \Upsilon_k^{-1}\left(\beta_{kl\,i} + \sum_{j=1}^{I}\lambda_{kj}\omega_{kl\,ji}\right)\widehat{p}_{kl\,i,t}^{\,k}$$

(A.95)

$$\widehat{p}_{k,t}^{\mathrm{EXP}} = \sum_{l \neq k}\sum_{i=1}^{I} \frac{P_{ki}C_{l\,ki} + \sum_{j=1}^{I} P_{ki}X_{l\,kji}}{P_k^{\mathrm{EXP}}\mathrm{EXP}_k}\widehat{p}_{l\,ki,t}^{\,k} = \sum_{l \neq k}\sum_{i=1}^{I} \frac{\mathcal{Y}_{l\,k}}{\Upsilon_k}\left(\beta_{l\,ki} + \sum_{j=1}^{I}\lambda_{l\,j}\omega_{l\,kji}\right)\widehat{p}_{l\,ki,t}^{\,k}$$

Gross Domestic Product and Net Exports. Let us now move to nominal GDP (34). In steady state, assuming zero net exports, $P_k^{\mathrm{EXP}}\mathrm{EXP}_k - P_k^{\mathrm{IMP}}\mathrm{IMP}_k = 0$, we can write $\mathcal{Y}_k = P_k^C C_k$. Using the household's budget constraint (A.22) in steady state, we can write

$$\text{(A.96)} \qquad \mathcal{Y}_k = P_k^C C_k = W_k N_k + \Pi_k = W_k N_k + \sum_{i=1}^{I}\left(1 - \frac{\psi_{ki}}{\mathcal{M}_{ki}}\right) P_{ki}Y_{ki}$$

where the last equality makes use of (A.50).

Log-linearizing the real GDP (35) definition,

$$\text{(A.97)} \quad \widehat{y}_{k,t} = \frac{P_k^C C_k}{\mathcal{Y}_k}\widehat{c}_{k,t} + \frac{P_k^{\mathrm{EXP}}\mathrm{EXP}_k}{\mathcal{Y}_k}\widehat{\mathrm{exp}}_{k,t} - \frac{P_k^{\mathrm{IMP}}\mathrm{IMP}_k}{\mathcal{Y}_k}\widehat{\mathrm{imp}}_{k,t} = \widehat{c}_{k,t} + \Upsilon_k\left(\widehat{\mathrm{exp}}_{k,t} - \widehat{\mathrm{imp}}_{k,t}\right)$$

where second equality uses that nominal consumption expenditures will be equal nominal GDP in steady state, and $\Upsilon_k = \frac{P_k^{\mathrm{EXP}}\mathrm{EXP}_k}{\mathcal{Y}_k} = \frac{P_k^{\mathrm{IMP}}\mathrm{IMP}_k}{\mathcal{Y}_k}$ is the export (or import) share of nominal GDP.

The nominal exports expression (32) can be log-linearized to:

$$\begin{aligned}
\widehat{\exp}_{k,t} &= \sum_{l\neq k}\sum_{i\in I}\left(\frac{P_{ki}C_{lki}}{P^{\text{EXP}}_{k}\text{EXP}_k}\widehat{c}_{lki,t} + \sum_{j=1}^{I}\frac{P_{ki}X_{lki}}{P^{\text{EXP}}_{k}\text{EXP}_k}\widehat{x}_{lkji,t}\right)\\
&= \sum_{l\neq k}\sum_{i\in I}\frac{y_{lk}}{\Upsilon_k}\left(\beta_{lki}\widehat{c}_{lki,t} + \sum_{j=1}^{I}\lambda_{lj}\omega_{lkji}\widehat{x}_{lkji,t}\right)
\end{aligned}\tag{A.98}$$

where the export share of nominal GDP is given by

$$\Upsilon_k = \frac{P^{\text{EXP}}_{k}\text{EXP}_k}{y_k} = \sum_{l\neq k}\sum_{i=1}^{I} y_{lk}\left[\beta_{lki} + \sum_{j=1}^{I}\lambda_{lj}\omega_{lkji}\right] = \left(\sum_{i=1}^{I}\lambda_{ki}\right) - \sum_{i=1}^{I}\left(\beta_{kki} + \sum_{j=1}^{I}\lambda_{kj}\omega_{kkji}\right)\tag{A.99}$$

$$= \frac{P^{\text{IMP}}_{k}\text{IMP}_k}{y_k} = \sum_{l\neq k}\sum_{i=1}^{I}\left[\beta_{kli} + \sum_{j=1}^{I}\lambda_{kj}\omega_{klji}\right]\tag{A.100}$$

Similarly, the nominal imports expression (33) can be log-linearized to

$$\begin{aligned}
\widehat{\text{imp}}_{k,t} &= \sum_{l\neq k}\sum_{i\in I}\left(\frac{P_{kli}C_{kli}}{P^{\text{IMP}}_{k}\text{IMP}_k}\widehat{c}_{kli,t} + \sum_{j=1}^{I}\frac{P_{kli}X_{klji}}{P^{\text{IMP}}_{k}\text{IMP}_k}\widehat{x}_{klji,t}\right)\\
&= \sum_{l\neq k}\sum_{i\in I}\Upsilon_k^{-1}\left(\beta_{kli}\widehat{c}_{kli,t} + \sum_{j=1}^{I}\lambda_{kj}\omega_{klji}\widehat{x}_{klji,t}\right)
\end{aligned}\tag{A.101}$$

Now we can combine the linearized expression for real gdp, que the expressions for real imports and exports:

$$\begin{aligned}
\widehat{y}_{k,t} &= \widehat{c}_{k,t} + \sum_{l\neq k}\sum_{i\in I}\left(\frac{P_{ki}C_{lki}}{y_k}\widehat{c}_{lki,t} + \sum_{j=1}^{I}\frac{P_{ki}X_{lkji}}{y_k}\widehat{x}_{lkji,t} - \frac{P_{kli}C_{kli}}{y_k}\widehat{c}_{kli,t} - \sum_{j=1}^{I}\frac{P_{kli}X_{klji}}{y_k}\widehat{x}_{klji,t}\right)\\
&= \widehat{c}_{k,t} + \sum_{l\neq k}\sum_{i\in I}\left(y_{lk}\beta_{lki}\widehat{c}_{lki,t} + \sum_{j=1}^{I}y_{lk}\lambda_{lj}\omega_{lkji}\widehat{x}_{lkji,t} - \beta_{kli}\widehat{c}_{kli,t} - \sum_{j=1}^{I}\lambda_{kj}\omega_{klji}\widehat{x}_{klji,t}\right)
\end{aligned}\tag{A.102}$$

Sectoral value added and real GDP. Define sectoral nominal value added as $\text{VA}^{\text{nom}}_{ki,t} = P_{ki,t}Y_{ki,t} - \sum_{l=1}^{K}\sum_{j=1}^{I}(1+\tau_{klj,t})P_{lj,t}X_{klij,t}$ and its associated price index as $P^{\text{VA}}_{ki,t} = \text{VA}^{\text{nom}}_{ki,t}/(P_{ki}Y_{ki,t} -$

$\sum_{l=1}^{K}\sum_{j=1}^{I}P_{lj}X_{klij,t})$ where the denominator contains constant prices. Define sectoral real value added as $\mathrm{VA}_{ki,t}=\mathrm{VA}^{\mathrm{nom}}_{ki,t}/P^{\mathrm{VA}}_{ki,t}=P_{ki}Y_{ki,t}-\sum_{l=1}^{K}\sum_{j=1}^{I}P_{lj}X_{klij,t}$ which, log-linearized, can be written as

$$\widehat{\mathrm{va}}_{ki,t}=\frac{P_{ki}Y_{ki}}{P_{ki}Y_{ki}-P^{X}_{ki}X_{ki}}\widehat{y}_{ki,t}-\sum_{l=1}^{K}\sum_{j=1}^{I}\frac{P_{lj}X_{klij}}{P_{ki}Y_{ki}-P^{X}_{ki}X_{ki}}\widehat{x}_{klij,t}=\frac{1}{1-\vartheta_{ki}}\widehat{y}_{ki,t}-\sum_{l=1}^{K}\sum_{j=1}^{I}\frac{\omega_{klij}}{1-\vartheta_{ki}}\widehat{x}_{klij,t} \tag{A.103}$$

We are now interested in showing that $\sum_{i=1}^{I}\lambda_{ki}(1-\vartheta_{ki})\widehat{\mathrm{va}}_{ki,t}=\widehat{y}_{k,t}$, where the RHS variable is real GDP (A.102). Using (29),

$$\begin{aligned}\sum_{i=1}^{I}\lambda_{ki}(1-\vartheta_{ki})\widehat{\mathrm{va}}_{ki,t}&=\sum_{i=1}^{I}\lambda_{ki}\widehat{y}_{ki,t}-\sum_{i=1}^{I}\sum_{l=1}^{K}\sum_{j=1}^{I}\lambda_{ki}\omega_{klij}\widehat{x}_{klij,t}\\&=\sum_{i=1}^{I}\sum_{l=1}^{K}\mathcal{Y}_{lk}\left(\beta_{lki}\widehat{c}_{lki,t}+\sum_{j=1}^{I}\lambda_{lj}\omega_{lkji}\widehat{x}_{lkji}\right)-\sum_{i=1}^{I}\sum_{l=1}^{K}\sum_{j=1}^{I}\lambda_{ki}\omega_{klij}\widehat{x}_{klij,t}\end{aligned} \tag{A.104}$$

Separating domestic and foreign absorption, we can write

$$\begin{aligned}\sum_{i=1}^{I}\lambda_{ki}(1-\vartheta_{ki})\widehat{\mathrm{va}}_{ki,t}=&\sum_{i=1}^{I}\left(\beta_{kki}\widehat{c}_{kki,t}+\sum_{j=1}^{I}\lambda_{kj}\omega_{kkji}\widehat{x}_{kkji,t}\right)+\sum_{i=1}^{I}\sum_{l\neq k}^{K}\mathcal{Y}_{lk}\left(\beta_{lki}\widehat{c}_{lki,t}+\sum_{j=1}^{I}\lambda_{lj}\omega_{lkji}\widehat{x}_{lkji,t}\right)\\&-\sum_{i=1}^{I}\sum_{j=1}^{I}\lambda_{ki}\omega_{kkij}\widehat{x}_{kkij,t}-\sum_{i=1}^{I}\sum_{l\neq k}^{K}\sum_{j=1}^{I}\lambda_{ki}\omega_{klij}\widehat{x}_{klij,t}.\end{aligned} \tag{A.105}$$

Now note that, by relabeling indices, $\sum_{i=1}^{I}\sum_{j=1}^{I}\lambda_{ki}\omega_{kkij}\widehat{x}_{kkij,t}=\sum_{i=1}^{I}\sum_{j=1}^{I}\lambda_{kj}\omega_{kkji}\widehat{x}_{kkji,t}$, so domestic intermediate input use cancels out and

$$\begin{aligned}\sum_{i=1}^{I}\lambda_{ki}(1-\vartheta_{ki})\widehat{\mathrm{va}}_{ki,t}=&\sum_{i=1}^{I}\beta_{kki}\widehat{c}_{kki,t}+\sum_{i=1}^{I}\sum_{l\neq k}^{K}\mathcal{Y}_{lk}\left(\beta_{lki}\widehat{c}_{lki,t}+\sum_{j=1}^{I}\lambda_{lj}\omega_{lkji}\widehat{x}_{lkji,t}\right)\\&-\sum_{i=1}^{I}\sum_{l\neq k}^{K}\sum_{j=1}^{I}\lambda_{ki}\omega_{klij}\widehat{x}_{klij,t}.\end{aligned} \tag{A.106}$$

Using the definition of aggregate consumption, $\widehat{c}_{k,t}=\sum_{l=1}^{K}\sum_{i=1}^{I}\beta_{kli}\widehat{c}_{kli,t}$, it follows that

$$\sum_{i=1}^{I}\beta_{kki}\widehat{c}_{kki,t}=\widehat{c}_{k,t}-\sum_{l\neq k}^{K}\sum_{i=1}^{I}\beta_{kli}\widehat{c}_{kli,t}. \tag{A.107}$$

Similarly, relabeling indices in the imported intermediates term gives

$$\sum_{i=1}^{I}\sum_{l\neq k}^{K}\sum_{j=1}^{I}\lambda_{ki}\omega_{kl\,ij}\widehat{x}_{kl\,ij,t} = \sum_{l\neq k}^{K}\sum_{i=1}^{I}\sum_{j=1}^{I}\lambda_{kj}\omega_{kl\,ji}\widehat{x}_{kl\,ji,t}. \tag{A.108}$$

Substituting the previous expressions yields

$$\begin{aligned}\sum_{i=1}^{I}\lambda_{ki}(1-\vartheta_{ki})\widehat{\mathrm{va}}_{ki,t} = \widehat{c}_{k,t} + \sum_{l\neq k}^{K}\sum_{i=1}^{I}\Bigg[&y_{l\,k}\beta_{l\,ki}\widehat{c}_{l\,ki,t} + \sum_{j=1}^{I}y_{l\,k}\lambda_{l\,j}\omega_{l\,kji}\widehat{x}_{l\,kji,t} \\ &- \beta_{kl\,i}\widehat{c}_{kl\,i,t} - \sum_{j=1}^{I}\lambda_{kj}\omega_{kl\,ji}\widehat{x}_{kl\,ji,t}\Bigg],\end{aligned} \tag{A.109}$$

where the right-hand side is precisely real GDP in (A.102). Therefore,

$$\sum_{i=1}^{I}\lambda_{ki}(1-\vartheta_{ki})\widehat{\mathrm{va}}_{ki,t} = \widehat{y}_{k,t}. \tag{A.110}$$

B. Additional Figures and Tables

Table 1 lists the 44 NACE sectors used in the analysis.

TABLE 1. NACE Rev. 2 sectors used in the Analysis

NACE Code	**Sector Name**
A.01–02	Agriculture, forestry and fishing
A.03	Fishing and aquaculture
B.05–06	Mining of coal, lignite, crude petroleum and natural gas
B.07–08	Mining of metal ores and other mining and quarrying
B.09	Mining support service activities
C.10–12	Manufacture of food, beverages and tobacco
C.13–15	Manufacture of textiles, wearing apparel and leather products
C.16	Manufacture of wood and products of wood and cork
C.17–18	Manufacture of paper, and printing and media reproduction
C.19	Manufacture of coke and refined petroleum products

Continued on next page

NACE Code	Sector Name
C.20	Manufacture of chemicals and chemical products
C.21	Manufacture of basic pharmaceutical products
C.22	Manufacture of rubber and plastic products
C.23	Manufacture of other non-metallic mineral products
C.24	Manufacture of basic metals
C.25	Manufacture of fabricated metal products
C.26	Manufacture of computer, electronic and optical products
C.27	Manufacture of electrical equipment
C.28	Manufacture of machinery and equipment n.e.c.
C.29	Manufacture of motor vehicles, trailers and semi-trailers
C.30	Manufacture of other transport equipment
C.31–33	Other manufacturing and repair and installation of machinery
D.35	Electricity, gas, steam and air conditioning supply
E.36–39	Water supply; sewerage, waste management and remediation
F.41–43	Construction
G.45–47	Wholesale and retail trade; repair of motor vehicles
H.49	Land transport and transport via pipelines
H.50	Water transport
H.51	Air transport
H.52	Warehousing and support activities for transportation
H.53	Postal and courier activities
I.55–56	Accommodation and food service activities
J.58–60	Publishing, audiovisual and broadcasting activities
J.61	Telecommunications
J.62–63	IT and other information services
K.64–66	Financial and insurance activities
L.68	Real estate activities
M.69–75	Professional, scientific and technical activities
N.77–82	Administrative and support service activities
O.84	Public administration and defence; compulsory social security

Continued on next page

NACE Code	Sector Name
P.85	Education
Q.86–88	Human health and social work activities
R.90–93	Arts, entertainment and recreation
S.94–96	Other personal service activities