A Behavioral Hybrid New Keynesian Model: Quantifying the Importance of Belief Formation Frictions

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May 31, 2026

Recent evidence points towards significant belief formation frictions and forecast sluggishness. In this paper, we build a bounded rationality New Keynesian model, estimated to match the degree of forecast sluggishness present in the data. We find that bounded rationality induces enough myopia and intrinsic persistence, diminishing the influence of consumption habits and price indexation. Additionally, the bounded rationality model generates impulse response dynamics to monetary policy shocks that resemble those observed in empirical estimations. This study highlights the significance of bounded rationality in capturing real-world dynamics and provides valuable insights into the role of belief formation frictions in macroeconomic modeling.

Keywords: New Keynesian, Bounded Rationality, Bayesian Estimation.

JEL Classifications: E27, E52, E71.

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1. Introduction

"Despite the advances in theoretical modeling, accompanying econometric analysis of the 'new Phillips curve' has been rather limiting [...]. The work to date has generated some useful findings, but these findings have raised some troubling questions about the existing theory."

J. Galí and M. Gertler, *Inflation dynamics: A structural econometric analysis (1999)*.

An important characteristic of the standard New Keynesian (NK) model is that it can be synthesized in a system of two first-order stochastic difference equations that are easy to interpret: the Dynamic IS curve for the demand side, and the Phillips curve for the supply side. Every slope in these curves is a combination of different parameters in the model, namely the discount factor, the degree of risk aversion, the Frisch elasticity and the Calvo-inaction probability. As a result, by estimating the slopes of the final system of equations, one can retrieve the structural parameters of the model. However, when the monetary economics literature performed such analyses, some estimated parameters were at odds with microeconometric studies.

Reconciling the NK theory with the data has proven to be a difficult exercise. One of the main criticisms to the benchmark NK model is that it is purely forward-looking, and therefore lacks the ability to capture any sort of endogenous persistence in output and inflation (see Galí and Gertler 1999; Fuhrer and Moore 1995; Fuhrer 2010; Christiano et al. 2005; Altig et al. 2011). As a result, the model does not produce the intrinsic persistence (and hump-shaped responses) that we observe in the data. The main approach to enforce intrinsic persistence in the model is to include backward-looking households and firms, either assuming a backward-looking utility function for households or sticky price indexation for firms. Unfortunately, the parameter values that characterize the frictions required to produce the degree of intrinsic persistence that the data suggests are at odds with the micro evidence (see Galí and Gertler 1999; Nakamura and Steinsson 2008; Bils and Klenow 2004; Havranek et al. 2017). To reconcile these differences between empirics and theory, we put forward a behavioral NK model, similar in spirit to the one described in Gabaix (2020), extended with *external habit persistent* households and *price-indexing* firms. We show that the combination of backward-looking agents *and* bounded rationality (BR) helps to reduce the discrepancy between macro and micro estimates.

Our contribution to the literature is threefold. First, we extend the behavioral NK setting in Gabaix (2020) to allow for household habit persistence and firm price indexation, inducing intrinsic persistence in the model dynamics. Second, we estimate the structural parameters behind the coefficients in the behavioral Dynamic IS (DIS) and hybrid NK Phillips curves using Bayesian techniques. Third, we also find empirical evidence for considerable BR behavior, supporting the deviation from the standard fully rational behavioral framework. A salient feature of our model is that it can be easily reduced to the ones described in Galí and Gertler (1999), Galí (2008) or Gabaix (2020) by turning off certain key parameters such as the degree of habit persistence, the degree of price indexation, or the BR parameter. As a result, our model nests those frameworks and allows us to easily compare estimates.

The first approach to induce intrinsic persistence in the NK framework dates back to Fuhrer and Moore (1995); Galí and Gertler (1999). The authors focus on the supply side of the model and estimate two different (micro-founded) NK Phillips curves: the standard curve (NKPC) and the Hybrid curve (H-NKPC), which has a backward-looking component. In an empirical exercise, they show that the H-NKPC produces dynamics closer to what the data suggests. However, even in the hybrid version, the structural parameter estimates are at odds with the micro evidence. For example, the discount factor estimate at a quarterly frequency is generally below 0.95 (see Galí and Gertler 1999). In subsequent research, Christiano et al. (2005) induce inflation persistence by assuming that firms that cannot re-optimize their prices update them according to past inflation. Other approaches able to generate intrinsic inflation persistence can be found in Roberts (1997) and Milani (2007), through the modeling of adaptive expectations and learning, respectively; in the form of sticky information as in Mankiw and Reis (2002); incomplete information as in Woodford (2003); Angeletos et al. (2021); by relaxing the Calvo assumption of a random selection of firms that are able to change their prices as in Sheedy (2010), or in a model of heterogeneous firms that features both rational and naïve agents as in Cornea-Madeira et al. (2019), which generate intrinsic persistence and myopia in aggregate inflation dynamics. Importantly, their mean estimate of myopia, 0.353, is similar to the estimates presented in this paper.[1] Likewise, Christiano et al.

[1]Other studies have stressed alternative mechanisms. For example, Grauwe (2011) generates non-fundamental (animal spirit) business cycles by introducing optimistic and pessimistic agents. In the same vein, Hommes and Lustenhouwer (2019) describe how Central Bank credibility can be affected by the share of rational and naïve agents, each agent type optimally decided at the individual level. They provide the conditions under which a self-fulfilling liquidity trap can occur, and how Central Bank credibility affects the equilibrium. Finally, Madeira (2014) introduces employment frictions and finds that such a characteristic helps to get a better estimate for the parameter of price stickiness.

(2005) also extend the backward-looking behavior to households by including internal habits.[2] They find that the degree of habits necessary to match the impulse response after a monetary shock is three or four times larger than the one estimated in the micro literature (Havranek et al. 2017). From these extensions we take the lesson that adding a backward-looking behavior is no panacea, at least on its own, for a reconciliation between micro and macro estimates.

We include household habit persistence in the light of Christiano et al. (2005) and Blanchard et al. (2015). Christiano et al. (2005) find a quantitatively important degree of household habit persistence for the US. Most importantly, they show that including habit persistence is critical to obtain hump-shaped impulse responses in the model, as the empirical VAR literature has observed. Given that our intention is to build a model that is closer to the data, we follow their approach in order to consistently estimate the behavioral DIS curve. We include backward-looking firms along the lines of Galí and Gertler (1999) and Christiano et al. (2005).[3] This is done in order to obtain a hybrid NK Phillips curve that is closer to empirical evidence, in the sense that it also includes lags of inflation and helps explain its persistence. The motivation for this is mostly empirical, since previous studies have found the inflation equation to be largely inertial.[4]

Our departure from the standard Full-Information Rational Expectations (FIRE) assumption is motivated by empirical evidence. Using survey data from households' and firms' expectations, Coibion and Gorodnichenko (2015) test for the null of FIRE, which is rejected in the data. However, their empirical findings are inconclusive on the direction of the FIRE departure, whether it is Full-Information or Rational Expectations that is rejected. This leaves room for different extensions beyond FIRE, some being more empirically robust than others. One notable approach that lies between the models of less than full information and the models of less than full rationality has been developed in a series of papers by Gabaix (2014, 2016, 2020), which provides an operational, tractable framework by incorporating the behavioral assumption that the decision makers allocate their attention optimally according to a simplified version of the full model, where their utility is replaced by a linear-quadratic approximation, and

[2] Christiano et al. (2005) model the backward-looking behavior by means of internal habits (each agent cares about its own consumption growth). In this paper, we instead focus on external habits (each agent cares about the difference between its consumption today and aggregate consumption yesterday). We take this route motivated by the meta-analysis in Havranek et al. (2017).

[3] Hajdini (2022) introduces myopia as in Gabaix (2020) and finds a smaller role for habits as the endogenous persistence arises due to the expectations themselves. The expectations implied by the model can account for many recently documented facts on expectations (see Angeletos et al. 2021).

[4] Since the seminal paper by Fuhrer and Moore (1995), a sizable literature has tried to estimate the NKPC. See, Mavroeidis et al. (2014), for an extensive review.

then solve the full model with this partial attention vector. The framework captures some of the essential features of the rational inattention framework, namely the underreaction of beliefs and actions, while it allows tractability for dynamic models beyond linear-quadratic forms.

In this paper, we follow this strand of the literature by assuming an attention coefficient that the decision-makers assign to a piece of newly arriving information, so that the posterior expectation is a convex combination of the prior mean and the realization (i.e. full-information) value. We follow this reduced-form approach since our core interest is to reconcile the theory with empirical evidence, and this behavioral approximation of a limited attention model affords us to arrive at the simple closed-form solutions that are typical of the standard New Keynesian model while incorporating the first-order effects of limited inattention. We calibrate the cognitive discounting parameter to match the empirical evidence on forecast underrevision (Coibion and Gorodnichenko 2015). Cognitive discounting, as presented in Gabaix (2020), is successful in producing myopia but does not produce intrinsic persistence *on its own*. We find that the cognitive discount factor, *together* with habit persistence and price indexation, is key to obtain macro estimates that align with their micro counterpart, since the cognitive discount factor increases the relative weight of the past (intrinsic persistence) and reduces the weight of the future (myopia).[5]

For the estimation of the structural parameters, and being able to compare our New Keynesian models, we follow a Bayesian approach as in Fernández-Villaverde and Rubio-Ramírez (2004), Rabanal and Rubio-Ramírez (2005) and Milani (2007). This approach has some advantages over limited-information methods such as the generalized method of moments (GMM). For example, Bayesian estimation mitigates the misspecification problem and allows a transparent comparison across models. In particular, we estimate

[5]Kortelainen et al. (2016) estimate a standard New Keynesian model using survey expectations data for the euro data and show that the use of this type of data improves the quality of the macro estimates. Instead, we focus on the US and directly match the degree of forecast sluggishness present in the survey data. Second, in the same spirit, Henzel and Wollmershauser (2008) estimate a hybrid New Keynesian Phillips curve and evaluate the role of expectations by using data from the CESifo World Economic Survey. Importantly, they provide empirical evidence that the hybrid version is more adequate for understanding the effects of monetary policy on the macroeconomy. We extend their framework and argue that, in addition to the backward-looking dimension, bounded rationality is supported by empirical evidence. Third, Chou et al. (2023) compare the performance of the standard New Keynesian model with a variety of models under inattention, ranging from sticky to imperfect information. They conclude that models under information frictions improve the replication of certain stylized facts, as compared to the standard setup. Lastly, Murat Arslan (2008) argues that models under imperfect information outperform the benchmark New Keynesian model in terms of the dynamics of the impulse response functions. We extend both of these analyses by calibrating the information-related parameters to specifically match the degree of forecast underrevision observed in the data.

four different models using US data: (i) the standard NK model, (ii) the hybrid NK model, (iii) the behavioral NK model, and (iv) the behavioral hybrid NK model. We find that the latter model reports estimates that are closer to those of the micro empirical evidence, with a larger log data density. Likewise, in order to test the ability of our set of models to replicate empirical impulse-response functions, we also estimate a monetary policy shock by means of a Bayesian vector autoregression (VAR) model using narrative sign restrictions as in Antolín-Díaz and Rubio-Ramírez (2018). We find that only our Behavioral NK model with both habit formation and backward-looking firms is able to generate, at the same time, hump-shaped responses and enough inflation persistence as we observe in the data. We complement this analysis by studying the impulse response functions after demand (natural rate) and supply (cost-push) shocks.

The paper proceeds as follows. In section 2 we introduce the behavioral model. In section 3 we estimate the structural parameters of the model. In section 4 we discuss our findings. Section 5 concludes the paper.

2. The Behavioural Agents and Firms Setting

2.1. Bounded Rationality Assumptions

Before introducing a behavioral version of the New Keynesian model, we here briefly explain the cognitive discounting approach à la Gabaix (2016, 2020) that we operationalize in this paper. Let $X_t \in \Omega$ be the state vector at period t, that might include exogenous shocks, and $\varepsilon_t \in E$ is an additive stochastic noise with zero mean.

Now let $G\colon \Omega \times E \to \Omega$ be the function that represents the equilibrium law of motion for the state variable, $X_{t+1} = G(X_t, \varepsilon_{t+1})$. Let us assume that the deterministic economy has a unique non-exploratory steady-state, and is denoted by X. Here, the cognitive discounting assumption states that the agents do not fully internalize the expected equilibrium deviations from the steady state by partially anchoring their belief to the steady state. Let $\bar{m} \in [0, 1]$ denote the degree of cognitive discounting, and let $G^B\colon \Omega \times E \to \Omega$, $G^B(X_t, \varepsilon_t) = \bar{m}G(X_t, \varepsilon_t) + (1 - \bar{m})X$ denote the law of motion perceived by the behavioral agent. For notational simplicity, in the rest of this section, we will assume that the state vector is de-meaned, i.e. the steady state is given by the zero vector; however, the analysis holds true for the generic case as well. Under this assumption the above expression simplifies to $G^B(X_t, \varepsilon_t) = \bar{m}G(X_t, \varepsilon_t)$. The linearization of the actual law of motion and renormalization gives $X_{t+1} = \Gamma X_t + \varepsilon_{t+1}$ for some matrix Γ. Likewise, the perceived law of motion by the behavioral agents linearizes to $X_{t+1} = \bar{m}(\Gamma X_t + \varepsilon_{t+1})$.

However, since the noise parameter has zero mean, we have the following relation between the expectation of a behavioral agent, denoted by the expectation operator with a superscript B, and the rational expectation, $\mathbb{E}_t^B[X_{t+1}] = \bar{m}\Gamma X_t = \bar{m}\mathbb{E}_t[X_{t+1}]$. Likewise, iterating for k periods we obtain $\mathbb{E}_t^B[X_{t+k}] = \bar{m}^k\mathbb{E}_t[X_{t+k}]$. Throughout the paper, we will assume that all forecasts, made by households or firms and across different macroeconomic variables, are cognitively discounted by the same factor $\overline{m}$.[6]

2.2. Households

Rational Agents. We consider a population of households that is treated as a continuum of unit mass. Each household chooses its consumption and labor supply level for each period. We assume identical preferences over expected lifetime utility and hence omit indexing for notational ease. The preference of a representative household can be given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(C_t - h\overline{C}_{t-1})^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right] \tag{1}$$

where C_t is a consumption index given by $C_t \equiv \left(\int_0^1 C_{it}^{\frac{\epsilon_t - 1}{\epsilon_t}} di \right)^{\frac{\epsilon_t}{\epsilon_t - 1}}$, with C_{it} denoting the quantity of good $i \in [0,1]$ consumed by the household in period t, N_t denotes employment or labor supply, $\overline{C}_{t-1}$ denotes the average consumption level in the economy (which is taken as given by the individual household), σ is the intertemporal elasticity of substitution, $1/\varphi$ is the Frisch elasticity and ϵ_t denotes the elasticity of substitution among goods varies over time.[7] The period-t consumption utility of each household is affected by a reference level, which we assume to be given by a linear function of the average consumption level in the previous period. Thus, the household preferences exhibit a *keeping up with the Joneses* element.[8] The parameter $h \in [0,1]$ represents the sensitivity towards this reference point. The household's budget constraint is given by

$$P_t C_t + Q_t B_t = B_{t-1} + W_t N_t + T_t \tag{2}$$

[6]In Appendix A.6 we show that intrinsic persistence can be microfounded via beliefs, by assuming $\mathbb{E}_t^B X_{t+h} = \overline{m}^h \mathbb{E}_t X_{t+h} + (1 - \overline{m}^h) X_{t-1}$.

[7]We microfound cost-push shocks by allowing for a time-varying elasticity of substitution between varieties of goods.

[8]The consequences of such an assumption are similar to assuming habit persistence, albeit simplifying the computation.

where P_t is the price of the consumption good, B_t stands for bond holdings at the household, Q_t is the price of each bond, W_t is the wage rate for each unit of labor supply and T_t are transfers to households. We show in Appendix A.1 that the demand for good i is given by

$$Y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\epsilon_t} Y_t \tag{3}$$

where $Y_t = C_t$ (since we are in a representative household economy), and the aggregate price index P_t is given by $P_t = \left(\int_0^1 P_{it}^{1-\epsilon_t} di\right)^{\frac{1}{1-\epsilon_t}}$. The optimization problem of the household is represented as maximizing lifetime utility (1) subject to its budget constraint (2) and the usual transversality condition $\lim_{t\to\infty} \beta^t u'(C_t)B_t = 0$. The rational household optimality conditions, derived in Appendix A.1, are

$$\frac{W_t}{P_t} = \frac{N_t^{\varphi}}{(C_t - h\overline{C}_{t-1})^{-\sigma}} \tag{4}$$

$$Q_t = \beta \mathbb{E}_t \left[\left(\frac{C_{t+1} - h\overline{C}_t}{C_t - h\overline{C}_{t-1}} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right] \tag{5}$$

Notice that, since households are identical and of unit mass, we can take the average consumption of the past period as the consumption of the representative agent in that period, $C_t = \overline{C}_t$ for all periods t.

Behavioral Agents. The behavioral households exhibit cognitive discounting as described in Section 2.1, hence their mean expectation of the stochastic variables in the economy is dampened towards its steady-state values compared to the expectation of a rational agent. This effect is even more nuanced for events that are far into the future. We can rewrite condition (5) as

$$Q_t = \beta \mathbb{E}_t^B \left[\left(\frac{C_{t+1} - hC_t}{C_t - hC_{t-1}} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right] \tag{6}$$

The labor supply condition is unaffected: since it is an intratemporal condition, cognitive discounting plays no role here. Fully rational and behavioral households do not differ in intratemporal considerations, but in their perception of the future. On the other hand, the Euler condition now has a different expectation operator. The log-linearized version of both optimality conditions, derived in Appendix A.2, is

$$\widehat{w}_t - \widehat{p}_t = \varphi \widehat{n}_t + \frac{\sigma}{1-h}\widehat{c}_t - \frac{\sigma h}{1-h}\widehat{c}_{t-1} \tag{7}$$

$$\widehat{c}_t = \frac{h}{1+h}\widehat{c}_{t-1} + \frac{1}{1+h}\overline{m}\mathbb{E}_t\widehat{c}_{t+1} - \frac{1-h}{\sigma(1+h)}\left(\widehat{i}_t - \overline{m}\mathbb{E}_t\pi_{t+1}\right) \tag{8}$$

where a hat on top of a variable denotes the log deviation from the steady state, $\widehat{x}_t = (X_t - X)/X$, and $\widehat{i}_t = -\log Q_t$ is the short-term nominal interest rate. Here we have made use of the BR assumptions described in the previous section, setting $\mathbb{E}_t^B\widehat{c}_{t+1} = \overline{m}\mathbb{E}_t\widehat{c}_{t+1}$ and $\mathbb{E}_t^B\pi_{t+1} = \overline{m}\mathbb{E}_t\pi_{t+1}$. The Euler condition (8) can be rewritten in terms of the output gap as the Behavioral DIS (BDIS) curve

$$\widetilde{y}_t = \lambda_b\widetilde{y}_{t-1} + \lambda_f\mathbb{E}_t\widetilde{y}_{t+1} + \lambda_r\left(\widehat{i}_t - \overline{m}\mathbb{E}_t\pi_{t+1} - r_t^n\right) \tag{9}$$

where $\lambda_b = \frac{h}{1+h}$, $\lambda_f = \frac{1}{1+h}\overline{m}$, $\lambda_r = -\frac{1-h}{\sigma(1+h)}$, r_t^n is the natural interest rate and follows an AR(1) process; and a tilde denotes the log deviation with respect to the natural level $\widetilde{x}_t = \widehat{x}_t - x_t^n$.[9]

2.3. Firms

There is a continuum of firms with unit mass, each producing a different type of good. Good i is produced by a monopolistic firm i with technology

$$Y_{it} = A_t N_{it} \tag{10}$$

where A_t represents the level of technology, assumed to be common across firms. Given $Y_t = C_t$ and $Y_{it} = C_{it}$,[10] we know that the final good is produced competitively in quantity Y_t. Each firm chooses the price level of the good that it produces. Prices are set subject to a Calvo-style friction (in each period, a firm is only allowed to reset its price with probability $1-\theta$, independent of the time elapsed since it last adjusted its price). Thus, in each period a measure $1-\theta$ of producers reset their prices freely. However – and departing from the standard NK setting – it is assumed that when a firm is unable to reoptimize, its price is partially indexed to past inflation as in Christiano et al. (2005),

[9] We define the natural level as the equilibrium under no pricing frictions, which we consider as the demand shock.

[10] No firm will choose to produce more that what is demanded.

i.e.,

$$P_{it} = P_{it-1}\Pi_{t-1}^{\omega} \tag{11}$$

where $\Pi_t = \frac{P_t}{P_{t-1}}$ is the gross rate of inflation between $t-1$ and t, and ω is the elasticity of prices with respect to past inflation.[11] As a result, a firm that last reset its price in period t will in period $t+k$ have a nominal price of $P_t^*\chi_{t,t+k}$, where $\chi_{t,t+k} = \begin{cases} \Pi_t^{\omega}\Pi_{t+1}^{\omega}\Pi_{t+2}^{\omega}\cdots\Pi_{t+k-1}^{\omega} & \text{if } k \geq 1 \\ 1 & \text{if } k = 0 \end{cases}$.

Rational Agents. The rational firm's problem is to maximize its discounted profit stream

$$\mathbb{E}_0 \sum_{t=0}^{\infty} Q_t[P_{it}Y_{it} - W_t N_{it}] \tag{12}$$

subject to the sequence of demand constraints (3) and technology constraints (10). We can rewrite the objective function (profits) as $P_{it}Y_{it} - W_t N_{it} = P_{it}Y_{it} - W_t\frac{Y_{it}}{A_t} = \left[P_{it} - P_t\text{MC}_t\right]\left[\frac{P_{it}}{P_t}\right]^{-\epsilon_t} Y_t$, where the marginal costs are defined as $\text{MC}_t = (W_t/P_t)(\partial Y_t/\partial N_t) = W_t/(P_t A_t)$.

Consider a firm reoptimizing its price at time t. Let the firm's optimal price be denoted $P_t^*(i)$, such that in this setting at time $t+k$ its price will be $P_{it}^*\chi_{t,t+k}$. Ignoring states in which reoptimization is allowed, its maximization program is $\max_{P_{it}^*} \mathbb{E}_t \sum_{k=0}^{\infty} \theta^k Q_{t+k}[P_{it}^*\chi_{t,t+k} - P_{t+k}\text{MC}_{t+k}]\left[\frac{P_{it}^*\chi_{t,t+k}}{P_t}\right]^{-\epsilon_t} Y_{t+k}$ which yields the following first-order condition,[12]

$$P_{it}^* = \frac{\mathbb{E}_t \sum_{k=0}^{\infty}(\theta\beta)^k (C_{t+k} - hC_{t+k-1})^{-\sigma} C_{t+k} P_{t+k}^{\epsilon_t} P_{t+k-1}^{-\omega\epsilon_t} \text{MC}_{t+k}\mathcal{M}_{t+k}}{\mathbb{E}_t \sum_{k=0}^{\infty}(\theta\beta)^k (C_{t+k} - hC_{t+k-1})^{-\sigma} C_{t+k} P_{t+k}^{\epsilon_t - 1} P_{t+k-1}^{\omega(1-\epsilon)}} P_{t-1}^{\omega} \tag{13}$$

where we have used the Euler condition (6), $\chi_{t,t+k} = \left(\frac{P_{t+k-1}}{P_{t-1}}\right)^{\omega}$ and $\mathcal{M}_t = \frac{\epsilon_t}{\epsilon_t - 1}$, which stands for the mark-up. Notice that with flexible prices (i.e., $\theta = 0$), the optimal pricing condition (13) collapses to the familiar monopolistic competition price-setting rule

$$P_{it}^* = \mathcal{M}_t P_t \text{MC}_t \tag{14}$$

[11] This assumption is equivalent to the more ad-hoc derivation of backward-looking firms in Galí and Gertler (1999). However, since firms are identical, its consequences are equivalent: ω could be interpreted as the share of backward-looking firms.

[12] A detailed derivation can be found in Appendix A.3.

where (14) is the frictionless mark-up. Since all firms who get to reset are facing an identical environment (i.e., we can treat them as if they were a representative firm), they choose to set the same price: $P^*_{it} = P^*_t \;\; \forall i$, and $\mathcal{M}_t = \frac{1}{\mathrm{MC}_t}$. The log-linearized version of the optimal pricing condition (13) is

$$(15) \quad p^*_t = p_t + (1-\theta\beta)\mathbb{E}_t \sum_{k=0}^{\infty}(\theta\beta)^k \left[(\pi_{t+1} + \ldots + \pi_{t+k}) - \omega(\pi_t + \ldots + \pi_{t+k-1}) + \widehat{\mathrm{mc}}_{t+k} + \widehat{\mu}_{t+k}\right]$$

where $\widehat{\mathrm{mc}}_t = \mathrm{mc}_t - \mathrm{mc} = \mathrm{mc}_t + \mu$, $\mu = \log \mathcal{M}$, and $\widehat{\mu}_t = \mu_t - \mu$, which is assumed to follow an AR(1) process. That is, a resetting firm will choose a price that corresponds to the desired mark-up over a convex combination of current and expected future prices and nominal marginal costs, in addition to the prices in the previous period.

Behavioral Agents. The behavioral firm faces the same problem, with a less accurate view of reality. Most importantly, the behavioral firm perceives the future via the cognitive discounting mechanism discussed in Section 2.1. To be precise, we model that at time t, the firm perceives the future inflation and marginal costs at date $t + k$ as $\mathbb{E}^B_t[\pi_{t+k}] = \overline{m}^k \mathbb{E}_t[\pi_{t+k}]$ and $\mathbb{E}^B_t[\widehat{\mathrm{mc}}_{t+k}] = \overline{m}^k \mathbb{E}_t[\widehat{\mathrm{mc}}_{t+k}]$. Note that we assume a common cognitive discount factor across households and firms, since households own the firms, which inherit their belief formation frictions. The equivalent condition of equation (15) for a behavioral firm is

(16)

$$p^*_t = p_t + (1-\theta\beta)\sum_{k=0}^{\infty}(\theta\beta\overline{m})^k \mathbb{E}_t \left[(\pi_{t+1} + \ldots + \pi_{t+k}) - \omega(\pi_t + \ldots + \pi_{t+k-1}) + \widehat{\mathrm{mc}}_{t+k} + \widehat{\mu}_{t+k}\right]$$

where the future is additionally discounted by a cognitive discount factor $\overline{m}$.

Aggregate Price Dynamics and the Behavioral Hybrid New Keynesian Phillips Curve. In this economy, in every period, there are two types of firms: those allowed to reset their price and those who are not, whose price is updated with previous aggregate inflation. We can describe the price dynamics as $P_t = \left[\Pi_{t-1}^{\omega(1-\epsilon_t)}\theta P_{t-1}^{1-\epsilon_t} + (1-\theta)(P^*_t)^{1-\epsilon_t}\right]^{\frac{1}{1-\epsilon_t}}$. All firms resetting their price in any given period will choose the same price because they face an identical problem. A log-linear approximation to the aggregate price index around a zero inflation steady-state, derived in Appendix A.4, yields

(17) $$\pi_t = \theta\omega\pi_{t-1} + (1-\theta)(p_t^* - p_{t-1}).$$

After some algebra relegated to Appendix A.5, a rearrangement of expressions (16) and (17) yields the Behavioral Hybrid NK Phillips curve in terms of marginal costs

(18) $$\pi_t = \gamma_b\pi_{t-1} + \gamma_\mu\widehat{\text{mc}}_t + \gamma_f\mathbb{E}_t\pi_{t+1} + \gamma_\mu\widehat{\mu}_t$$

where $\gamma_b = \frac{\omega}{1+\omega\beta\overline{m}\left[\theta+(1-\theta)\frac{1-\theta\beta}{1-\theta\beta\overline{m}}\right]}$, $\gamma_\mu = \frac{(1-\theta)(1-\theta\beta)}{\theta\left\{1+\omega\beta\overline{m}\left[\theta+(1-\theta)\frac{1-\theta\beta}{1-\theta\beta\overline{m}}\right]\right\}}$, and $\gamma_f = \frac{\beta\overline{m}\left[\theta+(1-\theta)\frac{1-\theta\beta}{1-\theta\beta\overline{m}}\right]}{1+\omega\beta\overline{m}\left[\theta+(1-\theta)\frac{1-\theta\beta}{1-\theta\beta\overline{m}}\right]}$.

In order to obtain the Behavioural Hybrid NK Phillips curve in terms of the output gap, recall that in this economy with technological progress, $\text{MC}_t = W_t/(A_tP_t)$. Taking logs, we can write $\text{mc}_t = w_t - p_t - a_t$. Additionally, firm technology implies $y_t = a_t + n_t$ and the aggregate resource constraint implies $y_t = c_t$. Finally, thanks to the labor supply condition (4) we know $w_t - p_t = \varphi n_t + \frac{\sigma}{1-h}c_t - \frac{\sigma h}{1-h}c_{t-1}$. Using these four expressions together yields

(19) $$\widehat{\text{mc}}_t = \left(\varphi + \frac{\sigma}{1-h}\right)\widetilde{y}_t - \frac{\sigma h}{1-h}\widetilde{y}_{t-1}$$

Introducing (19) into (18) leads to the Behavioral Hybrid NK Phillips curve

(20) $$\pi_t = \gamma_b\pi_{t-1} + \alpha_b\widetilde{y}_{t-1} + \alpha_c\widetilde{y}_t + \gamma_f\mathbb{E}_t\pi_{t+1} + \gamma_\mu\widehat{\mu}_t$$

where $\alpha_b = -\gamma_\mu\frac{\sigma h}{1-h}$ and $\alpha_c = \gamma_\mu\left(\varphi + \frac{\sigma}{1-h}\right)$.

2.4. Closing the Model

The Behavioral Hybrid NK Phillips curve (20), together with the Behavioral Dynamic IS curve (9) and a reaction function for the monetary authority (an ad-hoc inertial Taylor rule with an AR(1) monetary policy shock)

(21) $$\widehat{i}_t = (1-\rho_r)(\phi_\pi\pi_t + \phi_y\widetilde{y}_t) + \rho_r\widehat{i}_{t-1} + e_t$$

constitute the Behavioral New Keynesian framework with *keeping up with the Joneses* households and hybrid firms.

2.5. Sensitivity Analysis

To understand how the new parameter $\overline{m}$ is affecting the model dynamics, let us see how the composite parameters are influenced by its introduction. Let us start with the Dynamic IS curve. Once we relax $\overline{m}$ and allow it to be in the closed unit interval, we find that $\lambda_f = \frac{\overline{m}}{1-h}$ and $-\lambda_r\overline{m} = \frac{(1-h)\overline{m}}{\sigma(1+h)}$, while the other two composite parameters are unaffected by $\overline{m}$. One can see that both affected composite parameters, λ_f and $-\lambda_r\overline{m}$, are increasing in the degree of attention $\overline{m}$. Introducing inattention generates household myopia and reduces the importance of the parameters interacting with forward-looking variables, doing so without affecting the parameters linked to past and contemporaneous variables.

Turning now to the Phillips curve, the main difference relies on the fact that composite parameters interacting with both backward-looking and contemporaneous variables are now also affected by $\overline{m}$. Extending the model to BR generates intrinsic persistence in the Phillips curve, since cognitive discounting also interacts with price-indexation. Let us start with the backward-looking inflation term. Relaxing $\overline{m}$ by allowing it to be in the closed unit interval, we find that γ_b is decreasing in attention: $\partial\gamma_b/\partial\overline{m} < 0$ as long as $(1-\beta\theta)(1-\beta\theta^2\overline{m}^2) + \beta\theta^2(1-\overline{m})^2 > 0$, which is always satisfied. The backward-looking output gap term α_b is now affected by inattention, and is decreasing in $\overline{m}$. Again, inattention and intrinsic persistence coming from price indexation lead to *more* intrinsic persistence. The forward looking term γ_f is increasing in $\overline{m}$: with inattentive firms the relative importance of composite parameters is transferred from forward-looking to backward-looking ones. Besides, the composite parameter interacting with the contemporaneous output gap, α_c, is decreasing in $\overline{m}$.

3. Estimation

This section lays out the approach we follow for the estimation of our structural parameters of interest through Bayesian techniques. First, we discuss the time series data we use and their transformation. Second, we describe our estimation procedure, that is, prior selection, and evaluation of the likelihood function using the Kalman Filter and the Metropolis-Hastings algorithm for finding posterior distributions as well as moments for our structural parameters.

There are some advantages associated with full-information methods such as Bayesian estimation.[13] For example, Bayesian approaches can improve the estimator precision

[13] Mavroeidis et al. (2014) present a survey of studies using limited-information methods for the estima-

and can lessen identification problems, at least asymptotically; can reduce the risk of misspecification and can deal with model uncertainty and, finally, the results can be easily compared to the point estimates from standard Bayesian VARs.[14]

3.1. The Data

We estimate the model using three US time series at the quarterly frequency: 1) the log of real GDP per capita, 2) the log-difference of the CPI inflation rate, and 3) the nominal interest rate.[15] In particular, to proxy for the output gap, we apply a one-sided HP filter to the log of real GDP per capita. We demean both the inflation rate and the nominal interest rate, the effective Federal Funds rate. The underlying data comes from FRED.[16]

We use two different samples that differ regarding their time spans. Our main sample starts in 1955:I and ends in 2007:III. For robustness, we repeat our estimation for the sample starting in 1985:I until 2007:III.

3.2. A Bayesian Approach

We need to specify the prior distributions for the structural parameters. Using prior information and the observable variables, we apply the Kalman Filter to evaluate the likelihood function of each model and the Metropolis-Hastings algorithm to draw from the posterior distributions and estimate their moments.[17]

Prior Selection. The prior distribution for the parameters is standard (see e.g., Smets and Wouters 2007) and reported in Table 1. For the subjective discount factor, β, we use a Beta distribution with mean 0.99 and standard deviation 0.001 (similar to Boehl

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tion of the New Keynesian Phillips curve.

[14]See Rabanal and Rubio-Ramírez (2005) and Fernández-Villaverde and Rubio-Ramírez (2004), for instance, for a more detailed discussion.

[15]We also consider GDP deflator as the price index in the estimation, in table A1. We find similar estimates of BR. In the spirit of Bouakez et al. (2005), we use the per capita series to control for population growth.

[16]We obtain real GDP from the U.S. Bureau of Economic Analysis (retrieved from FRED), "Real Gross Domestic Product [GDPC1]"; the price index from the U.S. Bureau of Labor Statistics (retrieved from FRED), "Consumer Price Index for All Urban Consumers: All Items in U.S. City Average [CPIAUCSL]"; and the nominal interest rate from the Board of Governors of the Federal Reserve System (retrieved from FRED), "Effective Federal Funds Rate [FEDFUNDS]". To convert real GDP in per capita terms we use population from the U.S. Bureau of Labor Statistics (retrieved from FRED), "Population Level [CNP16OV]". Regarding the latter, we have employed the trailing moving average (4 quarters) of the series, which is robust to the critique in Edge et al. (2013).

[17]As usual, the posterior distribution can be approximated by the product of the prior and the likelihood function.

et al. 2022; Kulish et al. 2017).[18] Along the lines of Smets and Wouters (2007), for price stickiness, θ, and price indexation, ω, we also choose a Beta distribution with mean 0.5 and standard deviation 0.10 (0.15 for ω).[19] For habit persistence, h, we set a Beta distribution with mean 0.70 and standard deviation 0.15. We also employ the same prior distribution as in Smets and Wouters (2007) to estimate σ and φ. For the parameters entering the Taylor rule, we use a Normal distribution with mean 1.5 and standard deviation 0.15 for the response to changes in inflation, ϕ_{π}. Likewise, for the response to deviations from potential output, ϕ_y, we set a normally distributed prior with mean 0.15 and standard deviation 0.10. As in Smets and Wouters (2007), we use a Beta distribution for the persistence parameters using a mean of 0.5 and a standard deviation of 0.2. Finally, for the standard deviation of the shocks we use an Inverse Gamma Distribution with a mean of 0.1 and infinite standard deviation.

Bayesian Inference. We solve the model and estimate the remaining parameters for each specification using Dynare.[20] We use the CMA-ES algorithm for computing the mode which is robust to multiple local maxima (Hansen et al. 2003). To sample and estimate the moments of the posterior distributions, we use a Markov Chain Monte Carlo with 500.000 draws from the Metropolis-Hastings algorithm and burn-in the first 125.000 (25%). The acceptance rate was around 23%. Since we only employ one chain for the Metropolis-Hastings algorithm to reduce estimation time, convergence is checked using the test proposed by Geweke (1991).

Estimation Algorithm and Belief Formation Frictions. We give a belief formation interpretation to $\overline{m}$ by matching the forecast underrevision coefficient in Coibion and Gorodnichenko (2015). Under the full-information rational-expectations (FIRE), $\overline{m} = 1$ in our case, the model produces no co-movement between ex-ante forecast errors, measured as the difference between the realization of future inflation and the time-t forecast, and forecast revisions, measured by the difference between time–t and time–$t-1$ forecast. That is, the forecast revision does not consistently predict the forecast error. Otherwise, the agent would incorporate this information in his or her information set. Therefore, a positive co-movement between ex-ante forecast errors and forecast revisions suggests that the FIRE assumption is violated. Using survey data on US consumers', firms', and

[18] To be precise, our prior standard deviation for β is the same as Kulish et al. (2017), but our prior mean is slightly different and equal to 0.99.

[19] This prior would imply that the average length of price contracts is 6 months.

[20] See Adjemian et al. (2011) for more details.

professional forecasters' forecasts, Coibion and Gorodnichenko (2015) document a significant sluggishness in responses to new information, measured by the positive co-movement of ex-ante forecast errors and forecast revisions. That is, after a positive revision of annual inflation forecasts between two time periods, agents consistently under-predict inflation with their revised forecast. Our model is consistent with this empirical observation through BR: whenever $\overline{m} < 1$, agents forecasts under-predict future inflation.

Starting from a linear grid of $\overline{m}$ in the closed unit interval, we estimate the rest of the parameters for each guessed value of $\overline{m}$. We then simulate the theoretical framework using the point estimates of the parameters, and construct two time series of the model-implied ex-ante average forecast error, forecast $\text{error}_t = \pi_{t+3,t} - \mathbb{E}_t^B \pi_{t+3,t}$ and the average forecast revision, $\text{revision}_t = \mathbb{E}_t^B \pi_{t+3,t} - \mathbb{E}_{t-1}^B \pi_{t+3,t}$, where $\pi_{t+3,t}$ is the price level growth rate between period $t+3$ and $t-1$. We then compute the Coibion and Gorodnichenko (2015) estimate following Angeletos et al. (2021); Gallegos (2023), $\beta_{CG} = \frac{\mathbb{C}(\text{forecast error}_t, \text{revision}_t)}{\mathbb{V}(\text{revision}_t)}$. Finally, we select the $\overline{m}$ that minimizes the distance between the theoretical coefficient, and the estimated underrevision coefficient using data up to 2007:III, 1.2306. Additionally, we perform an exercise in which we directly estimate $\overline{m}$ together with the rest of the model parameters, obtaining an implied Coibion and Gorodnichenko (2015) coefficient.

4. Findings

This section discusses the estimation results for our parameters of interest and their implications for the business cycle. First, we examine the ability of our model specifications to reconcile previous empirical estimates and compare the fit to the data through their log data densities calculated using the Laplace approximation as in Geweke (1991). Second, we consider whether our analytical models can replicate the responses of output gap and inflation to a monetary policy shock estimated by means of a Bayesian VAR model using narrative sign restrictions.

4.1. Posterior Distributions and Moments

Standard New Keynesian Model. Table 1 displays the main results.[21] We report the posterior mean and the 90% error bands. We begin by estimating an otherwise standard

[21]We find that the discount factor coefficient, β, is weakly identified in the data.

New Keynesian model using the benchmark sample that goes from 1955:I to 2007:III. The first column reports the estimation of the standard NK model. That is, we estimate the model restricting $h = \omega = 0$ and $\overline{m} = 1$. In this standard framework there is no aggregate intrinsic persistence in the system since $\lambda_b = \gamma_b = \alpha_b = 0$, and the model exhibits an extreme forward-looking behavior. We find that this extreme forward-looking behavior produces the smallest log data density. As a result, this basic model is the least preferred among the four considered.

Hybrid New Keynesian Model. Our estimate of $h = 0.834$ is on the upper bound of the estimated values in the literature. In a meta-analysis, Havranek et al. (2017) find that the standard value of external habits in the macro literature is around 0.7, while the micro-consistent estimate is 0.4. There is less micro-empirical evidence on the true value of ω, which we estimate to be 0.820, since this form of indexation is a model artifact. This value is not excessively different from the standard assumed value of 1 in the literature (see e.g., Christiano et al. 2005; Auclert et al. 2020). On top of these, our estimate of θ is in the upper range in the micro literature, although aligned with the macro literature. Bils and Klenow (2004); Nakamura and Steinsson (2008) find a median price duration of 4.5-11 months in US micro data. Galí (2008) sets $\theta = 0.75$ to match an implied duration of 1 year. Auclert et al. (2020) estimate θ between 0.88 and 0.93 from macro data, implying a price duration of 12-14 quarters. The model is flexible to allow for intrinsic persistence, and produces a larger log data density relative to the standard New Keynesian model.

Behavioral New Keynesian Model. In the BR but only forward-looking case, the cognitive discount factor $\overline{m}$ does not interact with the degree of external habits h and the Calvo price rigidity parameter θ, as it appears only in front of expectations. This version produces the smallest Calvo price rigidity, although not significantly different from the other versions. In order to account for the persistence of endogenous variables in the data, the estimated first-order autocorrelation of the shocks is larger than in the hybrid version, and closer to the estimates of the basic NK. Regarding the degree of inattention in the economy, we obtain $\overline{m} = 0.46$, which results in a model-implied forecast underrevision coefficient of 1.2313 (compared to 1.2306 the Coibion and Gorodnichenko 2015 coefficient, using data up to 2007:III).[22] We also find that the estimated variance of

[22] Cornea-Madeira et al. (2019) estimate a myopia coefficient of 0.353, Ilabaca et al. (2020) estimate a cognitive discount factor that is around 0.5, and Andrade et al. (2019) report a value of 0.67 using maximum likelihood inference.

	Prior Distribution		Posterior Distribution			
		Mean	*1955:I-2007:III*			
		(S.d)	**NK**	**HNK**	**BNK**	**BHNK**
β	*Beta*	0.99	0.990	0.990	0.990	0.990
		(0.001)	(0.988, 0.992)	(0.988, 0.992)	(0.988, 0.992)	(0.988, 0.992)
σ	*Normal*	1.5	2.543	1.631	2.028	1.281
		(0.37)	(2.093, 2.977)	(1.061, 2.188)	(1.576, 2.464)	(0.699, 1.838)
φ	*Normal*	2	1.475	1.406	1.417	1.437
		(0.75)	(0.500, 2.340)	(0.500, 2.202)	(0.500, 2.267)	(0.500, 2.290)
ϕ_π	*Normal*	1.50	1.452	1.348	1.347	1.348
		(0.15)	(1.264, 1.633)	(1.160, 1.540)	(1.162, 1.547)	(1.141, 1.546)
ϕ_y	*Normal*	0.15	0.401	0.349	0.377	0.334
		(0.10)	(0.303, 0.498)	(0.248, 0.448)	(0.277, 0.479)	(0.223, 0.442)
θ	*Beta*	0.50	0.908	0.939	0.889	0.909
		(0.10)	(0.879, 0.939)	(0.925, 0.953)	(0.854, 0.924)	(0.881, 0.940)
h	*Beta*	0.70	—	0.834	—	0.650
		(0.15)	(—)	(0.751, 0.921)	(—)	(0.491, 0.811)
ω	*Beta*	0.50	—	0.820	—	0.781
		(0.15)	(—)	(0.732, 0.913)	(—)	(0.705, 0.855)
$\overline{m}$	*Implied*	—	1	1	0.46	0.46
		—	(—)	(—)	(—)	(—)
ρ_i	*Beta*	0.50	0.813	0.834	0.846	0.860
		(0.20)	(0.782, 0.843)	(0.803, 0.869)	(0.811, 0.878)	(0.827, 0.894)
ρ_d	*Beta*	0.50	0.838	0.380	0.864	0.697
		(0.20)	(0.789, 0.885)	(0.221, 0.533)	(0.813, 0.915)	(0.604, 0.792)
ρ_s	*Beta*	0.50	0.821	0.086	0.807	0.068
		(0.20)	(0.762, 0.886)	(0.012, 0.156)	(0.746, 0.867)	(0.010, 0.122)
ρ_{e_i}	*Beta*	0.50	0.130	0.171	0.160	0.167
		(0.20)	(0.042, 0.213)	(0.071, 0.270)	(0.063, 0.255)	(0.066, 0.262)
σ_d	*Inv. gamma*	0.10	0.212	0.270	0.480	0.479
		(∞)	(0.165, 0.259)	(0.217, 0.321)	(0.436, 0.526)	(0.434, 0.523)
σ_s	*Inv. gamma*	0.10	0.099	0.239	0.288	0.318
		(∞)	(0.072, 0.126)	(0.214, 0.263)	(0.260, 0.314)	(0.291, 0.345)
σ_i	*Inv. gamma*	0.10	0.216	0.209	0.207	0.207
		(∞)	(0.197, 0.235)	(0.192, 0.226)	(0.190, 0.224)	(0.190, 0.223)
	Log data density		–376.641	–358.229	–360.443	–352.573

Note: Results are reported at the posterior mean. 90% confidence intervals in parenthesis. The model-implied forecast-underrevision coefficients are 1.2313 (BNK) and 1.2308 (BHNK). The baseline forecast-underrevision reported in Coibion and Gorodnichenko (2015) is 1.2306.

TABLE 1. Estimated Structural Parameters

the exogenous shocks is larger than in the case without BR. Extending the benchmark model to inattention, either in the form of BR à la Gabaix (2020), sticky information à la Mankiw and Reis (2002), or noisy information à la Angeletos and Huo (2018), produces lower volatility of endogenous variables since forecasts are less volatile themselves (anchored to steady-state or priors). As a result, given the unconditional variance of endogenous variables in the data, a greater volatility of exogenous shocks is required (vis à vis a model without inattention).[23] This extension results in a log data density of –360.443.

Behavioral Hybrid New Keynesian Model. In this case, the cognitive discount factor $\overline{m}$ interacts with the degree of external habits h and the Calvo price rigidity parameter θ backward-, contemporaneous and forward-looking terms. As a result, relaxing the cognitive discount factor helps match the other parameters to their micro empirical estimates. We estimate $h = 0.650$, $\theta = 0.909$ and $\omega = 0.781$. Regarding the degree of inattention in the economy, we obtain $\overline{m} = 0.46$, which results in a model-implied forecast underrevision coefficient of 1.2308. This extension yields the largest log data density, –352.573.

Robustness Checks. We conduct several robustness checks to the BHNK framework. First, we replace the CPI data for the GDP Deflator, given that the model is a closed one-good economy (see Table A1, column 4). We find a similar estimate of $\overline{m}$, with a smaller price indexation coefficient. The remaining estimates are stable and we do not observe any considerable differences. Second, we directly estimate $\overline{m}$ together with the rest of the model parameters (see Table A1, column 5). We find $\overline{m} = 0.365$, which implies a forecast underrevision coefficient of 1.443, with other estimates being similar to our benchmark exercise.

Third, we modify our sample to the 1985:I-2007:III period (see Table A1, column 6). The literature has found evidence of the fall in the persistence of inflation (Fuhrer 2010), the flattening of the Phillips curve (Rubbo 2019; del Negro et al. 2020; Hazell et al. 2022), a fall in the volatility of macroeconomic variables (McConnell and Perez-Quiros 2000), and heterogeneous changes in belief formation frictions (Coibion and Gorodnichenko 2015; Gallegos 2023) in this period. As argued before, the empirical evidence suggests

[23] To see this formally, consider the more general case of the HBNK. Starting from (9), we can write the natural interest rate as

$$
\begin{aligned}
r_t^n &= i_t - \overline{m}\mathbb{E}_t\pi_{t+1} + \lambda_r^{-1}\left[\left(-\widehat{y}_t + \lambda_b\widehat{y}_{t-1} + \lambda_f\widehat{y}_{t+1}\right) - \left(-y_t^n + \lambda_b y_{t-1}^n + \lambda_f y_{t+1}^n\right)\right] \\
&= \rho + \lambda_r^{-1}\left(y_t^n - \lambda_b y_{t-1}^n - \lambda_f y_{t+1}^n\right)
\end{aligned}
$$

and the its variance is given by $\mathbb{V}(r_t^n) = \frac{\sigma^2}{(1-h)^2}\left[(1+h)^2 + h^2 + \overline{m}^2\right)\mathbb{V}(y_t^n)$, where $\mathbb{V}(y_t^n)$ is orthogonal to $\overline{m}$ (see equation A20). Therefore, an increase in BR reduced the volatility of the natural rate. We thank an anonymous referee for suggesting the discussion.

that the (intrinsic) persistence of inflation fell in the 1980s, which is reflected by the lower estimates of ω (which generates intrinsic persistence in our framework) and ρ_s (which generates extrinsic persistence). On top of this, Angeletos et al. (2021) and Gallegos (2023) find evidence of a fall in the forecast-underrevision coefficient since the mid-1980s. For this reason, we do not target any Coibion and Gorodnichenko (2015) estimate, and instead estimate directly $\overline{m}$. We obtain an estimate of 0.391, which together with the lack of intrinsic persistence of inflation, implies a forecast-underrevision coefficient of 0.506, consistent with the empirical fingings in Angeletos et al. (2021); Gallegos (2023).

Lastly, we repeat the exercise performed in table 1, but targeting the underrevision coefficient of output growth expectations in Coibion and Gorodnichenko (2015), 0.752, using data up to 2007:III. We find a larger implied $\overline{m}$ as a result of the smaller underrevision coefficient. Interestingly, the rest of estimates are stable and we do not observe any considerable differences.

In Appendix A.8 we extend the medium-scale DSGE model in Smets and Wouters (2007) to BR, and then estimate it using the priors in the original paper. We find that BR helps in reconciling parameters with their micro estimates.

4.2. Monetary Policy Shocks: Narrative VAR vs. NK Models

Empirical Impulse Response Functions. In order to compare our set of models to the data, we also estimate a VAR model and reproduce the impulse responses after an expansionary 25 bp monetary policy shock. First, we identify the VAR monetary policy shock by means of sign restrictions. We follow Uhlig (2005) and assume that an expansionary monetary policy shock is the one that reduces the nominal rate and rises output gap and inflation for the first two quarters.[24]

In addition to pure sign restrictions, we impose narrative sign restrictions as in Antolín-Díaz and Rubio-Ramírez (2018). Therefore, it is required that the identified monetary policy shock series and the historical decomposition are constrained on particular dates. In particular, we consider the Volcker reform in 1979:IV as a period of an exogenous monetary policy change. For this event we impose the following restrictions:

- ***Narrative Restriction 1:*** The monetary policy shock must be positive for the observation in 1979:IV.

[24] We do not include non-borrowed and total reserves in order to have the same variables as in the NK models. However, we have checked that our results are almost identical when including these two extra variables. The timing restriction is similar to the one in Uhlig (2005).

- ***Narrative Restriction 2:*** The monetary policy shock is the most important contributor to the observed changes in the federal funds rate in 1979:IV.

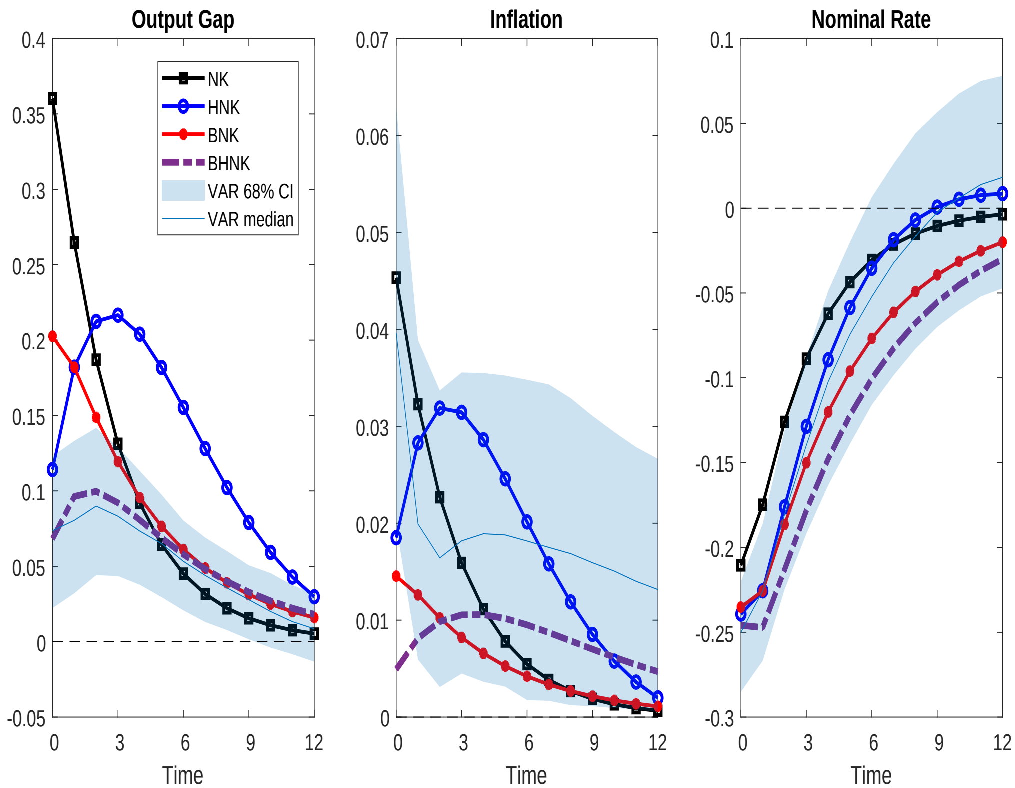
The VAR includes the same observables as in the theoretical model over the period 1955:I through 2007:III. It features three lags and is estimated by Bayesian methods under a conjugate normal inverse-wishart prior following Antolín-Díaz and Rubio-Ramírez (2018).

Theoretical Impulse Response Functions. A failure of the standard model is that it does not produce hump-shaped impulse responses after an exogenous monetary policy shock, which is at odds with empirical macro evidence (see e.g., Christiano et al. 2005 and Altig et al. 2011). Applied macro studies generally find that the peak effect of output after a monetary policy shock occurs after 2-8 quarters, whereas in the standard model without intrinsic persistence the peak effect occurs instantaneously, and the impulse responses are monotonically decreasing over time. In Figure 1, we plot the impulse response functions (IRFs) of output, inflation and nominal interest rates after an expansionary monetary policy shock of 25 bp in the standard NK (black line) and their empirical counterparts (blue line, with an associated confidence interval). The counterfactual shape of the impulse responses motivate our departure from the benchmark model.

Figure 1 also displays the IRFs of the HNK framework (blue line with circles). The inclusion intrinsic persistence leads to strong hump-shaped responses for output gap and inflation following an exogenous monetary policy shock, but larger in magnitude than what the empirical evidence suggests.

When we introduce behavioral features into the HNK model, labelled "BHNK" (violet and dashed line), we observe that cognitive discounting dampens the aggregate response to a monetary policy shock. Adding the backward-looking behavior together with cognitive discounting helps obtain impulse responses that are closer to their empirical counterpart — the reaction is smaller, more persistent and exhibit hump-shaped dynamics. Intuitively, because there is intrinsic persistence due to price indexation, less attentive firms' actions will be determined by past aggregates to a larger extent.

For completeness, we also report the results for the BNK model without intrinsic persistence (red line). We observe that the exclusion of external habits and inflation indexation implies that the IRFs are not hump-shaped. Therefore, we conclude that we need both cognitive discounting and intrinsic persistence, first to match the empirical estimates for certain parameters of interest, and second to obtain hump-shaped IRFs and initially muted responses for both output gap and inflation. In particular, the strong



Note: The dynamic paths for the variables are reported under different model specifications after an expansionary 25 bp monetary policy shock: (i) a standard NK model in black lines (squares), (ii) a hybrid NK model in blue lines (circles), (iii) a behavioral NK model in red lines (asterisks), and (iv) a behavioral hybrid NK model in purple lines (dashed). The VAR-based monetary policy shock is identified by means of narrative sign restrictions as in Antolín-Díaz and Rubio-Ramírez (2018). The horizontal axis displays the time which is measured in quarters. Vertical axis values refer to deviations from steady state in percentage.

FIGURE 1. Dynamic Responses to a Monetary Policy Shock

inflation persistence obtained in VAR frameworks is exclusively present in the BHNK model.

We perform an additional exercise in which we compare the model performance along an additional dimension. In figures A1-A2, we plot the dynamic responses of the output gap, the inflation rate, and the nominal interest rate after aggregate demand and supply shocks in the four versions of the economy; together with the empirical VAR-based IRFs recursively identified, ordering last the nominal interest rate. In the demand (natural rate shock) case (figure A1) we find that the output gap of all four versions of the model behave remarkably close to their empirical counterpart. With respect to inflation, we find that the HBNK setup produces dynamics closest to the empirical IRF, with inattention producing additional intrinsic persistence (compared to the HNK), although the data requires even more intrinsic persistence. We find that the rest of frameworks are quite off in terms of their inflation response. Finally, in terms of the nominal interest rate, we find that the BNK model produces the closest dynamics to the empirical IRF. In a comparison HBNK vs. HNK, we find that the HNK does not produce the necessary persistence, compared to the HBNK. In the supply (cost-push shock) case (figure A2) we find that the output gap reaction in our benchmark setup, the HBNK, is excessively small (in absolute term). In that case, both the BNK and the HNK setups have closer dynamics to their empirical counterpart. We do not find significant heterogeneity in the inflation dynamics. Finally, in terms of the nominal interest rate response, we find that only the BR versions of the model produce the persistence that the data requires; although the four versions predict an excessively large response of nominal rates.

5. Conclusion

The benchmark NK model is purely forward looking and lacks the ability to capture the intrinsic persistence in output and inflation that we observe in the data. In order to avoid this, the literature has included backward-looking agents, either assuming a backward-looking utility function for households or sticky price indexation for firms. Unfortunately, the parameter values that characterize the frictions required to produce the degree of intrinsic persistence that the data suggests are at odds with empirical evidence. In this paper, we harmonize these discrepancies between empirics and theory by building and estimating a New Keynesian model augmented with backward-looking agents *and* cognitive discounting. We find strong evidence for aggregate myopia, with a cognitive discount factor estimate of 0.46 at a quarterly frequency, producing the largest log data density.

For the estimation of the structural parameters, we follow a Bayesian approach that allows a transparent comparison across models. We estimate four different models: the standard NK model, the hybrid NK model, the behavioral NK model, and the behavioral hybrid NK model. We show that cognitive discounting is successful in producing myopia but does not produce intrinsic persistence on its own. We find that the cognitive discount factor, *together* with habit persistence and price indexation, is key to obtain macro estimates that align better with their micro counterpart. Finally, in order to test the ability of our set of models to replicate empirical impulse-response functions, we compare them with an estimated monetary policy shock. We find that only our

Behavioral NK model with both habit formation and backward-looking firms is able to generate, at the same time, hump-shaped responses and enough output and inflation persistence as we observe in the data.

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Appendix A. Appendix for "online" publication

A.1. Demand for good i, Aggregate Price Index and Optimality Conditions

The representative household derives utility from consumption of different goods, indexed $i \in I = [0,1]$, according to the consumption index. Let $\mathcal{C} = \{C_t \in \mathcal{L}^1 \colon C_t \colon I \to \mathbb{R}$ is quasi-concave and Borel measurable, $t \in \mathbb{Z}_+\}$ be the set of consumption choice functions over the set of goods I in the economy at a given period t.

Given the price function $P_t \colon I \to \mathbb{R}_+$ with $\|P_t\|_\infty < \infty$, and for a fixed endowment $Z_t \in \mathbb{R}_+$, the representative household's maximization problem at period t is $\widetilde{C}_t = \max_{C_t \in \mathcal{C}} \left[\int_0^1 C_t(i)^{\frac{\epsilon_t - 1}{\epsilon_t}} di\right]^{\frac{\epsilon_t}{\epsilon_t - 1}}$, subject to the budget constraint:

$$\int_0^1 P_t(i) C_t(i) \; di \leq Z_t \tag{A1}$$

which will be satisfied with equality in the optimum. The derivative of the Lagrangian with respect to $C_t(i)$, the consumption level of good i, yields $\left[\int_0^1 C_t(i)^{\frac{\epsilon_t - 1}{\epsilon_t}} di\right]^{\frac{1}{\epsilon_t - 1}} C_t(i)^{-\frac{1}{\epsilon_t}} - \lambda_t P_t(i) = 0 \implies \widetilde{C}_t^{\frac{1}{\epsilon_t}} C_t(i)^{-\frac{1}{\epsilon_t}} = \lambda_t P_t(i)$ where λ_t is the sequence of Lagrange multipliers attached to the sequence of restrictions (A1). By dividing the last expression for two different goods $i, j \in I$, we find relation between the optimal consumption levels of two different goods:

$$C_t(i) = \left[\frac{P_t(j)}{P_t(i)}\right]^{\epsilon_t} C_t(j) \tag{A2}$$

and inserting (A2) into (A1),

$$Z_t = \int_0^1 P_t(i) \left[\frac{P_t(j)}{P_t(i)}\right]^{\epsilon_t} C_t(j) \; di \implies C_t(j) = \frac{Z_t P_t(j)^{-\epsilon_t}}{\int_0^1 P_t(i)^{1-\epsilon_t} di} \tag{A3}$$

we obtain an expression for the optimal consumption levels of almost all goods in terms of prices and the initial endowment. Integrating the last equation over all goods gives the optimal aggregate consumption level $\widetilde{C}_t = \left[\int_0^1 \left(\frac{Z_t P_t(i)^{-\epsilon_t}}{\int_0^1 P_t(i)^{1-\epsilon_t} di}\right)^{\frac{\epsilon_t - 1}{\epsilon_t}} di\right]^{\frac{\epsilon_t}{\epsilon_t - 1}} = Z_t \left[\int_0^1 P_t(i)^{1-\epsilon_t} di\right]^{\frac{1}{\epsilon_t - 1}}$.

Now, let's define $\widetilde{P}_t$ as the unit cost of the aggregate consumption level $\widetilde{C}_t$ at endowment level Z, $\widetilde{P}_t\widetilde{C}_t = Z_t$. Hence,

$$\text{(A4)} \qquad \widetilde{P}_t Z_t \left[\int_0^1 P_t(i)^{1-\epsilon_t} di\right]^{\frac{1}{\epsilon_t - 1}} = Z_t \implies \widetilde{P}_t = \left[\int_0^1 P_t(i)^{1-\epsilon_t} di\right]^{\frac{1}{1-\epsilon_t}}$$

where (A4) is the price index. Inserting (A4) into (A3), $C_t(j) = \frac{Z_t P_t(j)^{-\epsilon_t}}{\widetilde{P}_t^{1-\epsilon_t}} = \frac{Z_t}{\widetilde{P}_t}\left[\frac{\widetilde{P}_t}{P_t(i)}\right]^{\epsilon_t}$. And finally, replacing Z_t we find the desired optimal consumption for good i in terms of the aggregate good and the aggregate price:

$$\text{(A5)} \qquad C_t(i) = \left[\frac{P_t(i)}{\widetilde{P}_t}\right]^{-\epsilon_t} \widetilde{C}_t$$

With market clearing and a representative household setting, $C_t(i) = Y_t(i)$ and $\widetilde{C}_t = \widetilde{Y}_t$, and we obtain expression (3). Since we deal with the aggregate quantities in the rest of the paper, with a slight abuse of notation we drop the tilde from the aggregate terms.

Finally, in order to obtain the optimality conditions we form the Lagrangian, $\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(C_t - h\overline{C}_{t-1})^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} + \lambda_t \left[B_{t-1} + W_t N_t + T_t - P_t C_t - Q_t B_t\right]\right]$. The FOCs with respect to C_t, B_t and N_t yield,

$$\begin{aligned} C_t : &\quad \lambda_t P_t = (C_t - h\overline{C}_{t-1})^{-\sigma} \\ N_t : &\quad \lambda_t W_t = N_t^{\varphi} \\ B_t : &\quad \lambda_t Q_t = \lambda_{t+1} \end{aligned}$$

Combining them and cancelling the lagrange multiplier λ_t we obtain the optimality conditions $\frac{W_t}{P_t} = \frac{N_t^{\varphi}}{(C_t - h\overline{C}_{t-1})^{-\sigma}}$ and $Q_t = \beta \mathbb{E}_t \left[\left(\frac{C_{t+1} - h\overline{C}_t}{C_t - h\overline{C}_{t-1}}\right)^{-\sigma} \frac{P_t}{P_{t+1}}\right]$.

A.2. Log-linearization of Behavioural Household's Optimality Conditions

We now proceed to log-linearize (4) and (6). Starting with (4), taking a first order Taylor approximation around a zero-inflation steady state, we obtain, $\widehat{w}_t - \widehat{p}_t = \varphi \widehat{n}_t + \frac{\sigma}{1-h}\widehat{c}_t - \frac{\sigma h}{1-h}\widehat{c}_{t-1}$. Turning to (6), we first take logs, $\log Q_t = \log \beta + \mathbb{E}_t^B\{-\sigma \log(C_{t+1} - hC_t) + \sigma \log(C_t - hC_{t-1}) + \log P_t - \log P_{t+1}\}$. Since $Q_t = 1/(1+i_t)$, one can show that $i_t \approx -\log Q_t$. We then write $\rho = -\log \beta$ and $\pi_{t+1} = \log \frac{P_{t+1}}{P_t}$. Let us now log-linearize the terms that include consumption, $\log[C_{t+1} - hC_t] \approx \log[(1-h)C] + \frac{1}{(1-h)C}[C_{t+1} - C] - \frac{h}{(1-h)C}[C_t - C] =$

$\log[(1-h)C] + \frac{1}{1-h}\widehat{c}_{t+1} - \frac{h}{1-h}\widehat{c}_t$. Proceeding in a similar manner with the other consumption term, and plugging into the above expression leads to

$$0 = \mathbb{E}_t^B \left\{ i_t - \rho - \frac{\sigma}{1-h}\left[\widehat{c}_{t+1} - (1+h)\widehat{c}_t + h\widehat{c}_{t-1}\right] - \pi_{t+1} \right\} \tag{A6}$$

Under cognitive discounting, $\mathbb{E}_t^B x_{t+k} = \overline{m}^k \mathbb{E}_t x_{t+k}$ for any variable x. Hence, $0 = \widehat{i}_t - \frac{\sigma}{1-h}\overline{m}\mathbb{E}_t\widehat{c}_{t+1} - \frac{(1+h)\sigma}{1-h}c_t - \frac{h\sigma}{1-h}c_{t-1} - \overline{m}\mathbb{E}_t\pi_{t+1}$, where we have defined $\widehat{i}_t = i_t - i = i_t - \rho$. Rewritting this last expression leads to (8). Written in natural terms and denoting the real interest rate as $r_t = \widehat{i}_t - \overline{m}\mathbb{E}_t\pi_{t+1}$, the previous equation yields $\widehat{c}_t^n = \frac{h}{1+h}\widehat{c}_{t-1}^n + \frac{1}{1+h}\overline{m}\mathbb{E}_t\widehat{c}_{t+1}^n - \frac{1-h}{\sigma(1+h)}r_t^n$. Since $\widehat{c}_t = \widehat{y}_t = \widehat{c}_t^n = \widehat{y}_t^n$, we can rewrite it in terms of the output gap $\widetilde{y}_t = y_t - y_t^n$ and it yields (9).

A.3. Solving the Firm Problem

We can rewrite condition (13) as

$$\begin{aligned} P_t^*(i) &= \frac{\mathbb{E}_t \sum_{k=0}^{\infty}(\theta\beta)^k (C_{t+k} - hC_{t+k-1})^{-\sigma} C_{t+k} P_{t+k}^{\epsilon_{t+k}-1} P_{t+k-1}^{-\omega\epsilon_{t+k}} \frac{W_{t+k}}{A_{t+k}} \mathcal{M}_{t+k}}{\mathbb{E}_t \sum_{k=0}^{\infty}(\theta\beta)^k (C_{t+k} - hC_{t+k-1})^{-\sigma} C_{t+k} P_{t+k}^{\epsilon_{t+k}-1} P_{t+k-1}^{\omega(1-\epsilon_{t+k})}} P_{t-1}^{\omega} = \\ &= \frac{\mathbb{E}_t \sum_{k=0}^{\infty}(\theta\beta)^k (C_{t+k} - hC_{t+k-1})^{-\sigma} C_{t+k} P_{t+k}^{\epsilon_{t+k}} P_{t+k-1}^{-\omega\epsilon_{t+k}} MC_{t+k} \mathcal{M}_{t+k}}{\mathbb{E}_t \sum_{k=0}^{\infty}(\theta\beta)^k (C_{t+k} - hC_{t+k-1})^{-\sigma} C_{t+k} P_{t+k}^{\epsilon_t-1} P_{t+k-1}^{\omega(1-\epsilon_{t+k})}} P_{t-1}^{\omega} \end{aligned} \tag{A7}$$

where we have used $MC_{t+k} = \frac{W_{t+k}}{A_{t+k}P_{t+k}}$. With flexible prices, (A7) collapses to

$$P_t^*(i) = \mathcal{M}_t \frac{(C_t - hC_{t-1})^{-\sigma} C_t P_t^{\epsilon_t} P_{t-1}^{-\omega\epsilon_t} MC_t}{(C_t - hC_{t-1})^{-\sigma} C_t P_t^{\epsilon_t-1} P_{t-1}^{\omega(1-\epsilon_t)}} P_{t-1}^{\omega} = \mathcal{M}_t P_t MC_t \tag{A8}$$

where (A8) is the frictionless mark-up. To simplify computation, we now log-linearize (A7). Separating both sides,

$$\begin{aligned} &P_t^* \mathbb{E}_t \sum_{k=0}^{\infty}(\theta\beta)^k (C_{t+k} - hC_{t+k-1})^{-\sigma} C_{t+k} P_{t+k}^{\epsilon_{t+k}-1} P_{t+k-1}^{\omega(1-\epsilon_{t+k})} = \\ &= \mathbb{E}_t \sum_{k=0}^{\infty}(\theta\beta)^k (C_{t+k} - hC_{t+k-1})^{-\sigma} C_{t+k} P_{t+k}^{\epsilon_{t+k}} P_{t+k-1}^{-\omega\epsilon_{t+k}} MC_{t+k} P_{t-1}^{\omega} \mathcal{M}_{t+k} \end{aligned} \tag{A9}$$

We know that, in steady-state, $P_t^* = P_t = P_{t-1} = P$, $\Pi_t = \Pi = 1$, $C_t = C$, $Q_{t,t+k} = \beta^k$ and $MC_t = MC$. It lasts to find MC. To obtain it, we can write (A7) in steady-state and solve for MC, $P = \mathcal{M}P\text{MC}$. Hence, $MC = \frac{1}{\mathcal{M}}$. Before log-linearizing, divide (A9) by P_{t-1},

$$\frac{P_t^*}{P_{t-1}}\mathbb{E}_t\sum_{k=0}^{\infty}(\theta\beta)^k(C_{t+k}-hC_{t+k-1})^{-\sigma}C_{t+k}P_{t+k}^{\epsilon_{t+k}-1}P_{t+k-1}^{\omega(1-\epsilon_{t+k})} =$$

(A10)

$$= \mathbb{E}_t\sum_{k=0}^{\infty}(\theta\beta)^k(C_{t+k}-hC_{t+k-1})^{-\sigma}C_{t+k}P_{t+k}^{\epsilon_{t+k}}P_{t+k-1}^{-\omega\epsilon_{t+k}}MC_{t+k}P_{t-1}^{\omega-1}\mathcal{M}_{t+k}$$

Log-linearizing the LHS,

$$\frac{P_t^*}{P_{t-1}}\mathbb{E}_t\sum_{k=0}^{\infty}(\theta\beta)^k(C_{t+k}-hC_{t+k-1})^{-\sigma}C_{t+k}P_{t+k}^{\epsilon-1}P_{t+k-1}^{\omega(1-\epsilon)} \simeq$$

$$\begin{aligned}
&\simeq \sum_{k=0}^{\infty}(\theta\beta)^k C^{1-\sigma}(1-h)^{-\sigma}P^{-(1-\epsilon)(1-\omega)}+\\
&+\frac{1}{P}\sum_{k=0}^{\infty}(\theta\beta)^k C^{1-\sigma}(1-h)^{-\sigma}P^{-(1-\epsilon)(1-\omega)}(P_t^*-P)-\\
&-\frac{P}{P^2}\sum_{k=0}^{\infty}(\theta\beta)^k C^{1-\sigma}(1-h)^{-\sigma}P^{-(1-\epsilon)(1-\omega)}(P_{t-1}-P)+\\
&+\sum_{k=0}^{\infty}(\theta\beta)^k C^{1-\sigma}(1-h)^{-\sigma}(\epsilon-1)P^{\epsilon-2}P^{\omega(1-\epsilon)}(P_{t+k}-P)+\\
&+\sum_{k=0}^{\infty}(\theta\beta)^k C^{1-\sigma}(1-h)^{-\sigma}P^{\epsilon-1}\omega(1-\epsilon)P^{\omega(1-\epsilon)-1}(P_{t+k-1}-P)+\\
&+\sum_{k=0}^{\infty}(\theta\beta)^k\{\underbrace{(-\sigma)[C(1-h)]^{-\sigma-1}C+[C(1-h)]^{-\sigma}}_{C^{-\sigma}(1-h)^{-\sigma-1}(1-h-\sigma)}\}P^{-(1-\epsilon)(1-\omega)}(C_{t+k}-C)+\\
&+\sum_{k=0}^{\infty}(\theta\beta)^k\underbrace{(-\sigma)[C(1-h)]^{-\sigma-1}(-h)C}_{\sigma hC^{-\sigma}(1-h)^{-\sigma-1}}P^{-(1-\epsilon)(1-\omega)}(C_{t+k-1}-C)+\\
&+\sum_{k=0}^{\infty}(\theta\beta)^k C^{1-\sigma}(1-h)^{-\sigma}P^{-(1-\epsilon)(1-\omega)}\ln(P^{1-\omega})(\epsilon_{t+k}-\epsilon)=\\
&=\sum_{k=0}^{\infty}(\theta\beta)^k C^{1-\sigma}(1-h)^{-\sigma}P^{-(1-\epsilon)(1-\omega)}\Big\{1+p_t^*-p-p_{t-1}+p-(1-\epsilon)(p_{t+k}-p)+\\
&+\omega(1-\epsilon)(p_{t+k-1}-p)+\left(1-\frac{\sigma}{1-h}\right)(c_{t+k}-c)+\frac{\sigma h}{1-h}(c_{t+k-1}-c)+\ln(P^{1-\omega})(\epsilon_{t+k}-\epsilon)\Big\}
\end{aligned}$$

Log-linearizing the RHS,

$$\mathbb{E}_t \sum_{k=0}^{\infty}(\theta\beta)^k (C_{t+k} - hC_{t+k-1})^{-\sigma} C_{t+k} P_{t+k}^{\epsilon_{t+k}} P_{t+k-1}^{-\omega\epsilon_{t+k}} MC_{t+k} P_{t-1}^{\omega-1} \mathcal{M}_{t+k} \simeq$$

$$\begin{aligned}
\simeq & \mathcal{M}\sum_{k=0}^{\infty}(\theta\beta)^k C^{1-\sigma}(1-h)^{-\sigma} P^{-(1-\epsilon)(1-\omega)} MC+ \\
& + \mathcal{M}\sum_{k=0}^{\infty}(\theta\beta)^k C^{1-\sigma}(1-h)^{-\sigma} P^{\epsilon} P^{-\omega\epsilon} MC(\omega-1) P^{\omega-2}(P_{t-1} - P)+ \\
& + \mathcal{M}\sum_{k=0}^{\infty}(\theta\beta)^k C^{1-\sigma}(1-h)^{-\sigma} \epsilon P^{\epsilon-1} P^{-\omega\epsilon} MC P^{\omega-1}(P_{t+k} - P)+ \\
& + \mathcal{M}\sum_{k=0}^{\infty}(\theta\beta)^k C^{1-\sigma}(1-h)^{-\sigma} P^{\epsilon}(-\omega\epsilon) P^{-\omega\epsilon-1} MC P^{\omega-1}(P_{t+k-1} - P)+ \\
& + \mathcal{M}\sum_{k=0}^{\infty}(\theta\beta)^k C^{-\sigma}(1-h)^{-\sigma-1}(1-h-\sigma) P^{-(1-\epsilon)(1-\omega)} MC(C_{t+k} - C)+ \\
& + \mathcal{M}\sum_{k=0}^{\infty}(\theta\beta)^k C^{-\sigma}(1-h)^{-\sigma-1}\sigma h P^{-(1-\epsilon)(1-\omega)} MC(C_{t+k-1} - C)+ \\
& + \mathcal{M}\sum_{k=0}^{\infty}(\theta\beta)^k C^{1-\sigma}(1-h)^{-\sigma} P^{-(1-\epsilon)(1-\omega)} (MC_{t+k} - MC)+ \\
& + \mathcal{M}\sum_{k=0}^{\infty}(\theta\beta)^k C^{1-\sigma}(1-h)^{-\sigma} P^{-(1-\epsilon)(1-\omega)} \ln(P^{1-\omega}) \mathrm{MC}(\epsilon_{t+k} - \epsilon)+ \\
& + \sum_{k=0}^{\infty}(\theta\beta)^k C^{1-\sigma}(1-h)^{-\sigma} P^{-(1-\epsilon)(1-\omega)} \mathrm{MC}(\mathcal{M}_{t+k} - \mathcal{M}) = \\
= & \sum_{k=0}^{\infty}(\theta\beta)^k C^{1-\sigma}(1-h)^{-\sigma} P^{-(1-\epsilon)(1-\omega)} \Big\{ 1 - (1-\omega)(p_{t-1} - p) + \epsilon(p_{t+k} - p) - \\
& - \omega\epsilon(p_{t+k-1} - p) + \left(1 - \frac{\sigma}{1-h}\right)(c_{t+k} - c) + \frac{\sigma h}{1-h}(c_{t+k-1} - c) + mc_{t+k} - mc+ \\
& + \ln(P^{1-\omega})(\epsilon_{t+k} - \epsilon) + (\mu_{t+k} - \mu) \Big\}
\end{aligned}$$

Noticing that in steady state $P = 1$, and setting LHS=RHS, eliminating $C^{1-\sigma}(1 -$

$h)^{-\sigma}P^{-(1-\epsilon)(1-\omega)}$ on both sides,

$$\sum_{k=0}^{\infty}(\theta\beta)^k\Big\{1+p_t^*-p-p_{t-1}+p-(1-\epsilon)(p_{t+k}-p)+\omega(1-\epsilon)(p_{t+k-1}-p)+$$
$$+\left(1-\frac{\sigma}{1-h}\right)(c_{t+k}-c)+\frac{\sigma h}{1-h}(c_{t+k-1}-c)+(\mu_{t+k}-\mu)\Big\}=$$
$$=\sum_{k=0}^{\infty}(\theta\beta)^k\Big\{1-(1-\omega)(p_{t-1}-p)+\epsilon(p_{t+k}-p)-\omega\epsilon(p_{t+k-1}-p)+$$
$$+\left(1-\frac{\sigma}{1-h}\right)(c_{t+k}-c)+\frac{\sigma h}{1-h}(c_{t+k-1}-c)+mc_{t+k}-mc+\mu_{t+k}-\mu\Big\}$$

Rearranging and cancelling terms, we end up with

$$\begin{aligned}
p_t^* &= (1-\theta\beta)\sum_{k=0}^{\infty}(\theta\beta)^k\mathbb{E}_t\left[p_{t+k}-\omega(\omega p_{t+k-1}-p_{t-1})+mc_{t+k}-mc+\mu_{t+k}-\mu\right]\\
&= (1-\theta\beta)\sum_{k=0}^{\infty}(\theta\beta)^k\mathbb{E}_t\left[p_{t+k}-\omega(\omega p_{t+k-1}-p_{t-1})+\widehat{mc}_{t+k}+\widehat{\mu}_{t+k}\right]\\
&= p_t+(1-\theta\beta)\sum_{k=0}^{\infty}(\theta\beta)^k\mathbb{E}_t\left[(p_{t+k}-p_t)-\omega(\omega p_{t+k-1}-p_{t-1})+\widehat{mc}_{t+k}+\widehat{\mu}_{t+k}\right]
\end{aligned}$$

which can be rewritten as (15).

A.4. Aggregate Price Dynamics

Let S_t denote the subset of firms not reoptimizing at time t,

$$P_t=\left[\int_0^1 P_t(i)^{1-\epsilon_t}di\right]^{\frac{1}{1-\epsilon_t}}=\Bigg\{\underbrace{\int_{S_t}[P_{t-1}(i)\Pi_{t-1}^{\omega}]^{1-\epsilon_t}di}_{\Pi_{t-1}^{\omega(1-\epsilon_t)}\int_{S_t}P_{t-1}(i)^{1-\epsilon_t}di}+\int_{S_t^C}(P_t^*)^{1-\epsilon_t}di\Bigg\}^{\frac{1}{1-\epsilon_t}}=$$

$$=\left[\Pi_{t-1}^{\omega(1-\epsilon_t)}\theta\int_0^1 P_{t-1}(i)^{1-\epsilon_t}di+(1-\theta)\int_0^1(P_t^*)^{1-\epsilon_t}di\right]^{\frac{1}{1-\epsilon_t}}=\left[\Pi_{t-1}^{\omega(1-\epsilon_t)}\theta P_{t-1}^{1-\epsilon_t}+(1-\theta)(P_t^*)^{1-\epsilon_t}\right]^{\frac{1}{1-\epsilon_t}}$$

Moving the exponent from the RHS to the LHS, and dividing in both sides by $P_{t-1}^{1-\epsilon_t}$,

(A11)

$$\left(\frac{P_t}{P_{t-1}}\right)^{1-\epsilon_t} = \Pi_{t-1}^{\omega(1-\epsilon_t)}\theta + (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\epsilon_t} \implies \Pi_t^{1-\epsilon_t} = \left(\frac{P_{t-1}}{P_{t-2}}\right)^{\omega(1-\epsilon_t)}\theta + (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\epsilon_t}$$

To simplify computation, I now log-linearize the left-hand side of (A11),

$$\Pi_t^{1-\epsilon_t} \simeq \Pi^{1-\epsilon} + (1-\epsilon)\Pi^{-\epsilon}\underbrace{(\Pi_t - \Pi)}_{\pi_t} = 1 + (1-\epsilon)\pi_t$$

since $\Pi = \frac{P}{P} = 1$. A log-linearization of the right-hand side around a zero-inflation steady-state yields

$$\begin{aligned}
\left(\frac{P_{t-1}}{P_{t-2}}\right)^{\omega(1-\epsilon_t)}\theta + (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\epsilon_t} \simeq{}& \left(\frac{P}{P}\right)^{\omega(1-\epsilon)}\theta + (1-\theta)\left(\frac{P^*}{P}\right)^{1-\epsilon} + \\
&+ (1-\theta)(1-\epsilon)P^{-\epsilon}P^{\epsilon-1}(P_t^* - P) + \\
&+ \left[\theta\omega(1-\epsilon)P^{\omega(1-\epsilon)-1}P^{-\omega(1-\epsilon)} - (1-\theta)(1-\epsilon)P^{1-\epsilon}P^{2-\epsilon}\right] \times \\
&\times (P_{t-1} - P) - \theta\omega(1-\epsilon)P^{\omega(1-\epsilon)}P^{-\omega(1-\epsilon)-1}(P_{t-2} - P) = \\
={}& \theta + 1 - \theta + (1-\theta)(1-\epsilon)\widehat{p}_t^* + \\
&+ [\theta\omega(1-\epsilon) - (1-\theta)(1-\epsilon)]\widehat{p}_{t-1} - \theta\omega(1-\epsilon)\widehat{p}_{t-2} = \\
={}& 1 + (1-\theta)(1-\epsilon)\widehat{p}_t^* - (1-\epsilon)[1-\theta(1+\omega)]\widehat{p}_{t-1} - \\
&- \theta\omega(1-\epsilon)\widehat{p}_{t-2} = \\
={}& 1 + (1-\theta)(1-\epsilon)\,p_t^* - (1-\epsilon)[1-\theta(1+\omega)]\,p_{t-1} - \\
&- \theta\omega(1-\epsilon)\,p_{t-2}
\end{aligned}$$

Writing $\widehat{x}_t = x_t - x$, all log prices are cancelled out.

$$\begin{aligned}
\text{LHS=RHS:}\quad 1{+}(1-\epsilon)\pi_t &= 1 + (1-\theta)(1-\epsilon)\,p_t^* - (1-\epsilon)[1-\theta(1+\omega)]\,p_{t-1} - \theta\omega(1-\epsilon)\,p_{t-2} \implies \\
\implies \pi_t &= (1-\theta)\,p_t^* - [1-\theta(1+\omega)]\,p_{t-1} - \theta\omega\,p_{t-2} \\
&= \theta\omega\pi_{t-1} + (1-\theta)(p_t^* - p_{t-1})
\end{aligned}$$

A.5. Deriving the Behavioural Hybrid New Keynesian Phillips Curve

Rewriting $\theta\beta\overline{m} = \delta$ and $\widetilde{mc}_t = \widehat{mc}_t + \widehat{\mu}_t$, the firm's problem optimality condition (16) reads

$$p_t^* = p_t + (1-\theta\beta)\sum_{k=0}^{\infty}\delta^k \mathbb{E}_t\left[\overline{m}(\pi_{t+1}+\ldots+\pi_{t+k}) - \omega\overline{m}(\pi_t+\ldots+\pi_{t+k-1}) + \overline{m}\widetilde{mc}_{t+k}\right]$$

(A12)

$$= p_t + (1-\theta\beta)\mathbb{E}_t\left[\overline{m}\sum_{k=0}^{\infty}\delta^k(\pi_{t+1}+\ldots+\pi_{t+k}) - \omega\overline{m}\sum_{k=0}^{\infty}\delta^k(\pi_t+\ldots+\pi_{t+k-1}) + \overline{m}\sum_{k=0}^{\infty}\delta^k\widetilde{mc}_{t+k}\right]$$

We can calculate the following

$$H_t = \sum_{k=1}^{\infty}\delta^k(\pi_{t+1}+\ldots+\pi_{t+k}) = \sum_{j=1}^{\infty}\pi_{t+j}\sum_{k=j}^{\infty}\delta^k = \sum_{j=1}^{\infty}\pi_{t+j}\frac{\delta^j}{1-\delta} = \frac{1}{1-\delta}\sum_{j=1}^{\infty}\pi_{t+j}\delta^j =$$

$$= \frac{1}{1-\delta}\sum_{k=0}^{\infty}\pi_{t+k}\delta^k 1_{\{k>0\}}$$

$$\widetilde{H}_t = \sum_{k=1}^{\infty}\delta^k(\pi_t+\ldots+\pi_{t+k-1}) = \sum_{j=1}^{\infty}\pi_{t+j-1}\sum_{k=j}^{\infty}\delta^k = \sum_{j=1}^{\infty}\pi_{t+j-1}\frac{\delta^j}{1-\delta} = \frac{1}{1-\delta}\sum_{j=1}^{\infty}\pi_{t+j-1}\delta^j =$$

$$= \frac{1}{1-\delta}\sum_{k=0}^{\infty}\pi_{t+k-1}\delta^k 1_{\{k>0\}}$$

Rewriting (A12),

$$p_t^* - p_t = (1-\theta\beta)\mathbb{E}_t\left[\overline{m}H_t - \omega\overline{m}\widetilde{H}_t + \overline{m}\sum_{k=0}^{\infty}\delta^k\widetilde{mc}_{t+k}\right]$$

$$= (1-\theta\beta)\mathbb{E}_t\left[\overline{m}\frac{1}{1-\delta}\sum_{k=0}^{\infty}\pi_{t+k}\delta^k 1_{\{k>0\}} - \omega\overline{m}\frac{1}{1-\delta}\sum_{k=0}^{\infty}\pi_{t+k-1}\delta^k 1_{\{k>0\}} + \overline{m}\sum_{k=0}^{\infty}\delta^k\widetilde{mc}_{t+k}\right]$$

$$= (1-\theta\beta)\mathbb{E}_t\sum_{k=0}^{\infty}\delta^k\left[\overline{m}\frac{1}{1-\delta}\pi_{t+k}1_{\{k>0\}} - \omega\overline{m}\frac{1}{1-\delta}\pi_{t+k-1}1_{\{k>0\}} + \overline{m}\widetilde{mc}_{t+k}\right]$$

$$= \mathbb{E}_t\sum_{k=0}^{\infty}\delta^k\left[\overline{m}\frac{1-\theta\beta}{1-\delta}\pi_{t+k}1_{\{k>0\}} - \omega\overline{m}\frac{1-\theta\beta}{1-\delta}\pi_{t+k-1}1_{\{k>0\}} + \overline{m}(1-\theta\beta)\widetilde{mc}_{t+k}\right]$$

(A13)

$$= \mathbb{E}_t \sum_{k=0}^{\infty} \delta^k \left[\widetilde{m}_\pi \pi_{t+k} 1_{\{k>0\}} - \omega \widetilde{m}_\pi \pi_{t+k-1} 1_{\{k>0\}} + \widetilde{m}_\mu \widetilde{mc}_{t+k}\right]$$

where $\widetilde{m}_\pi = \overline{m}\frac{1-\theta\beta}{1-\delta}$ and $\widetilde{m}_\mu = \overline{m}(1-\theta\beta)$. Rewriting the price evolution expression (17),

$$p_t^* - p_{t-1} + p_t - p_t = \frac{\pi_t - \theta\omega\pi_{t-1}}{1-\theta} \implies p_t^* - p_t = \frac{\theta}{1-\theta}(\pi_t - \omega\pi_{t-1})$$

Hence, we can rewrite (A13) as

$$\frac{\theta}{1-\theta}(\pi_t - \omega\pi_{t-1}) = \mathbb{E}_t \sum_{k=0}^{\infty} \delta^k \left[\widetilde{m}_\pi \pi_{t+k} 1_{\{k>0\}} - \omega \widetilde{m}_\pi \pi_{t+k-1} 1_{\{k>0\}} + \widetilde{m}_\mu \widetilde{mc}_{t+k}\right] \tag{A14}$$

Let us now introduce the forward operator F such that $F^k x_t = x_{t+k}$. Using the forward operator, we can write

$$\sum_{k=0}^{\infty} \delta^k x_{t+k} = \sum_{k=0}^{\infty} \delta^k F^k x_t = \sum_{k=0}^{\infty} (\delta F)^k x_t = \frac{x_t}{1-\delta F} \tag{A15}$$

Rewriting (A14) using (A15)

$$\begin{aligned}
\frac{\theta}{1-\theta}(\pi_t - \omega\pi_{t-1}) &= \widetilde{m}_\pi \mathbb{E}_t \left[\sum_{k=0}^{\infty} \delta^k \pi_{t+k} 1_{\{k>0\}}\right] - \omega \widetilde{m}_\pi \mathbb{E}_t \left[\sum_{k=0}^{\infty} \delta^k \pi_{t+k-1} 1_{\{k>0\}}\right] + \widetilde{m}_\mu \mathbb{E}_t \left[\sum_{k=0}^{\infty} \delta^k \widetilde{mc}_{t+k}\right] \\
&= \widetilde{m}_\pi \mathbb{E}_t \left[\sum_{k=0}^{\infty} (\delta F)^k \pi_t 1_{\{k>0\}}\right] - \omega \widetilde{m}_\pi \mathbb{E}_t \left[\sum_{k=0}^{\infty} (\delta F)^k \pi_{t-1} 1_{\{k>0\}}\right] + \widetilde{m}_\mu \mathbb{E}_t \left[\sum_{k=0}^{\infty} (\delta F)^k \widetilde{mc}_t\right] \\
&= \widetilde{m}_\pi \mathbb{E}_t \left[\sum_{k=0}^{\infty} (\delta F)^k \pi_t - \pi_t\right] - \omega \widetilde{m}_\pi \mathbb{E}_t \left[\sum_{k=0}^{\infty} (\delta F)^k \pi_{t-1} - \pi_{t-1}\right] + \widetilde{m}_\mu \mathbb{E}_t \left[\sum_{k=0}^{\infty} (\delta F)^k \widetilde{mc}_t\right] \\
&= \widetilde{m}_\pi \mathbb{E}_t \left[\frac{\pi_t}{1-\delta F} - \pi_t\right] - \omega \widetilde{m}_\pi \mathbb{E}_t \left[\frac{\pi_{t-1}}{1-\delta F} - \pi_{t-1}\right] + \widetilde{m}_\mu \mathbb{E}_t \left[\frac{\widetilde{mc}_t}{1-\delta F}\right] \\
&= \widetilde{m}_\pi \mathbb{E}_t \left[\frac{\delta F \pi_t}{1-\delta F}\right] - \omega \widetilde{m}_\pi \mathbb{E}_t \left[\frac{\delta F \pi_{t-1}}{1-\delta F}\right] + \widetilde{m}_\mu \mathbb{E}_t \left[\frac{\widetilde{mc}_t}{1-\delta F}\right]
\end{aligned}$$

Premultiplying by $(1-\delta F)$,

$$\frac{\theta}{1-\theta}(1-\delta F)(\pi_t - \omega\pi_{t-1}) = \widetilde{m}_\pi \mathbb{E}_t [\delta F \pi_t] - \omega \widetilde{m}_\pi \mathbb{E}_t [\delta F \pi_{t-1}] + \widetilde{m}_\mu \mathbb{E}_t \left[\widetilde{mc}_t\right]$$

which can be rearranged to (18). Let us now derive the Behavioural Hybrid New Keynesian Phillips curve. We have the following expressions

$$mc_t = w_t - p_t - a_t \tag{A16}$$

$$y_t = a_t + n_t \tag{A17}$$

$$w_t - p_t = \varphi n_t + \frac{\sigma}{1-h} c_t - \frac{\sigma h}{1-h} c_{t-1} \tag{A18}$$

$$c_t = y_t \tag{A19}$$

Hence, we can write

$$\begin{aligned} mc_t = w_t - p_t - a_t = \varphi n_t + \frac{\sigma}{1-h} c_t - \frac{\sigma h}{1-h} c_{t-1} - a_t = \varphi(y_t - a_t) + \frac{\sigma}{1-h} c_t - \frac{\sigma h}{1-h} c_{t-1} - a_t \\ = \varphi(y_t - a_t) + \frac{\sigma}{1-h} y_t - \frac{\sigma h}{1-h} y_{t-1} - a_t = \left(\varphi + \frac{\sigma}{1-h}\right) y_t - \frac{\sigma h}{1-h} y_{t-1} - (1+\varphi) a_t \end{aligned}$$

In the natural equilibrium (with no price frictions), the marginal cost is $mc_t^r = -\mu_t = \left(\varphi + \frac{\sigma}{1-h}\right) y_t^n - \frac{\sigma h}{1-h} y_{t-1}^n - (1+\varphi) a_t$, which we can rewrite as

$$y_t^n = \frac{\sigma h}{\varphi(1-h) + \sigma} y_{t-1}^n - \frac{1-h}{\varphi(1-h)+\sigma} \mu_t + \frac{(1+\varphi)(1-h)}{\varphi(1-h)+\sigma} a_t \tag{A20}$$

hence, we can write $\widehat{mc}_t = mc_t - mc = \left(\varphi + \frac{\sigma}{1-h}\right) \widetilde{y}_t - \frac{\sigma h}{1-h} \widetilde{y}_{t-1}$ which, inserted into the (18), yields the Behavioural Hybrid New Keynesian Phillips curve (20).

A.6. Intrinsic Myopia

Consider instead a framework without habit formation nor price indexation, but with the following belief formation process. For any variable $\widehat{x}_t$, the BR forecast of such variable at horizon h is instead given by $\mathbb{E}_t^B \widehat{x}_{t+h} = \overline{m}^h \mathbb{E}_t \widehat{x}_{t+h} + (1-\overline{m}^h)\widehat{x}_{t-1}$ for $h > 0$, which implies that expectations of objects more in the future are more anchored to the past.

Let us first derive the DIS curve. Starting from (A6) without habit formation, we can write $\widehat{c}_t = -\frac{1}{\sigma}(i_t - \rho - \mathbb{E}_t^B \pi_{t+1}) + \mathbb{E}_t^B \widehat{c}_{t+1} = -\frac{1}{\sigma}\left[i_t - \rho - \overline{m}\mathbb{E}_t \pi_{t+1} - (1-\overline{m})\pi_{t-1}\right] + \overline{m}\mathbb{E}_t \widehat{c}_{t+1} + (1 - \overline{m})\widehat{c}_{t-1}$, and we can finally write the intrinsic myopia DIS curve as

$$\widetilde{y}_t = -\frac{1}{\sigma}\left[\widehat{i}_t - \overline{m}\mathbb{E}_t \pi_{t+1} - (1-\overline{m})\pi_{t-1} - r_t^n\right] + \overline{m}\mathbb{E}_t \widetilde{y}_{t+1} + (1-\overline{m})\widetilde{y}_{t-1}$$

Let us move to the supply side. Rewriting condition (15) without price indexation, but with intrinsic myopia,

$$
\begin{aligned}
p_t^* - p_{t-1} &= (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k \mathbb{E}_t^B \widetilde{\mathrm{mc}}_{t+k} + \sum_{k=0}^{\infty}(\beta\theta)^k \mathbb{E}_t^B \pi_{t+k} + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k \mathbb{E}_t^B \widehat{\mu}_{t+k} \\
&= \frac{\beta\theta(1-\overline{m})}{1-\beta\theta\overline{m}}\widetilde{\mathrm{mc}}_{t-1} + \frac{\beta\theta(1-\overline{m})}{(1-\beta\theta)(1-\beta\theta\overline{m})}\pi_{t-1} + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta\overline{m})^k \mathbb{E}_t \widetilde{\mathrm{mc}}_{t+k} + \sum_{k=0}^{\infty}(\beta\theta\overline{m})^k \mathbb{E}_t \pi_{t+k}
\end{aligned}
$$

where we have used $\sum_{k=0}^{\infty}(\beta\theta)^k \mathbb{E}_t^B \widehat{x}_{t+h} = \sum_{k=0}^{\infty}(\beta\theta\overline{m})^k \mathbb{E}_t \widehat{x}_{t+k} + \frac{\beta\theta(1-\overline{m})}{(1-\beta\theta)(1-\beta\theta\overline{m})}\widehat{x}_{t-1}$. If we take out past and present elements of each summation operator, the equation can be written more compactly as a difference equation,

$$
\begin{aligned}
p_t^* - p_{t-1} &= \frac{\beta\theta(1-\overline{m})}{1-\beta\theta\overline{m}}\widetilde{\mathrm{mc}}_{t-1} + \frac{\beta\theta(1-\overline{m})}{(1-\beta\theta)(1-\beta\theta\overline{m})}\pi_{t-1} + (1-\beta\theta)\widetilde{\mathrm{mc}}_t + \pi_t \\
&+ (1-\beta\theta)\sum_{k=1}^{\infty}(\beta\theta\overline{m})^k \mathbb{E}_t \widetilde{\mathrm{mc}}_{t+k} + \sum_{k=1}^{\infty}(\beta\theta\overline{m})^k \mathbb{E}_t \pi_{t+k} \\
&= \frac{\beta\theta(1-\overline{m})}{1-\beta\theta\overline{m}}\widetilde{\mathrm{mc}}_{t-1} + \frac{\beta\theta(1-\overline{m})}{(1-\beta\theta)(1-\beta\theta\overline{m})}\pi_{t-1} + (1-\beta\theta)\widetilde{\mathrm{mc}}_t + \pi_t \\
&+ \beta\theta\overline{m}\left[(1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta\overline{m})^k \mathbb{E}_t \widetilde{\mathrm{mc}}_{t+k+1} + \sum_{k=0}^{\infty}(\beta\theta\overline{m})^k \mathbb{E}_t \pi_{t+k+1}\right] \\
&= \frac{\beta\theta(1-\overline{m})}{1-\beta\theta\overline{m}}\widetilde{\mathrm{mc}}_{t-1} + \frac{\beta\theta(1-\overline{m})}{(1-\beta\theta)(1-\beta\theta\overline{m})}\pi_{t-1} + (1-\beta\theta)\widetilde{\mathrm{mc}}_t + \pi_t \\
&+ \beta\theta\overline{m}\left[\mathbb{E}_t(p_{t+1}^* - p_t) - \frac{\beta\theta(1-\overline{m})}{1-\beta\theta\overline{m}}\widetilde{\mathrm{mc}}_t - \frac{\beta\theta(1-\overline{m})}{(1-\beta\theta)(1-\beta\theta\overline{m})}\pi_t\right] \\
&= \beta\theta\overline{m}\mathbb{E}_t(p_{t+1}^* - p_t) + \left[1-\beta\theta-\beta\theta\overline{m}\frac{\beta\theta(1-\overline{m})}{1-\beta\theta\overline{m}}\right]\widetilde{\mathrm{mc}}_t + \left[1-\beta\theta\overline{m}\frac{\beta\theta(1-\overline{m})}{(1-\beta\theta)(1-\beta\theta\overline{m})}\right]\pi_t \\
&+ \frac{\beta\theta(1-\overline{m})}{1-\beta\theta\overline{m}}\widetilde{\mathrm{mc}}_{t-1} + \frac{\beta\theta(1-\overline{m})}{(1-\beta\theta)(1-\beta\theta\overline{m})}\pi_{t-1}
\end{aligned}
$$

Inserting the aggregate price dynamics (17), we can write

$$
\begin{aligned}
\frac{1}{1-\theta}\pi_t &= \frac{\beta\theta\overline{m}}{1-\theta}\mathbb{E}_t\pi_{t+1} + \left[1-\beta\theta-\beta\theta\overline{m}\frac{\beta\theta(1-\overline{m})}{1-\beta\theta\overline{m}}\right]\widetilde{\mathrm{mc}}_t + \left[1-\beta\theta\overline{m}\frac{\beta\theta(1-\overline{m})}{(1-\beta\theta)(1-\beta\theta\overline{m})}\right]\pi_t \\
&+ \frac{\beta\theta(1-\overline{m})}{1-\beta\theta\overline{m}}\widetilde{\mathrm{mc}}_{t-1} + \frac{\beta\theta(1-\overline{m})}{(1-\beta\theta)(1-\beta\theta\overline{m})}\pi_{t-1}
\end{aligned}
$$

rearranging elements,

$$
\begin{aligned}
\pi_t &= \frac{\overline{m}\beta(1-\beta\theta)(1-\beta\theta\overline{m})}{1-\beta\theta\{1+\overline{m}[1-\beta+\beta\overline{m}(1-\theta)]\}}\mathbb{E}_t\pi_{t+1} + \frac{(1-\theta)(1-\beta\theta)\{1-\beta\theta[1+\overline{m}(1-\beta\theta\overline{m})]\}}{\theta\{1-\beta\theta[1+\overline{m}-\beta\overline{m}(1-\overline{m})]+\overline{m}^2\beta^2\theta^2\}}\widetilde{\mathrm{mc}}_t \\
&+ \frac{(1-\overline{m})\beta(1-\beta\theta)(1-\theta)}{1-\beta\theta\{1+\overline{m}[1-\beta+\beta\overline{m}(1-\theta)]\}}\widetilde{\mathrm{mc}}_{t-1} + \frac{(1-\overline{m})\beta(1-\theta)}{1-\beta\theta\{1+\overline{m}[1-\beta+\beta\overline{m}(1-\theta)]\}}\pi_{t-1} \\
&= \frac{\overline{m}\beta(1-\beta\theta)(1-\beta\theta\overline{m})}{1-\beta\theta\{1+\overline{m}[1-\beta+\beta\overline{m}(1-\theta)]\}}\mathbb{E}_t\pi_{t+1} + \frac{(1-\theta)(1-\beta\theta)\{1-\beta\theta[1+\overline{m}(1-\beta\theta\overline{m})]\}}{\theta\{1-\beta\theta[1+\overline{m}-\beta\overline{m}(1-\overline{m})]+\overline{m}^2\beta^2\theta^2\}}(\widehat{\mathrm{mc}}_t + \widehat{\mu}_t) \\
&+ \frac{(1-\overline{m})\beta(1-\beta\theta)(1-\theta)}{1-\beta\theta\{1+\overline{m}[1-\beta+\beta\overline{m}(1-\theta)]\}}(\widehat{\mathrm{mc}}_{t-1} + \widehat{\mu}_{t-1}) + \frac{(1-\overline{m})\beta(1-\theta)}{1-\beta\theta\{1+\overline{m}[1-\beta+\beta\overline{m}(1-\theta)]\}}\pi_{t-1}
\end{aligned}
$$

Finally, introducing (19) without habit formation, we can write the intrinsic myopia NK Phillips curve,

$$
\begin{aligned}
\pi_t &= \frac{\overline{m}\beta(1-\beta\theta)(1-\beta\theta\overline{m})}{1-\beta\theta\{1+\overline{m}[1-\beta+\beta\overline{m}(1-\theta)]\}}\mathbb{E}_t\pi_{t+1} + \frac{(1-\theta)(1-\beta\theta)\{1-\beta\theta[1+\overline{m}(1-\beta\theta\overline{m})]\}}{\theta\{1-\beta\theta[1+\overline{m}-\beta\overline{m}(1-\overline{m})]+\overline{m}^2\beta^2\theta^2\}}[(\sigma+\varphi)\widetilde{y}_t + \widehat{\mu}_t] \\
&+ \frac{(1-\overline{m})\beta(1-\beta\theta)(1-\theta)}{1-\beta\theta\{1+\overline{m}[1-\beta+\beta\overline{m}(1-\theta)]\}}[(\sigma+\varphi)\widetilde{y}_{t-1} + \widehat{\mu}_{t-1}] + \frac{(1-\overline{m})\beta(1-\theta)}{1-\beta\theta\{1+\overline{m}[1-\beta+\beta\overline{m}(1-\theta)]\}}\pi_{t-1}
\end{aligned}
$$

A.7. Robustness Checks

	Prior Distribution		Posterior Distribution			
		Mean (S.d)	**GDP Deflator**	**Estimating $\overline{m}$**	**1985:I-2007:III**	**Targeting Output**
β	*Beta*	0.99	0.990	0.990	0.990	0.990
		(0.001)	(0.989, 0.992)	(0.988, 0.992)	(0.988, 0.992)	(0.988, 0.992)
σ	*Normal*	1.50	1.280	1.239	1.261	1.353
		(0.37)	(0.703, 1.829)	(0.634, 1.777)	(0.663, 1.817)	(0.783, 1.917)
φ	*Normal*	2	1.425	1.446	1.409	1.446
		(0.75)	(0.500, 2.257)	(0.500, 2.306)	(0.500, 2.269)	(0.500, 2.298)
ϕ_π	*Normal*	1.50	1.430	1.346	1.388	1.352
		(0.15)	(1.219, 1.642)	(1.143, 1.547)	(1.134, 1.638)	(1.150, 1.547)
ϕ_y	*Normal*	0.15	0.340	0.333	0.288	0.336
		(0.10)	(0.231, 0.451)	(0.225, 0.444)	(0.134, 0.456)	(0.229, 0.445)
θ	*Beta*	0.50	0.930	0.904	0.873	0.922
		(0.10)	(0.912, 0.953)	(0.870, 0.939)	(0.828, 0.921)	(0.898, 0.948)
h	*Beta*	0.70	0.654	0.638	0.623	0.701
		(0.15)	(0.500, 0.813)	(0.473, 0.808)	(0.420, 0.816)	(0.559, 0.851)
ω	*Beta*	0.50	0.077	0.781	0.268	0.777
		(0.15)	(0.0243 0.129)	(0.707, 0.857)	(0.114, 0.419)	(0.705, 0.854)
$\overline{m}$	*Implied*	—	0.51	0.365	0.391	0.65
		—	(—)	(0.153, 0.568)	(0.167, 0.603)	(—)
ρ_i	*Beta*	0.50	0.853	0.861	0.934	0.859
		(0.20)	(0.815, 0.891)	(0.828, 0.895)	(0.904, 0.960)	(0.825, 0.893)
ρ_d	*Beta*	0.50	0.684	0.709	0.752	0.646
		(0.20)	(0.591, 0.783)	(0.610, 0.806)	(0.628, 0.874)	(0.546, 0.753)
ρ_s	*Beta*	0.50	0.830	0.069	0.070	0.068
		(0.20)	(0.770, 0.895)	(0.010, 0.124)	(0.041, 0.336)	(0.010, 0.125)
ρ_{e_i}	*Beta*	0.50	0.206	0.167	0.571	0.164
		(0.20)	(0.099, 0.313)	(0.066, 0.261)	(0.422, 0.723)	(0.065, 0.256)
σ_d	*Inv. gamma*	0.10	0.461	0.524	0.294	0.394
		(∞)	(0.416, 0.505)	(0.414, 0.630)	(0.217, 0.365)	(0.349, 0.439)
σ_s	*Inv. gamma*	0.10	0.175	0.339	0.329	0.285
		(∞)	(0.158, 0.193)	(0.284, 0.390)	(0.277, 0.381)	(0.259, 0.310)
σ_i	*Inv. gamma*	0.10	0.212	0.207	0.081	0.207
		(∞)	(0.194, 0.229)	(0.190, 0.224)	(0.071, 0.091)	(0.190, 0.223)
	Log data density		−283.853	−352.369	−24.153	−354.076

Note: Results are reported at the posterior mean. 90% confidence intervals in parenthesis. The model-implied forecast-underrevision coefficients are 1.2362 (column 4), 1.4432 (column 5), 0.5061 (column 6) and 0.7428 (column 7). In columns 5 and 6, which directly estimate $\overline{m}$, we assume a prior Beta distribution with mean (S.d.) of 0.50 (0.15). In column 7, the forecast-underrevision coefficient refers to the one implied by output gap (a value of 0.7523 in the data following Coibion and Gorodnichenko 2015).

TABLE A1. Estimated Structural Parameters: Robustness Checks

A.8. Smets and Wouters (2007) with Bounded Rationality

A.8.1. Theory

Throughout this section, we outline the differences between the large scale DSGE model in Smets and Wouters (2007) and its BR extension. We assume that all structural shocks follow the same stochastic processes as in Smets and Wouters (2007). We follow their notation for ease of comparison. For a detailed description of all parameters (except for $\overline{m}$), we refer the reader to Smets and Wouters (2007).

The aggregate resource constraint, [1] in their paper, is identical since it does not involve BR expectations. Their consumption Euler equation [2] is changed to

$$c_t = \frac{h/\overline{\gamma}}{1+h/\overline{\gamma}}c_{t-1} + \frac{\overline{m}}{1+h/\overline{\gamma}}\mathbb{E}_t c_{t+1} - \frac{(\sigma_c-1)W_*^h L_*/C_*}{\sigma_c(1+h/\overline{\gamma})}(\overline{m}\mathbb{E}_t l_{t+1} - l_t) - \frac{1-h/\overline{\gamma}}{\sigma_c(1+h/\overline{\gamma})}(r_t - \overline{m}\mathbb{E}_t\widehat{\pi}_{t+1} + \varepsilon_t^b)$$

where $\overline{m}$ appears in front of the BR expectation operators. Their investment Euler equation [3] is changed to

$$i_t = \frac{1}{1+\beta\overline{\gamma}^{1-\sigma_c}}i_{t-1} + \frac{\beta\overline{\gamma}^{1-\sigma_c}\overline{m}}{1+\beta\overline{\gamma}^{1-\sigma_c}}\mathbb{E}_t i_{t+1} + \frac{1}{(1+\beta\overline{\gamma}^{1-\sigma})\overline{\gamma}^2\varphi}q_t + \varepsilon_t^i$$

where $\overline{m}$ appears in front of BR expected investment. Their arbitrage equation for the value of capital [4] is changed to

$$q_t = \beta\overline{\gamma}^{-\sigma_c}(1-\delta)\overline{m}\mathbb{E}_t q_{t+1} + [1-\beta\overline{\gamma}^{-\sigma_c}(1-\delta)]\overline{m}\mathbb{E}_t r_{t+1}^k - (r_t - \overline{m}\mathbb{E}_t\pi_{t+1} + \varepsilon_t^b)$$

where $\overline{m}$ appears in front of the BR expectation operators. The aggregate production function [5], capital laws of mation [6] and [8], capital utilization [7], and price mark-up [9] are identical since they do not involve BR expectations. The Smets and Wouters (2007) equivalent of our expression (16) (see pp. 18 in their Online Appendix) is given by

$$p_t^* = p_t + (1-\xi_p\beta\overline{\gamma}^{1-\sigma_c})\sum_{k=0}^{\infty}(\xi_p\beta\overline{\gamma}^{1-\sigma_c}\overline{m})^k\mathbb{E}_t\Big[(\pi_{t+1}+\cdots+\pi_{t+k}) - \omega(\pi_t+\cdots+\pi_{t+k-1}) + \frac{1}{(\phi_p-1)\varepsilon_p+1}\left(\widehat{\mathrm{mc}}_{t+k} + \widehat{\lambda}_{p,t+k}\right)\Big]$$

Price level dynamics in the Smets and Wouters (2007) follow the same dynamics as in (17). Following the steps in Appendix A.5, we can write the Smets and Wouters (2007) equivalent of our expression (18):

$$\pi_t = \pi_1 \pi_{t-1} + \pi_2 \mathbb{E}_t \pi_{t+1} - \pi_3 \mu_t^p + \pi_3 \widehat{\lambda}_{p,t}$$

which is the equivalent to condition [10] in their paper, where

$$\pi_1 = \frac{\iota_p}{1 + \iota_p \beta \overline{\gamma}^{1-\sigma_c} \overline{m} \left[\xi_p + (1 - \xi_p) \frac{1-\xi_p \beta \overline{\gamma}^{1-\sigma_c}}{1-\xi_p \beta \overline{\gamma}^{1-\sigma_c} \overline{m}} \right]}$$

$$\pi_2 = \frac{\beta \overline{\gamma}^{1-\sigma_c} \overline{m} \left[\xi_p + (1 - \xi_p) \frac{1-\xi_p \beta \overline{\gamma}^{1-\sigma_c}}{1-\xi_p \beta \overline{\gamma}^{1-\sigma_c} \overline{m}} \right]}{1 + \iota_p \beta \overline{\gamma}^{1-\sigma_c} \overline{m} \left[\xi_p + (1 - \xi_p) \frac{1-\xi_p \beta \overline{\gamma}^{1-\sigma_c}}{1-\xi_p \beta \overline{\gamma}^{1-\sigma_c} \overline{m}} \right]}$$

$$\pi_3 = \frac{(1 - \xi_p)(1 - \xi_p \beta \overline{\gamma}^{1-\sigma_c})}{\xi_p[(\phi_p - 1)\varepsilon_p + 1] \left\{ 1 + \beta \overline{\gamma}^{1-\sigma_c} \overline{m} \left[\xi_p + (1 - \xi_p) \frac{1-\xi_p \beta \overline{\gamma}^{1-\sigma_c}}{1-\xi_p \beta \overline{\gamma}^{1-\sigma_c} \overline{m}} \right] \right\}}$$

where in their paper $\varepsilon_t^p = \pi_3 \widehat{\lambda}_{p,t}$. Similarly, the wage Phillips curve, equivalent to their condition [13], is given by

$$w_t = w_1 w_{t-1} + (1 - w_1)(\mathbb{E}_t w_{t+1} + \mathbb{E}_t \pi_{t+1}) - w_2 \pi_t + w_3 \pi_{t-1} - w_4 \mu_t^w + \varepsilon_t^w$$

where

$$w_1 = \frac{1}{1 + \beta \overline{\gamma}^{1-\sigma_c} \overline{m} \left[\xi_w + (1 - \xi_w) \frac{1-\xi_w \beta \overline{\gamma}^{1-\sigma_c}}{1-\xi_w \beta \overline{\gamma}^{1-\sigma_c} \overline{m}} \right]}$$

$$w_2 = \frac{1 + \beta \overline{\gamma}^{1-\sigma_c} \iota_w}{1 + \beta \overline{\gamma}^{1-\sigma_c} \overline{m} \left[\xi_w + (1 - \xi_w) \frac{1-\xi_w \beta \overline{\gamma}^{1-\sigma_c}}{1-\xi_w \beta \overline{\gamma}^{1-\sigma_c} \overline{m}} \right]}$$

$$w_3 = \iota_w w_1$$

$$w_4 = \frac{(1 - \xi_w)(1 - \xi_w \beta \overline{\gamma}^{1-\sigma_c})}{\xi_w[(\phi_w - 1)\varepsilon_w + 1] \left\{ 1 + \beta \overline{\gamma}^{1-\sigma_c} \overline{m} \left[\xi_w + (1 - \xi_w) \frac{1-\xi_w \beta \overline{\gamma}^{1-\sigma_c}}{1-\xi_w \beta \overline{\gamma}^{1-\sigma_c} \overline{m}} \right] \right\}}$$

The relation between the rental rate of capital and the capital-capital labor ratio and the real wage rate is identical to [11] in Smets and Wouters (2007), and so is the wage mark-up [12] and the nominal interest rate rule [14], since they do not involve BR expectations.

A.8.2. Empirics

We estimate the medium-scale DSGE model in Smets and Wouters (2007), extended with BR in the previous section. The model equations are outlined in Appendix A.8.1, and the empirical findings are reported in tables A2-A3. We report the estimation of the model when $\overline{m} = 1$ because the original replication codes contain an error (see the discussion by Johannes Pfeifer here), using the same priors. We also remove the exogenous MA shocks, which simplifies the computation of the forecast underrevision coefficient.

In terms of the model parameters, we find that the BR model produces estimates that are closer to those found in the literature. We focus our discussion on the parameters that Smets and Wouters (2007) mention as not so close to the data or the literature. The authors estimate a trend growth rate of 0.43, whereas the average growth rate of output per capita over the sample is 0.502. In our estimation under $\overline{m} = 1$, the discrepancy persists ($\overline{\gamma} = 0.386$). However, once we estimate $\overline{m}$ to match the forecast underreaction coefficient in Coibion and Gorodnichenko (2015), $\overline{\gamma} = 0.508$. The implied duration of prices in Smets and Wouters (2007) is 8.82 months, in the upper range of the estimates in the micro-data (Nakamura and Steinsson 2008). Under BR, we estimate a price duration of 6.32 months, in the mid-range of the estimates reported in their table 1. Finally, Smets and Wouters (2007) estimate an excessive sensitivity of nominal interest rates to inflation (2.04, far from the estimates in Clarida et al. 2000). In our estimation under $\overline{m} = 1$, the inconsistency lingers: we find a Taylor rule coefficient of 1.833. Under BR, the estimated coefficient is lowered to 1.472, closer to the standard value assumed in the literature.

We find that adding more realistic elements improves the fit. In particular, the log data density of the rational model is –1624.463, whereas the log data density of the BR model is –1642.878. However, this model comparison exercise does not put any weight on expectation data: since we are only comparing the model performance with data on endogenous variables, we are not considering a penalty for not matching the evidence on the positive co-movement between ex-ante forecast errors and forecast revisions (Coibion and Gorodnichenko 2015). For instance, the estimation of the BR model yields $\overline{m} = 0.28$, which in turn implies an underrevision coefficient of 1.222. The baseline forecast-underrevision reported in Coibion and Gorodnichenko (2015) is 1.2306.

	Prior Distribution		Posterior Distribution	
		Mean	*1966:I to 2004:IV*	
		(S.d)	**SW**	**BSW**
φ	*Normal*	4	4.341	2.262
		(1.50)	(2.4451, 6.2669)	(2.0000, 2.6143)
σ_c	*Normal*	1.50	1.526	0.513
		(0.375)	(1.1334, 1.9435)	(0.3845, 0.6369)
h	*Beta*	0.70	0.537	0.743
		(0.10)	(0.3947, 0.6435)	(0.6602, 0.8279)
ξ_w	*Beta*	0.50	0.936	0.799
		(0.15)	(0.9200, 0.9500)	(0.7330, 0.8672)
σ_l	*Normal*	2	2.333	2.169
		(0.75)	(1.3518, 3.2770)	(1.1040, 3.2399)
ξ_p	*Beta*	0.50	0.538	0.525
		(0.10)	(0.5000, 0.5837)	(0.5000, 0.5559)
ι_w	*Beta*	0.50	0.661	0.211
		(0.15)	(0.5427, 0.7805)	(0.1068, 0.3151)
ι_p	*Beta*	0.50	0.093	0.107
		(0.15)	(0.0301, 0.1523)	(0.0324, 0.1737)
ψ	*Beta*	0.50	0.496	0.314
		(0.15)	(0.3246, 0.6663)	(0.1384, 0.4860)
Φ	*Normal*	1.25	1.684	1.664
		(0.125)	(1.5560, 1.8042)	(1.5247, 1.8035)
r_π	*Normal*	1.50	1.833	1.472
		(0.25)	(1.5350, 2.1353)	(1.1088, 1.8093)
ρ	*Beta*	0.75	0.913	0.937
		(0.10)	(0.8882, 0.9373)	(0.9118, 0.9634)
r_y	*Normal*	0.125	0.154	0.103
		(0.05)	(0.0955, 0.2143)	(0.0339, 0.1705)
$r_{\Delta y}$	*Normal*	0.125	0.172	0.096
		(0.05)	(0.1344, 0.2111)	(0.0675, 0.1236)
$\overline{\pi}$	*Gamma*	0.625	0.932	0.735
		(0.10)	(0.7664, 1.1032)	(0.6218, 0.8484)
$100(\beta^{-1} - 1)$	*Gamma*	0.25	0.216	0.254
		(0.10)	(0.0756, 0.2741)	(0.0979, 0.4011)
$\overline{l}$	*Normal*	0	0.298	–1.061
		(2)	(-1.4166, 2.1641)	(-3.0317, 0.9002)
$\overline{\gamma}$	*Normal*	0.40	0.216	0.508
		(0.10)	(0.3349, 0.4468)	(0.4838, 0.5311)
α	*Normal*	0.30	0.172	0.139
		(0.05)	(0.1400, 0.2029)	(0.1083, 0.1701)
$\overline{m}$	*Implied*	—	1	0.28
		—	(—)	(—)
	Log data density		–1624.463	–1642.878

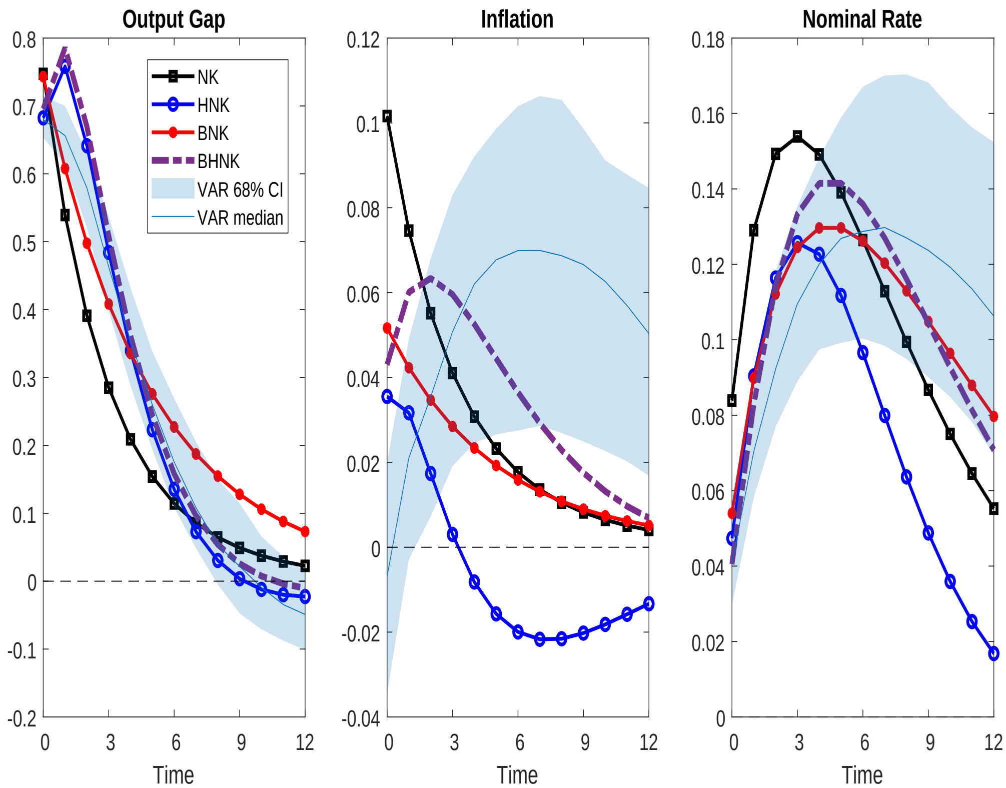
Note: Results are reported at the posterior mean. 90% confidence intervals in parenthesis. The model-implied forecast-underrevision coefficients are 0 (SW) and 1.2220 (BSW). The baseline forecast-underrevision reported in Coibion and Gorodnichenko (2015) is 1.2306.

TABLE A2. Estimated Structural Parameters: SW Models

	Prior Distribution		Posterior Distribution	
		Mean	*1966:I to 2004:IV*	
		(S.d)	**SW**	**BSW**
σ_a	*Inv. gamma*	0.10	0.519	0.521
		(2)	(0.4738, 0.5666)	(0.4754, 0.5655)
σ_b	*Inv. gamma*	0.10	0.146	0.816
		(2)	(0.0946, 0.1789)	(0.7251, 0.9015)
σ_g	*Inv. gamma*	0.10	0.742	0.743
		(2)	(0.6857, 0.7984)	(0.6833, 0.7990)
σ_I	*Inv. gamma*	0.10	0.453	1.360
		(2)	(0.3674, 0.5279)	(1.3604, 1.5980)
σ_r	*Inv. gamma*	0.10	0.228	0.208
		(2)	(0.2065 0.2479)	(0.1913, 0.2243)
σ_p	*Inv. gamma*	0.10	0.137	0.371
		(2)	(0.0909, 0.1829)	(0.3405, 0.4007)
σ_w	*Inv. gamma*	0.10	0.265	0.516
		(2)	(0.2321, 0.2996)	(0.4714, 0.5602)
ρ_a	*Beta*	0.50	0.994	0.972
		(0.20)	(0.9868, 0.9995)	(0.9544, 0.9904)
ρ_b	*Beta*	0.50	0.794	0.927
		(0.20)	(0.7314, 0.9164)	(0.8853, 0.9728)
ρ_g	*Beta*	0.50	0.993	0.975
		(0.20)	(0.9876, 0.9995)	(0.9571, 0.9930)
ρ_I	*Beta*	0.50	0.822	0.917
		(0.20)	(0.7258, 0.9255)	(0.8743, 0.9624)
ρ_r	*Beta*	0.50	0.143	0.161
		(0.20)	(0.0510, 0.2271)	(0.0656, 0.2549)
ρ_p	*Beta*	0.50	0.721	0.706
		(0.20)	(0.5911, 0.8425)	(0.6187, 0.8047)
ρ_w	*Beta*	0.50	0.176	0.465
		(0.20)	(0.0748, 0.2733)	(0.3522, 0.5780)

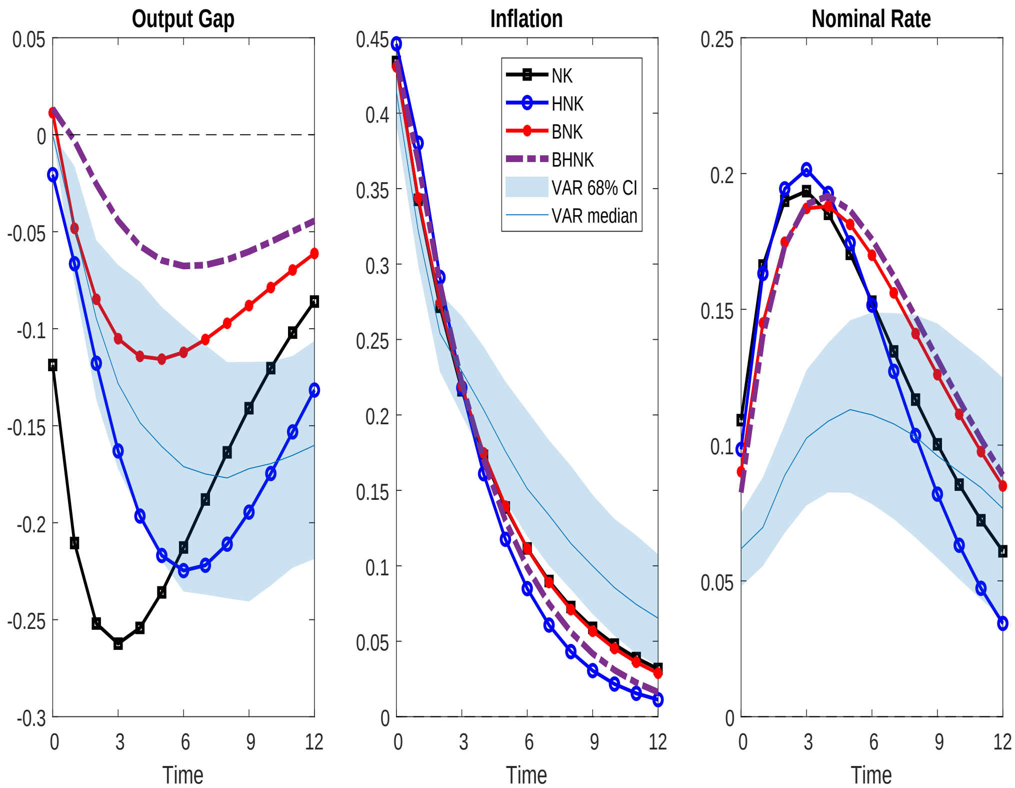
TABLE A3. Estimated Structural Parameters: SW Models

A.9. Impulse Response Functions to Demand and Supply Shocks



Note: The dynamic paths for the variables are reported under different model specifications after an aggregate demand shock: (i) a standard NK model in black lines (squares), (ii) a hybrid NK model in blue lines (circles), (iii) a behavioral NK model in red lines (asterisks), and (iv) a behavioral hybrid NK model in purple lines (dashed). The VAR-based demand shock is identified by means of a recursive identification, where the order of the variables is as follows: output gap, inflation and nominal interest rate. The horizontal axis displays the time which is measured in quarters. Vertical axis values refer to deviations from steady state in percentage.

FIGURE A1. Dynamic Responses to an Aggregate Demand Shock



Note: The dynamic paths for the variables are reported under different model specifications after an aggregate supply shock: (i) a standard NK model in black lines (squares), (ii) a hybrid NK model in blue lines (circles), (iii) a behavioral NK model in red lines (asterisks), and (iv) a behavioral hybrid NK model in purple lines (dashed). The VAR-based supply shock is identified by means of a recursive identification, where the order of the variables is as follows: output gap, inflation and nominal interest rate. The horizontal axis displays the time which is measured in quarters. Vertical axis values refer to deviations from steady state in percentage.

FIGURE A2. Dynamic Responses to an Aggregate Supply Shock