The Transmission of Foreign Shocks in a Networked Economy

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We analyze how production networks transmit foreign price shocks and reshape monetary policy trade-offs in an open-economy New Keynesian model with domestic and international input–output linkages. Analytically, we show that closing the output gap does not generally stabilize domestic inflation, as sector-level terms-of-trade movements and trade imbalances become additional drivers of inflation dynamics. Quantitatively, we study an international energy price shock in a model calibrated to major euro area countries and their trade partners. We find that production networks significantly amplify the cumulative headline inflation response and substantially worsen monetary policy trade-offs, as measured by the sacrifice ratio.

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1. Introduction

In recent years, the global economy has experienced supply-side shocks that have significantly affected inflation dynamics and macroeconomic stability. Prominent examples include energy price shocks, often triggered by geopolitical events, and increased production costs from supply-chain disruptions. Despite differing causes, these shocks share a key characteristic: they originate in specific sectors but quickly propagate through complex production networks and international supply chains, ultimately affecting the entire economy. Consequently, understanding how these shocks are transmitted through input-output (IO) linkages and spill over across countries and sectors has become a central focus of macroeconomic research.

In this context, this paper examines how production networks shape the transmission of foreign price shocks and the associated implications for monetary policy. Our first contribution is to derive analytical results in a New Keynesian small open-economy model with domestic and international IO linkages, showing how outcomes differ relative to one-sector open economies (Gali and Monacelli 2005; Corsetti *et al.* 2010) and to closed economies with production networks (La'O and Tahbaz-Salehi 2022; Rubbo 2023). We find that (*i*) production networks introduce a novel monetary policy trade-off arising from inefficient movements in sectoral terms-of-trade (ToT), which feed into domestic inflation both directly through marginal costs and indirectly through sectoral trade imbalances; and (*ii*) in contrast to the one-sector counterpart, international production networks steepen the Phillips curve relative to a closed economy. Our second contribution is to assess the quantitative importance of production networks in a larger model calibrated to the main euro area (EA) countries and their trading partners. Focusing on shocks to imported energy prices, we find first that production networks nearly double the cumulative response of inflation over time, with roughly half of this amplification accounted for by international production networks alone. Second, IO linkages worsen the trade-off faced by monetary policy, implying a larger sacrifice ratio.

More in detail, we first consider an analytically tractable small open-economy model with production networks in which fluctuations originate from shocks to the prices of imported goods.[1] Under logarithmic utility and a unit elasticity of substitution across goods—assumptions that we relax in the quantitative analysis—we derive the vector of domestic New Keynesian Phillips curves in terms of deviations from efficient allocations. We use these results to analyze how production networks shape shock transmission and the implications for monetary policy.

We show that production networks play a pivotal role in shaping transmission and generate monetary policy trade-offs absent in one-sector models.

[1]These shocks capture, for example, increases in the international price of energy or higher importing costs due to supply-side bottlenecks, which are exogenous from the perspective of a small open economy.

First, we document that international IO linkages steepen the slope of the Phillips curve relative to a closed economy. In a one-sector economy with unit elasticities of substitution and log utility, the degree of openness has no implications for the elasticity of domestic inflation with respect to movements in the output gap. By contrast, with IO linkages and multiple sectors, movements in the output gap induce shifts in sector-level net exports. As a result, part of the increase in domestic activity leaks abroad, requiring a larger increase in domestic real wages to sustain a given increase in employment and output. Consequently, marginal costs and domestic inflation rise further, increasing the elasticity of domestic prices with respect to output-gap movements.

Second, we show that inefficient movements in sectoral ToT propagate to domestic inflation over and above movements in the output gap. Consequently, and in contrast to the one-sector benchmark, domestic inflation can deviate from target even when the output gap is closed.[2] This occurs through the channels described below, which are naturally absent in closed economies with production networks.

On the one hand, movements in the ToT directly affect sectoral prices depending on a sector's exposure to the foreign economy. The resulting increase in the domestic price of imported production inputs further raises marginal costs. This occurs both directly, through a sector's use of intermediate goods, and indirectly through the production network, as the costs of the sector's suppliers also increase. That is, this channel would be present in a multi-sector economy with heterogeneous sectors even in the absence of cross-sector input–output linkages, but it is further reinforced by production-network linkages.

On the other hand, inefficient movements in ToT generate sectoral trade imbalances that transmit to domestic inflation through labor supply and real-wage adjustments. Two countervailing forces are at play. First, a deterioration in the ToT induces an expenditure-switching effect toward domestic goods, raising net exports. Second, offsetting this effect, the resulting increase in domestic demand feeds back into sectoral employment and increases the demand for imported goods. We show that, absent cross-sector production linkages, these forces exactly cancel, implying no effect on inflation. With production networks, however, this result breaks down because the sector experiencing the ToT deterioration also benefits from higher demand from other domestic sectors, generating sector-level movements in net exports that affect marginal costs through labor supply and wages. Hence, this channel fully relies on cross-sector relationships within the production network; absent these relationships, it would be absent.

A final component of the sectoral Phillips curve is the gap between lagged domestic prices

[2] In one-sector economies, ToT movements may also generate monetary-policy trade-offs in the presence of, for example, incomplete financial markets (Corsetti *et al.* 2010). Instead, in our framework, these trade-offs arise from the multi-sector structure with IO linkages.

and their efficient level, which captures the direct effect of international price shocks on inflation. This term is analogous to that derived in Rubbo (2023) for a closed economy with TFP shocks, which we extend to an open-economy setting with international price shocks. It incorporates the direct effect of the international price shock on domestic prices, mediated by sector-level exposure to foreign industries.

Lastly, we document that the international dimension of the economy further amplifies the persistence that production networks impart to shock propagation in closed-economy settings (Huang and Liu 2004; Pasten *et al.* 2020; Ghassibe 2021). Specifically, sectoral ToT introduce an additional source of price inertia and persistence, as producers inherit the price stickiness of their suppliers through marginal costs

In the second part of the paper, we consider a quantitative version of the model that allows for a more general parameterization of elasticities of substitution and country size.[3] We calibrate the model to 44 sectors per country and 6 regions: the four largest EA countries (Germany, France, Italy, and Spain), the rest of the EA, and the rest of the world. The model replicates observed trade flows across sectors and countries using IO tables from the OECD and Eurostat. We also use micro-level CPI data from Gautier *et al.* (2024) to calibrate heterogeneous sector- and country-specific price-adjustment probabilities, allowing the model to match the variation in price rigidity observed in the data.

Motivated by the recent energy price crisis, we study the transmission of an exogenous increase in the international price of energy and focus on the responses of EA variables.

First, we find that the presence of IO linkages matters quantitatively for the transmission of an increase in imported energy prices. We quantify and isolate the role that production networks play through a series of counterfactuals where we sequentially turn off domestic, international, and national and international production networks. We find that without national and international production networks, cumulative headline inflation would be roughly up to 60% of our baseline, which includes a fully fledged production network structure. In particular, we find that despite headline inflation rising similarly on impact—driven by the rise of energy prices—it stabilizes and dies out much faster when production networks are absent, in line with the intuition provided above.

Second, we then quantify how production networks shape monetary policy trade-offs, as measured by the sacrifice ratio. This measures the required change in the output gap required to bring inflation one percentage point down. We find that under a one-sector economy, the sacrifice ratio would be 20% smaller than in our fully-fledged economy with production networks. Therefore, ignoring this dimension would underestimate the cost of strict inflation in an open economy in the face of foreign shocks.

[3] In addition, we incorporate other quantitatively relevant dimensions, including incomplete financial markets, nominal wage rigidities, and domestic-currency pricing (Devereux and Engel 2003).

Related literature. Our paper contributes to several strands of the literature, at the intersection of the macroeconomic effects of production networks, the propagation of international macroeconomic shocks, and their monetary policy implications.

The seminal works of Acemoglu *et al.* (2012) and Baqaee and Farhi (2019) study the propagation of granular shocks in production networks under flexible prices, abstracting from their inflationary effects. Building on these contributions, Pasten *et al.* (2020), Ghassibe (2021), La'O and Tahbaz-Salehi (2022), Rubbo (2023), or Höynck (2025), incorporate input-output linkages into frameworks with nominal rigidities.[4] Our work incorporates several of the insights considered in this previous literature, and it extends them by considering how the international dimension of production networks additionally affects shock transmission and monetary policy trade-offs.

Our paper also contributes to the growing literature that incorporates input-output linkages into open-economy models. Baqaee and Farhi (2024) study shock propagation in an open-economy model with production networks, but with a limited role for nominal price rigidities and monetary policy. Comin *et al.* (2023) develop a more tractable small open-economy model with nominal rigidities, but focus on potentially binding capacity constraints, and Andrade *et al.* (2023) develop a three-sector small open-economy model à la Gali and Monacelli (2005) to study the propagation of productivity shocks. More recently, Kalemli-Özcan *et al.* (2025) study the propagation of tariffs in an open-economy framework with production networks, and Qiu *et al.* (2025) study optimal monetary policy in a static small open economy under the assumption of balanced trade. Relative to these papers, our work provides an analytical characterization of the transmission of more general foreign price shocks in a dynamic setting and their implications for monetary policy, while on the quantitative side focusing on international energy price shocks.

More generally, our work contributes to the ample literature studying monetary policy in open economies. In addition to the papers mentioned earlier, most of this work has focused on optimal monetary policy in one-sector models, like for example Corsetti and Pesenti (2005), Gali and Monacelli (2005), De Paoli (2009), Corsetti *et al.* (2010), and Egorov and Mukhin (2023). Our paper complements this earlier research by showing that accounting for input-output linkages in open-economy models can significantly affect the conduct of monetary policy.

Finally, we also contribute to the literature exploring the transmission of energy shocks. An earlier contribution is Bodenstein *et al.* (2008), which studied optimal monetary policy in a closed-economy model with an energy sector. Gagliardone and Gertler (2023) explore the origins of the inflation surge in the US using a closed-economy New Keynesian framework with oil. Auclert *et al.* (2023), Chan *et al.* (2024), and Bayer *et al.* (2023) explore the consequences

[4] For earlier work incorporating intermediate goods in frameworks with nominal rigidities see, for example, Basu (1995), Huang and Liu (2004), and Nakamura and Steinsson (2010).

of the recent energy crisis in open economy models with household heterogeneity.

Roadmap. The paper proceeds as follows. Section 2 presents the international input-output New Keynesian framework. In Section 3, we analytically inspect the transmission mechanism of foreign price shocks and the monetary policy implications. Later, in Section 4, we consider a quantitative version of the model and assess the macroeconomic consequences of shocks to the international price of energy in a networked economy, and quantify the implications for monetary policy. A final section concludes.

2. General Model

We consider a world economy composed of K countries, indexed by k. The core of our model is a production structure characterized by national and international production networks through IO linkages. Namely, each country is comprised of I production sectors, possibly heterogeneous within and between countries. Within each sector, there is a unit mass of monopolistically competitive firms indexed by $f \in (0,1)$ that produce using labor and intermediate goods produced by other domestic and foreign sectors. In addition, we allow for heterogeneous nominal price rigidities at the sectoral and country levels.

2.1. Households

There is a representative household in each country k that derives utility from consumption and disutility from labor according to the following per-period utility function:

$$U_t = U(C_{k,t}) - N_{k,t}^{1+\varphi}/(1+\varphi), \tag{1}$$

where $N_{k,t}$ is aggregate labor supplied by the household, $U(C_{k,t})$ is a constant-relative-risk-aversion utility function over aggregate consumption, $C_{k,t}$, and φ denotes the inverse of the Frisch elasticity. We allow for nominal wage rigidities à la Erceg *et al.* (2000), which if present give rise to the New Keynesian Wage Phillips Curve derived in Appendix B.4.

Aggregate consumption is defined as a constant-returns-to-scale (CRS) composite of sectoral consumption, each of which is itself a CRS aggregation of country-specific consumption goods. This nested structure is summarized by

$$C_{k,t} = \mathcal{C}_k\left(\{C_{ki,t}\}_{i=1}^{I}\right) \quad \text{and} \quad C_{ki,t} = \mathcal{C}_{ki}\left(\{C_{kl\,i,t}\}_{l=1}^{K}\right), \tag{2}$$

where $C_{ki,t}$ is consumption of sector i at time t, and $C_{kl\,i,t}$ is country's k household's consumption of good produced by industry i in country l. Finally, we let $C_{kl\,i,t}$ be a Dixit and Stiglitz

(1977) aggregator over differentiated goods produced by firms in sector i in country l:

(3) $$C_{kli,t} = \left(\int_0^1 C_{klif,t}^{(\epsilon_{ki}^p - 1)/\epsilon_{ki}^p} df \right)^{\epsilon_{ki}^p/(\epsilon_{ki}^p - 1)}$$

where ϵ_{ki}^p denotes the sectoral constant elasticity of substitution between good varieties.

The household faces the following per-period budget constraint:

(4) $$P_{k,t}^C C_{k,t} + \frac{B_{k,t}}{1+i_t} + \mathcal{E}_{k,t}^K \sum_{h \in \mathcal{H}} Q_t^h B_{k,t}^h \leq W_{k,t} N_{k,t} + \mathcal{E}_{k,t}^K \sum_{h \in \mathcal{H}} (Q_t^h + D_t^h) B_{k,t-1}^h + B_{k,t-1} + \Pi_{k,t} - T_{k,t}$$

where $W_{k,t}$ is the nominal wage received by the household, $\Pi_{k,t} = \sum_{i=1}^I \Pi_{ki,t}$ are the profits of domestic firms, and $T_{k,t}$ is a lump-sum tax or transfer. $P_{k,t}^C$ denotes the consumer price index in country k, implied by the consumption aggregators (2).

There are two sets of assets in the economy. The first of them, $B_{k,t}$, is a one-period nominal bond only traded domestically that pays a gross nominal interest rate $1+i_t$, set by the monetary authority. The second one is a set $\mathcal{H}$ of bonds traded internationally at price Q_t^h, with dividend payouts D_t^h.[5] These bonds are denominated in the currency of country K, and hence $\mathcal{E}_{k,t}^K$ is the bilateral nominal exchange rate between country k and country K.

Under the previous notation, we have the following allocation of consumption across sectors, countries, and differentiated goods:

(5) $$\frac{\partial \mathcal{C}_{ki,t}}{\partial C_{ki,t}} = \frac{P_{ki,t}^C}{P_{k,t}^C}, \quad \frac{\partial \mathcal{C}_{ki}}{\partial C_{kli,t}} = \frac{(1+\tau_{kli,t}) P_{kli,t}}{P_{ki,t}^C}, \quad \text{and} \quad C_{klif,t} = \left(\frac{P_{klif,t}}{P_{kli,t}} \right)^{-\epsilon_{ki}^p} C_{kli,t},$$

where $P_{ki,t}^C$ denotes the consumer price index of sector i faced by the household in country k, and $P_{kli,t} \equiv \left(\int_0^1 P_{klif,t}^{(\epsilon_{ki}^p - 1)/\epsilon_{ki}^p} df \right)^{\epsilon_{ki}^p/(\epsilon_{ki}^p - 1)}$ is the price in the currency of country k of the good produced by sector i in country l.[6]

The term $\tau_{kli,t}$ is an exogenous *price wedge* between the price set by industry i in country l exporting to country k, $P_{kli,t}$, and the gross price paid by domestic agents, $(1+\tau_{kli,t})P_{kli,t}$.[7]

[5] This specification follows Egorov and Mukhin (2023). Note if $\mathcal{S}$ is the set of possible states of nature, we have that financial markets are complete if $\mathcal{H} = \mathcal{S}$. On the other hand, by setting $\mathcal{H} = 1$ we have the usual specification of incomplete financial markets with a single bond traded internationally.

[6] Under the CRS assumptions, we have that $P_{k,t}^C C_{k,t} = \sum_{i=1}^I P_{ki,t}^C C_{ki,t}$ and $P_{ki,t}^C C_{ki,t} = \sum_{l=1}^K (1+\tau_{kli,t}) P_{kli,t} C_{kli,t}$.

[7] These theoretical price wedges are similar to import tariffs, since both elements distort international good prices faced by domestic agents. However, a key difference is that while the domestic government typically collects tariff revenues, and hence do not suppose a direct wealth transfer across countries, we assume that

Our motivation for these price wedges is to have a source of exogenous movements in import prices that are not necessarily triggered by changes in the economic activity of the exporting country. These are reminiscent of, for example, energy price shocks arising as a consequence of geopolitical tensions, or even supply-chain disruptions as the ones witnessed after the COVID-19 pandemic. More generally, one can think of these wedges as part of the transmission mechanism of any foreign shock that leads to changes in international goods' prices.

Finally, the first-order condition for households' savings in domestic bonds results in the following Euler equation, $U'(C_{k,t}) = \beta \mathbb{E}_t \left[U'(C_{k,t+1})(1+i_t)/(1+\pi^C_{k,t+1}) \right]$, where $1+\pi^C_{k,t+1} \equiv P^C_{k,t+1}/P^C_{k,t}$ is the gross consumer price inflation in country k, and β denotes the discount factor.

2.2. Firms

There are I industries in each economy, indexed by $i \in \{1, 2, ..., I\}$, and within each industry there is a unit mass of firms, indexed by $f \in (0, 1)$.

Production. Each firm f, in sector i and country k, produces a differentiated good $Y_{kif,t}$ using a CRS production function F_{ki} using labor $N_{kif,t}$ and a basket of intermediate goods $X_{kif,t}$ as inputs:

$$Y_{kif,t} = F_{ki}\left(N_{kif,t}, X_{kif,t}\right). \tag{6}$$

The bundle of intermediate goods $X_{kif,t}$ is defined similar to the household's consumption basket, given by a CRS aggregator of sectoral intermediate goods, which are themselves defined as a CRS aggregator of country-specific intermediate goods:

$$X_{kif,t} = \mathcal{X}_{ki}\left(\left\{X_{kijf,t}\right\}_{j=1}^{I}\right) \quad \text{and} \quad X_{kijf,t} = \mathcal{X}_{kij}\left(\left\{X_{kl\,ijf,t}\right\}_{l=1}^{K}\right), \tag{7}$$

where $X_{kijf,t}$ is firm's f demand in sector i of country k for goods produced in sector j, and $X_{kl\,ijf,t}$ is firm's f demand in sector i of country k for goods produced in sector j in country l. $X_{kl\,ijf,t}$ is itself a Dixit-Stiglitz aggregator over differentiated goods produced by firms in sector j in country l: $X_{kl\,ijf,t} = \left[\int_0^1 X_{kl\,ijff',t}^{(\epsilon^p_{kj}-1)/\epsilon^p_{kj}} df'\right]^{\epsilon^p_{ki}/(\epsilon^p_{ki}-1)}$.

wedge payments accrue to the exporting producer firms: for a shipment from l to k, the exporter receives the gross-of-wedge revenue $(1+\tau_{kl\,i,t})P_{kl\,i,t}$ per unit. Hence, τ generates a direct international transfer through firm profits. In this case the price wedges generate a direct wealth loss for the importing country and a wealth gain for the exporting country (see equations 13 and 17).

Cost minimization by firms delivers the following first-order conditions for labor and intermediate goods demands, $W_{k,t} = \mathrm{MC}_{ki,t}\partial F_{ki}/\partial N_{kif,t}$ and $P^X_{kij,t} = \mathrm{MC}_{ki,t}\partial F_{ki}/\partial X_{kijf,t}$, and the allocation of intermediate goods demand across sectors and countries:

$$\frac{\partial \mathcal{X}_{ki}}{\partial X_{kij,t}} = \frac{P^X_{kij,t}}{P^X_{ki,t}}, \quad \frac{\partial \mathcal{X}_{kij}}{\partial X_{kl\,ij,t}} = \frac{(1+\tau_{kl\,j,t})P_{kl\,j,t}}{P^X_{kij,t}}, \quad X_{kl\,ijf,t} = \left(\frac{P_{kljf,t}}{P_{kl\,j,t}}\right)^{-\epsilon^p_{kj}} X_{kij,t} \tag{8}$$

Above, $P^X_{ki,t}$ denotes the price index of the intermediate input bundle $X_{kif,t}$ faced by firms in sector i in country k, and $P^X_{kij,t}$ is the price index of the sectoral intermediate input $X_{kijf,t}$ faced by firms in sector i in country k. As in the case of households, the prices faced by domestic industries are subject to price wedges $\tau_{kl\,j,t}$.

Nominal marginal costs in sector i of country k are denoted by $\mathrm{MC}_{ki,t}$. Under CRS, all firms in a given sector choose the same combination of inputs, and $\mathrm{MC}_{ki,t}$ is common across firms and given by:

$$\mathrm{MC}_{ki,t} = \min_{N_{ki,t}, X_{kl\,ij,t}} W_{ki,t}N_{ki,t} + \sum_{j=1}^{I}\sum_{l=1}^{K}(1+\tau_{kl\,j,t})P_{kl\,j,t}X_{kl\,ij,t} \tag{9}$$

Price Setting. Firms set prices in a staggered manner (Calvo 1983). Specifically, firms in sector i of country k can reset their price with probability $1-\theta^p_{ki}$ each period. We allow for a country– and sector–specific probability of price adjustment.

We entertain two possibilities for the pricing decisions to foreign markets: producer currency pricing (PCP) or local currency pricing (LCP). Under PCP, the firm sets its export price in the domestic currency, and hence the selling price to the domestic and foreign markets coincide. A firm that is resetting its price chooses its optimal selling price to the domestic market, $\overline{P}_{ki,t}$, by solving the following problem:

$$\max_{\overline{P}_{ki,t}} \mathbb{E}_t \sum_{s=0}^{\infty} \mathrm{SDF}_{t,t+s}(\theta^p_{ki})^s \left[\sum_{l=1}^{K}\Big((1+\tau_{l\,ki,t+s})\overline{P}_{ki,t} - (1-\tau^p_{ki})MC_{ki,t+s}\Big)D_{l\,ki,t+s}\right], \tag{10}$$

where $\mathrm{SDF}_{t,t+s} = \beta^s U'(C_{k,t+s})/U'(C_{k,t})$ is the stochastic discount factor between periods t and $t+s$, and $\mathcal{D}_{l\,ki,t+s}$ denotes the demand for the firm's good from domestic agents given in equations (5) and (8). We further allow for having a country-sector specific production subsidy τ^p_{ki}.[8]

Under LCP, the firm sets its export price to country l in the currency of country l, de-

[8] Production subsidies will be useful to derive our analytical results in section 3, as they allow us to eliminate steady-state distortions arising from monopolistic competition (Galí 2015).

noted by $\overline{P}_{lki,t}$, solving the problem $\max_{\overline{P}_{lki,t}} \mathbb{E}_t \sum_{s=0}^{\infty} \text{SDF}_{t,t+s}(\theta^p_{ki})^s \left[\mathcal{E}^l_{k,t+s}(1+\tau_{lki,t+s})\overline{P}_{lki,t} - (1-\tau^p_{ki})\text{MC}_{ki,t+s}\right]\mathcal{D}_{lki,t+s}$, where $\mathcal{D}_{lki,t+s}$ is the demand from agents from country l for the firm's production.

Profits in sector i of country k are given by export- and domestic-market revenues net of costs, $\Pi_{ki,t} = \sum_{l=1}^{K}(1+\tau_{lki,t})P^m_{lki,t}D_{lki,t} - \text{MC}_{ki,t}Y_{ki,t}$, and aggregate profits rebated to households are $\Pi_{k,t} = \sum_{i=1}^{I}\Pi_{ki,t}$.

2.3. Government

The government is composed of a fiscal authority and a central bank. The fiscal authority issues domestic debt $B_{k,t}$, provides production subsidies to firms, and collects lump-sum taxes $T_{k,t}$ from the household in order to balance its budget constraint, $\frac{B_{k,t}}{1+i_{k,t}} + T_{k,t} = B_{k,t-1} + \sum_{i=1}^{I}\tau_{ki}\text{MC}_{ki,t}Y_{ki,t}$, where $Y_{ki,t}$ denotes the production of sector i in country k.

The central bank sets the nominal interest rate, $i_{k,t}$. The specific monetary policy rule is inconsequential for our analytical results in Section 3 as long as it ensures determinacy; we therefore defer its definition to Section 4.

2.4. Market Clearing and GDP

Market clearing in the goods market requires that the quantity produced of each good matches the quantity demanded at home and abroad, either for direct consumption or intermediate use. That is,

$$Y_{ki,t} = \sum_{l=1}^{K} C_{lki,t} + \sum_{l=1}^{K}\sum_{j=1}^{I} X_{lkji,t}. \tag{11}$$

Market clearing in the labor market requires that the aggregate labor supplied matches the sum of labor demand across sectors, for each country. That is,

$$N_{k,t} = \sum_{i=1}^{I} N_{ki,t}. \tag{12}$$

Finally, the aggregate resource constraint of the economy requires that the net foreign position of country k equals its trade balance:

$$\mathcal{E}^K_{k,t}\sum_{h\in\mathcal{H}} Q^h_t B^h_{k,t} - \mathcal{E}^K_{k,t}\sum_{h\in\mathcal{H}}(Q^h_t + D^h_t)B^h_{k,t-1} = P^{\text{EXP}}_{k,t}\text{EXP}_{k,t} - P^{\text{IMP}}_{k,t}\text{IMP}_{k,t}, \tag{13}$$

where the first term in the right-hand side of (13) is the total nominal exports of country k,

$$P^{\text{EXP}}_{k,t}\text{EXP}_{k,t} = \sum_{l\neq k}^{K}\sum_{i=1}^{I}\left[(1+\tau_{l\,ki,t})P^{k}_{l\,ki,t}C_{l\,ki,t} + \sum_{j=1}^{I}(1+\tau_{l\,ki,t})P^{k}_{l\,ki,t}X_{l\,kji,t}\right], \tag{14}$$

where $P^{m}_{l\,ki,t}$ denotes the price of a good from sector i, originating in country k, sold in country l, and invoiced in the currency of country m.

Under PCP, goods are invoiced in currency k,

$$P^{m}_{l\,ki,t} = \mathcal{E}^{l}_{m,t}P_{l\,i,t}, \tag{15}$$

and under LCP, where $P_{l\,ki,t}$ is invoiced in currency l,

$$P^{m}_{l\,ki,t} = \mathcal{E}^{l}_{m,t}P_{l\,ki,t}. \tag{16}$$

The second term in (13) is the total nominal imports,[9]

$$P^{\text{IMP}}_{k,t}\text{IMP}_{k,t} = \sum_{l\neq k}^{K}\sum_{i\in I}\left((1+\tau_{kl\,i,t})P^{k}_{kl\,i,t}C_{kl\,i,t} + \sum_{j=1}^{I}(1+\tau_{kl\,i,t})P^{k}_{kl\,i,t}X_{kl\,ji,t}\right). \tag{17}$$

Nominal GDP is defined as the sum of total household consumption and nominal net exports,

$$\mathcal{Y}_{k,t} = P^{C}_{k,t}C_{k,t} + P^{\text{EXP}}_{k,t}\text{EXP}_{k,t} - P^{\text{IMP}}_{k,t}\text{IMP}_{k,t}. \tag{18}$$

Defining the GDP deflator, $P_{kY,t}$, as the ratio between nominal GDP measured using time–t prices and nominal GDP measured using steady-state prices

$$P^{Y}_{k,t} = \frac{P^{C}_{k,t}C_{k,t} + P^{\text{EXP}}_{k,t}\text{EXP}_{k,t} - P^{\text{IMP}}_{k,t}\text{IMP}_{k,t}}{P^{C}_{k}C_{k,t} + P^{\text{EXP}}_{k}\text{EXP}_{k,t} - P^{\text{IMP}}_{k}\text{IMP}_{k,t}},$$

we have that real GDP is given by:[10]

$$Y_{k,t} = \mathcal{Y}_{k,t}/P^{Y}_{k,t}. \tag{19}$$

[9] Because wedge payments accrue to exporting producer firms, valuing trade flows at the gross price $(1+\tau)P$ in (14) and (17) matches micro-level receipts and payments, and the resulting movements in net foreign assets correspond to actual international transfers.

[10] Aggregate gross value added and real GDP coincide, up to first order, in the absence of time-varying taxes.

3. Analytical Insights

To derive the intuition behind the transmission of foreign shocks in an open economy with production networks, and the associated implications for monetary policy, we consider a simplified version of the model outlined in Section 2. The following assumptions not only render the model analytically tractable but also allow us to nest familiar cases in the literature in a transparent way, providing useful benchmarks to highlight the novel mechanisms presented in this paper.

Assumptions. First, we assume that there are only two countries $K = 2$, with the home country being small relative to the foreign country. To ease notation, we will denote foreign variables with an asterisk, *. Under the small open economy assumption, aggregate variables in the foreign economy are held constant at their steady-state values. Furthermore, we focus on the case where the preferences and the technology are symmetric across countries.

Second, we make the following functional assumptions about preferences, wage rigidities, and technology. Regarding preferences, we assume that utility (1) is logarithmic in consumption:

$$U(C_t) = \log C_t. \tag{20}$$

We further assume that wages are fully flexible, so that the labor supply condition is given by

$$N_{k,t}^{\varphi} = U'(C_{k,t}) W_{k,t} / P_{k,t}^{C}. \tag{21}$$

In addition, we assume that the functional form of all consumption (2) and intermediate (7) goods aggregators are Cobb and Douglas (1928): $\mathcal{C}(\{C_{i,t}\}_{i=1}^{I}) = \prod_{i=1}^{I} C_{i,t}^{\beta_i}$, $\mathcal{C}_i(\{C_{i,t}^{H}, C_{i,t}^{F}\}) = (C_{i,t}^{H})^{1-\zeta_i}(C_{i,t}^{F})^{\zeta_i}$,

$$\mathcal{X}_i(\{X_{ij,t}\}_{j=1}^{I}) = \prod_{j=1}^{I} X_{ij,t}^{\nu_j}, \quad \text{and} \quad \mathcal{X}_{ij}(\{X_{ij,t}^{H}, X_{ij,t}^{F}\}) = (X_{ij,t}^{H})^{1-\zeta_{ij}}(X_{ij,t}^{F})^{\zeta_{ij}}, \tag{22}$$

and that the elasticity of substitution between labor and intermediate goods in the production function (6) is unitary:

$$Y_{if,t} = N_{if,t}^{\alpha_i} X_{if,t}^{1-\alpha_i}. \tag{23}$$

Third, we assume that international financial markets are complete. Together with log utility (20), this implies the following risk-sharing condition:

$$C_t = Q_t C_t^*, \tag{24}$$

where Q_t is the real exchange rate defined by $Q_t = \mathcal{E}_t P_t^* / P_t^C$.

Finally, regarding pricing, we make the following assumptions. First, we assume PCP, so that the law of one price holds and (15) becomes

$$P_{i,t}^F = \mathcal{E}_t P_{i,t}^*, \tag{25}$$

where $P_{i,t}^F$ is the price of sectoral good i produced in the Foreign, denominated in domestic currency, and $P_{i,t}^*$ is the price of the same good denominated in the foreign currency. Second, we also assume that a set of production sectoral subsidies $\tau_i^p = 1/\epsilon_i$ is in place.[11]

3.1. Log-linearized Equilibrium Conditions

Under the previous set of assumptions, we derive a log-linear approximation of the model around a symmetric steady state with zero inflation.[12] In what follows, we focus on the set of equilibrium conditions of the domestic economy that underpin our discussion of the transmission of foreign shocks and the associated implications for monetary policy. We first derive the key equilibrium conditions under sticky prices and then consider the case of flexible prices, which helps us to illustrate the effects of nominal rigidities on the transmission of foreign shocks and the trade-offs faced by monetary policy. Regarding notation, we use lowercase letters to denote deviations of a variable from its steady state. For example, $y_t = \frac{Y_t - Y}{Y}$ denotes the deviation of domestic real GDP from its steady-state level Y. Furthermore, we use boldface letters and symbols (e.g., $\boldsymbol{x}$, $\boldsymbol{\Omega}$) to denote vectors and matrices throughout the text.

We first obtain the set of sectoral Phillips curves in the domestic economy by log-linearizing the first-order conditions associated with (10):

$$\boldsymbol{\pi}_t^H = \boldsymbol{\kappa}(\mathbf{mc}_t - \boldsymbol{p}_t^H) + \beta \mathbb{E}_t \boldsymbol{\pi}_{t+1}^H, \tag{26}$$

where $\boldsymbol{\pi}_t^H = [\pi_{1,t}^H, \dots, \pi_{I,t}^H]^\intercal = \boldsymbol{p}_t^H - \boldsymbol{p}_{t-1}^H$ is an $I \times 1$ vector containing the domestic sectoral inflation rates, $\boldsymbol{p}_t^H = [p_{1,t}^H, \dots, p_{I,t}^H]^\intercal$ is an $I \times 1$ vector of domestic sectoral prices, $\mathbf{mc}_t = [\mathrm{mc}_{1,t}, \dots, \mathrm{mc}_{I,t}]^\intercal$ is an $I \times 1$ vector of sectoral nominal marginal costs, and $\boldsymbol{\kappa} = \mathrm{diag}(\kappa_1, \dots, \kappa_I)$ is an $I \times I$ diagonal matrix containing the slopes of the sectoral Phillips

[11]These sectoral subsidies remove steady-state distortions arising from monopolistic competition and ensure that the flexible-price allocation is efficient.

[12]We relegate the derivation to Appendix A.

curves, with $\kappa_i = (1-\beta\theta_i)(1-\theta_i)/\theta_i$.

The heterogeneous frequencies of price adjustment imply sector-specific Phillips curve slopes, collected in $\boldsymbol{\kappa}$, which in turn generally give rise to sector-specific prices, $\boldsymbol{p}_t^H$, and inflation rates, $\boldsymbol{\pi}_t^H$. Moreover, note that differences in production structures across sectors, and in particular IO linkages, do not affect directly (26). However, these features are key determinants of nominal marginal costs across sectors, $\mathbf{mc}_t$. To see this, we log-linearize the solution to the cost-minimization problem (9) and obtain the following expression for sectoral nominal marginal costs:

$$\mathbf{mc}_t = \boldsymbol{\alpha} w_t + \boldsymbol{\Omega}_H \boldsymbol{p}_t^H + \boldsymbol{\Omega}_F \left(\boldsymbol{\tau}_t + \boldsymbol{p}_t^F \right) = \boldsymbol{\alpha} w_t + \boldsymbol{\Omega}\, \boldsymbol{p}_t^H + \boldsymbol{\Omega}_F \boldsymbol{s}_t, \tag{27}$$

where $\boldsymbol{\alpha} = [\alpha_1, \ldots, \alpha_I]^\top$ denotes the vector of sectoral labor shares, with $\alpha_i = WN_i/(P_iY_i)$; $\boldsymbol{\Omega}_H$ ($\boldsymbol{\Omega}_F$) is the domestic (foreign) $I \times I$ IO matrix,

$$\boldsymbol{\Omega}_H = \begin{bmatrix} \omega_{11}^H & \omega_{12}^H & \ldots & \omega_{1I}^H \\ \omega_{21}^H & \omega_{22}^H & \ldots & \omega_{2I}^H \\ \vdots & \vdots & \ddots & \vdots \\ \omega_{I1}^H & \omega_{I2}^H & \ldots & \omega_{II}^H \end{bmatrix}, \quad \boldsymbol{\Omega}_F = \begin{bmatrix} \omega_{11}^F & \omega_{12}^F & \ldots & \omega_{1I}^F \\ \omega_{21}^F & \omega_{22}^F & \ldots & \omega_{2I}^F \\ \vdots & \vdots & \ddots & \vdots \\ \omega_{I1}^F & \omega_{I2}^F & \ldots & \omega_{II}^F \end{bmatrix},$$

with elements $\omega_{ij}^H = P_jX_{ij}^H/(P_iY_i) = (1-\alpha_i)\nu_{ij}(1-\zeta_{ij})$ ($\omega_{ij}^F = P_jX_{ij}^F/(P_iY_i) = (1-\alpha_i)\nu_{ij}\zeta_{ij}$) that denote the domestic (foreign) IO shares; $\boldsymbol{p}_t^F = [p_{1,t}^F, \ldots, p_{I,t}^F]^\top$ is the $I \times 1$ vector of foreign sectoral prices in domestic currency, with elements $p_{i,t}^F = e_t + p_{i,t}^*$, and $\boldsymbol{\tau}_t = [\tau_{1,t}, \ldots, \tau_{I,t}]^\top$ is the vector of price wedges faced by the domestic agents.[13] Furthermore, $\boldsymbol{\Omega} = \boldsymbol{\Omega}_H + \boldsymbol{\Omega}_F$ denotes the total IO matrix, and $\boldsymbol{s}_t = [s_{1,t}, \ldots, s_{I,t}]^\top$ is an $I \times 1$ vector containing the sectoral ToT, $s_{i,t} = \tau_{i,t} + p_{i,t}^F - p_{i,t}^H$, given by:

$$\boldsymbol{s}_t = \boldsymbol{\tau}_t + \boldsymbol{p}_t^F - \boldsymbol{p}_t^H. \tag{28}$$

The expression for marginal costs in (27) helps to build intuition about how foreign price shocks propagate through the production network. When the price of imported input i increases due to a price wedge $\tau_{i,t}$, its impact on domestic sectors depends on their direct usage of that input, which is captured by the i-th column of the input–output matrix $\boldsymbol{\Omega}_F$.

This initial shock then triggers additional general-equilibrium effects. First, domestic firms adjust their prices $\boldsymbol{p}_t^H$ in response to higher costs, affecting other domestic producers through IO linkages captured by $\boldsymbol{\Omega}_H$. Second, the exchange rate e_t responds to the shock, influencing the domestic-currency prices of all foreign goods $\boldsymbol{p}_t^F$ and creating an additional

[13] The expression for $\boldsymbol{p}_{Fi,t}$ follows from log-linearizing the law of one price condition (25).

cost pressure on firms that use imported inputs. Third, changes in economic activity will also affect marginal costs through movements in wages, w_t.

Let us now build intuition on how wages, and hence marginal costs, are affected. We start by considering the log-linearized version of the labor supply condition (21):

$$w_t - p_t^C = c_t + \varphi n_t = c_t + \varphi y_t, \tag{29}$$

where the second equality uses that aggregate hours worked, n_t, equal aggregate real value added, y_t. Next, to obtain an expression for consumption, c_t, we rely on a log-linear approximation of GDP (19):

$$y_t = (1 + \varphi \boldsymbol{\lambda}^\top \boldsymbol{\Omega}_F \mathbf{1})^{-1} \left[c_t + (\boldsymbol{\beta}_F^\top + \boldsymbol{\lambda}^\top \boldsymbol{\Omega}_F) \boldsymbol{s}_t - \boldsymbol{\lambda}^\top \text{diag}\left(\boldsymbol{\Omega}_F \mathbf{1}\right) \boldsymbol{n}_t \right] \tag{30}$$

where $\boldsymbol{n}_t = [n_{1,t}, \ldots, n_{I,t}]^\top$ is an $I \times 1$ vector of sectoral employment levels, $\boldsymbol{\beta}_F = \left[P_i^F C_i^F/(P^C C), \ldots, P_I^F C_I^F/(P^C C)\right]^\top = [\beta_1 \zeta_1, \ldots, \beta_I \zeta_I]^\top$ denotes the $I \times 1$ vector of foreign consumption shares, $\boldsymbol{\lambda} = \left[P_i^H Y_i/(P^Y Y), \ldots, P_I^H Y_I/(P^Y Y)\right]^\top$ denotes the $I \times 1$ vector of Domar weights and satisfies $\boldsymbol{\lambda}^\top = \boldsymbol{\beta}^\top (\boldsymbol{I} - \boldsymbol{\Omega})^{-1}$, $\boldsymbol{\lambda}^\top \boldsymbol{\Omega}_F \mathbf{1} = \sum_{i=1}^I \sum_{j=1}^I P_i X_{ji}^F/(P^Y Y)$ denotes the share of imported intermediate goods over GDP, and the diagonal matrix $\text{diag}\left(\boldsymbol{\Omega}_F \mathbf{1}\right)$ collects, for each sector j, the total cost share of imported intermediate inputs. The consumption share vector $\boldsymbol{\beta} = \begin{bmatrix} \beta_1 & \beta_2 & \ldots & \beta_I \end{bmatrix}^\top = \boldsymbol{\beta}_H + \boldsymbol{\beta}_F$ contains elements $\beta_i = P_i C_i/(P_C C)$, and $\boldsymbol{\beta}_H = \left[P_i^H C_i^H/(P_C C), \ldots, P_I^H C_I^H/(P_C C)\right]^\top = \left[\beta_1(1-\zeta_1), \ldots, \beta_I(1-\zeta_I)\right]^\top$ denotes the $I \times 1$ vector of domestic consumption shares.

The above equation shows that real GDP is affected by three components: aggregate consumption, sectoral ToT, and sectoral employment levels. The last two terms in (30) capture movements in the domestic trade balance, arising from the aggregation of sector-specific trade imbalances.

First, movements in the sectoral ToT, $\boldsymbol{s}_t$, generate expenditure-switching effects. For example, an increase in $\tau_{i,t}$ that makes foreign goods in sector i relatively more expensive will generally reduce demand for foreign goods and increase demand for domestically produced goods. Importantly, because agents are differentially exposed to changes in sectoral prices, trade imbalances may differ across industries. This is evident from the coefficient on $\boldsymbol{s}_t$ in (30), which is given by the vector of foreign consumption shares, $\boldsymbol{\beta}_F$, and the input–output matrix relative to foreign producers, $\boldsymbol{\Omega}_F$, appropriately aggregated using the vector of Domar weights, $\boldsymbol{\lambda}$. Second, shocks may lead to changes in sectoral labor demand, $\boldsymbol{n}_t$, thereby also affecting demand for overall intermediate goods, $x_{i,t}$, and hence sectors' demand for imports.

Finally, the coefficient $(1 + \varphi \boldsymbol{\lambda}^\top \boldsymbol{\Omega}_F \mathbf{1})^{-1}$ is a general-equilibrium multiplier that captures second-round effects. Trade imbalances that induce wealth effects on labor supply (and hence on aggregate GDP) generate changes in wages, which in turn feed back into the demand for consumption and for imported intermediate goods.

Equation (30) can be further simplified. As shown in Appendix A.2, using the goods market-clearing conditions (11) we obtain the following expression for the vector of sectoral employment:

$$\boldsymbol{n}_t = \mathcal{N}_s \boldsymbol{s}_t - \mathcal{N}_\tau \boldsymbol{\tau}_t + (\mathcal{N}_y - \mathbf{1})\varphi y_t, \tag{31}$$

where $\mathcal{N}_s = \left(\boldsymbol{I} - \boldsymbol{\Lambda}^{-1}\boldsymbol{\Omega}_H^\top \boldsymbol{\Lambda}\right)^{-1}(\boldsymbol{I} - \boldsymbol{\Omega}_H)$ and $\mathcal{N}_\tau = \left(\boldsymbol{I} - \boldsymbol{\Lambda}^{-1}\boldsymbol{\Omega}_H^\top \boldsymbol{\Lambda}\right)^{-1}(\boldsymbol{I} - \boldsymbol{\Omega})$ are $I \times I$ matrices that summarize the elasticity of sectoral employment to sectoral ToT and to the price-wedge shocks, with $\boldsymbol{\Lambda} = \text{diag}(\lambda)$, while $\mathcal{N}_y = \left(\boldsymbol{I} - \boldsymbol{\Lambda}^{-1}\boldsymbol{\Omega}_H^\top \boldsymbol{\Lambda}\right)^{-1}\boldsymbol{\alpha}$ is a $I \times 1$ vector, and $\mathbf{1}$ is a $I \times 1$ vector of ones.

The terms in (31) reflect that expenditure switching toward domestic goods, induced by changes in the ToT, tends to increase labor demand across sectors, whereas the price wedge shock itself tends to reduce labor demand by depressing overall domestic demand. The term involving aggregate GDP in (31) captures the general-equilibrium effect of economy-wide labor supply on sectoral employment through the labor-supply condition. An increase in aggregate output raises the marginal disutility of labor and therefore tends to increase real wages, as implied by (29). The resulting effect on sectoral employment depends on the properties of $\mathcal{N}_y$. In the absence of domestic input-output linkages ($\boldsymbol{\Omega}_H = 0$), $\mathcal{N}_y = \boldsymbol{\alpha}$ and this channel reduces sectoral employment. With production networks, the sign and magnitude of the effect are ambiguous and reflect how aggregate output propagates through the network structure.

Introducing (31) into (30) we obtain:

$$y_t = \left(1 + \varphi \boldsymbol{\lambda}^\intercal \text{diag}\left(\boldsymbol{\Omega}_F \mathbf{1}\right) \mathcal{N}_y\right)^{-1} \left\{c_t + \left[\boldsymbol{\beta}_F^\intercal + \boldsymbol{\lambda}^\intercal \left(\boldsymbol{\Omega}_F - \text{diag}\left(\boldsymbol{\Omega}_F \mathbf{1}\right) \mathcal{N}_s\right)\right] \boldsymbol{s}_t - \boldsymbol{\lambda}^\intercal \text{diag}\left(\boldsymbol{\Omega}_F \mathbf{1}\right) \mathcal{N}_\tau \boldsymbol{\tau}_t\right\}. \tag{32}$$

Finally, equation (32) features the general-equilibrium multiplier $\left(1 + \varphi \boldsymbol{\lambda}^\intercal \text{diag}\left(\boldsymbol{\Omega}_F \mathbf{1}\right) \mathcal{N}_y\right)^{-1}$, which captures the feedback from aggregate GDP to labor supply and import demand through the network structure. The term $\boldsymbol{\lambda}^\intercal \text{diag}\left(\boldsymbol{\Omega}_F \mathbf{1}\right) \mathcal{N}_\tau$ governs the direct contribution of sectoral ToT shocks $\boldsymbol{\tau}_t$ to aggregate output. The overall effect of $\boldsymbol{\tau}_t$ on y_t is thus given by the product of this coefficient and the general-equilibrium multiplier.

Flexible-price equilibrium. We next consider the natural allocation, that is, the allocation that would prevail in the absence of nominal rigidities. We focus here on the main results and relegate the derivations to Appendix A.2. There, we also show that the natural equilibrium coincides with the solution to the social planner's problem. Therefore, the flexible-price equilibrium is efficient. In terms of notation, we denote variables associated with this equilibrium by a superscript n.

In the flexible-price equilibrium, where prices equal nominal marginal costs ($p_{i,t}^{H,n} = \text{mc}_{i,t}$) and nominal wages remain constant, the domestic prices adjust as follows:

$$\boldsymbol{p}_t^{H,n} = (\boldsymbol{I} - \boldsymbol{\Omega}_H)^{-1} \boldsymbol{\Omega}_F \boldsymbol{\tau}_t. \tag{33}$$

Equation (33) shows how, in a flexible-price equilibrium, domestic prices adjust in response to the price-wedge shocks $\boldsymbol{\tau}_t$. First, there is a direct effect of the shock on domestic prices, mediated by the direct exposure of domestic industries to foreign sectors, $\boldsymbol{\Omega}_F$. Second, there is a general-equilibrium effect captured by the Leontief inverse with respect to domestic producers, $(\boldsymbol{I} - \boldsymbol{\Omega}_H)^{-1}$. This term captures the fact that, as domestic producers adjust their prices, these adjustments feed back into marginal costs through the production network, leading to a further increase in domestic prices. Hence, the presence of input–output linkages in the production network amplifies the impact of the shock on domestic prices.

Once the response of domestic prices in the natural equilibrium (33) has been derived, it is straightforward to see that the response of the ToT is given by $\boldsymbol{s}_t^n = \left[\boldsymbol{I} - (\boldsymbol{I} - \boldsymbol{\Omega}_H)^{-1} \boldsymbol{\Omega}_F\right] \boldsymbol{\tau}_t$.

Finally, Appendix A.2 also shows that, under these efficient price adjustments, GDP in the flexible-price equilibrium remains unaffected by international price-wedge shocks:

$$y_t^n = 0. \tag{34}$$

3.2. Sectoral Phillips Curves

In this section, we examine the implications and relevance of domestic and foreign production networks for the transmission of foreign price-wedge shocks to domestic inflation in a small open economy.

One-Sector Small Open Economy. To more transparently identify the contribution of production networks to the transmission of foreign shocks, we first consider the New Keynesian Phillips curve in a one-sector economy in which production uses intermediate inputs. In Appendix A.2, we show that in this case the Phillips curve is given by:

$$\pi_t^H = \kappa(1 + \varphi)(1 - \omega_H)\widetilde{y}_t + \beta \mathbb{E}_t \pi_{t+1}^H, \tag{35}$$

where $\widetilde{y}_t$ denotes the output gap—computed as the log-deviation from the natural output prevailing under price flexibility— and ω_H denotes the share of domestically produced intermediate goods used in Home's production in this one-sector economy.

Equation (35) is analogous to the expression derived in Gali and Monacelli (2005) for a one-sector small open economy, but extended to allow for intermediate goods. Three features stand out. First, closing the output gap stabilizes domestic price inflation; that is, the so-called *divine coincidence* holds, and therefore the monetary authority does not face a trade-off. Second, the presence of intermediate goods ($\omega_H > 0$) reduces the slope of the Phillips curve in (35), in line with the results of Christiano (2016) and Rubbo (2019) for closed economies. Third, the slope of the Phillips curve is independent of the degree of openness of the economy: the slope in the open economy is the same as the slope that would arise in a one-sector closed economy with intermediate inputs.

The previous set of results, as shown in Gali and Monacelli (2005), relies on the assumption of unitary elasticities of substitution between domestic and foreign goods and on the assumption of log utility. Although these assumptions may seem strong—and indeed we relax them in Section 4 for our quantitative application— they provide a useful benchmark for analyzing the impact of production networks, to which we now turn.

Multi-sector Small Open Economy. In our baseline small open economy with production networks and multiple sectors, combining equations (26), (27), (29), and (32), the domestic sectoral Phillips curves can be written as[14]

$$\boldsymbol{\pi}_t^H = \mathcal{B}\left(1 + \varphi + \varphi \boldsymbol{\lambda}^\top \text{diag}\left(\boldsymbol{\Omega}_F \mathbf{1}\right) \mathcal{N}_y\right) \widetilde{y}_t + \boldsymbol{\Psi} \widetilde{\boldsymbol{s}}_t - \mathcal{V} \boldsymbol{\chi}_t + \beta (\boldsymbol{I} - \mathcal{V}) \mathbb{E}_t \boldsymbol{\pi}_{t+1}^H, \tag{36}$$

where $\widetilde{\boldsymbol{n}}_t = \begin{bmatrix} \widetilde{n}_{1,t} & \dots & \widetilde{n}_{I,t} \end{bmatrix}^\top$ denotes the vector of sectoral employment gaps, $\widetilde{\boldsymbol{s}}_t = \begin{bmatrix} \widetilde{s}_{1,t} & \dots & \widetilde{s}_{I,t} \end{bmatrix}^\top$ denotes the vector of sectoral ToT gaps, and the vector $\boldsymbol{\chi}_t \equiv \boldsymbol{p}_{t-1}^H - (\boldsymbol{I} - \boldsymbol{\Omega}_H)^{-1} \boldsymbol{\Omega}_F \boldsymbol{\tau}_t$ denotes the wedge between lagged domestic prices, $\boldsymbol{p}_{t-1}^H$, and the efficient response of domestic prices to the shocks, as given by equation (33).

The derivations of the matrices $\mathcal{B} = \boldsymbol{\Delta}(\boldsymbol{I} - \boldsymbol{\Omega}\boldsymbol{\Delta})^{-1}\boldsymbol{\alpha}/[1 - \boldsymbol{\beta}^\top \boldsymbol{\Delta}(\boldsymbol{I} - \boldsymbol{\Omega}\boldsymbol{\Delta})^{-1}\boldsymbol{\alpha}]$ and $\mathcal{V} = \left\{\boldsymbol{\Delta}(\boldsymbol{I} - \boldsymbol{\Omega}\boldsymbol{\Delta})^{-1} - \mathcal{B}[\boldsymbol{\lambda}^\top - \boldsymbol{\beta}^\top \boldsymbol{\Delta}(\boldsymbol{I} - \boldsymbol{\Omega}\boldsymbol{\Delta})^{-1}]\right\}(\boldsymbol{I} - \boldsymbol{\Omega})$ are equivalent to those in the closed-economy version of the model in Rubbo (2023), with $\boldsymbol{\Delta} = (\boldsymbol{I} + \boldsymbol{\kappa})^{-1}\boldsymbol{\kappa}$. Finally, $\boldsymbol{\Psi} \equiv \boldsymbol{\Delta}(\boldsymbol{I} - \boldsymbol{\Omega}\boldsymbol{\Delta})^{-1}\boldsymbol{\Omega}_F + \mathcal{B}\boldsymbol{\beta}^\top \boldsymbol{\Delta}(\boldsymbol{I} - \boldsymbol{\Omega}\boldsymbol{\Delta})^{-1}\boldsymbol{\Omega}_F + \mathcal{B}\boldsymbol{\lambda}^\top \left(\text{diag}\left(\boldsymbol{\Omega}_F \mathbf{1}\right) \mathcal{N}_s - \boldsymbol{\Omega}_F\right)$ denotes the slope of the Phillips curve with respect to the ToT gaps.

The vector of sectoral Phillips curves in (36) shows that production networks can play a key role in shaping both the transmission of international price shocks to domestic prices and the trade-offs faced by the monetary authority.

More precisely, and in contrast to the one-sector economy, it is apparent from (36) that closing the output gap, $\widetilde{y}_t$, does not in general stabilize inflation across domestic sectors, $\boldsymbol{\pi}_t^H$,

[14] We relegate the derivation to Appendix A.2.

because of sectoral wedges contained in χ_t and inefficient movements in the sectoral ToT, $\tilde{\boldsymbol{s}}_t$. We next describe how each of these elements transmits into domestic sectoral inflation and, consequently, how it shapes the trade-offs faced by the monetary authority.

We first focus on the slope of domestic sectoral inflation with respect to the output gap, given by $\mathcal{B}\left(1+\varphi+\varphi\boldsymbol{\lambda}^\top \text{diag}\left(\boldsymbol{\Omega}_F\mathbf{1}\right)\mathcal{N}_y\right)$. The first component, $\mathcal{B}\left(1+\varphi\right)$, is analogous to the slope in a closed economy with production networks, after appropriately recalibrating consumption and input–output shares. The intuition for this coefficient follows Rubbo (2019). In such a closed economy, $1+\varphi$ is the slope of labor supply, determining how an increase in employment translates into real wages. This increase in real wages feeds into marginal costs and inflation through two channels captured by $\mathcal{B}$. The first channel, $\boldsymbol{\Delta}(\boldsymbol{I}-\boldsymbol{\Omega}\boldsymbol{\Delta})^{-1}\boldsymbol{\alpha}$, captures the increase in nominal wages via the direct effect mediated by the labor share $\boldsymbol{\alpha}$, and via the indirect effect operating through the production network, $\boldsymbol{\Delta}(\boldsymbol{I}-\boldsymbol{\Omega}\boldsymbol{\Delta})^{-1}$. The latter term governs pass-through into prices by incorporating price stickiness, via $\boldsymbol{\Delta}$, and the indirect effect of higher prices of domestic suppliers through the rigidity-adjusted Leontief inverse, $(\boldsymbol{I}-\boldsymbol{\Omega}\boldsymbol{\Delta})^{-1}$. The resulting increase in prices due to higher nominal wages raises the consumer price level, thereby partially offsetting the initial increase in real wages. This is captured by the the second component present in $\mathcal{B}$, $1/[1-\boldsymbol{\beta}^\top\boldsymbol{\Delta}(\boldsymbol{I}-\boldsymbol{\Omega}\boldsymbol{\Delta})^{-1}\boldsymbol{\alpha}]$, which uses consumption shares, $\boldsymbol{\beta}^\top$, to aggregate the increase in sectoral prices induced by higher nominal wages, $\boldsymbol{\Delta}(\boldsymbol{I}-\boldsymbol{\Omega}\boldsymbol{\Delta})^{-1}\boldsymbol{\alpha}$, into the consumer price level.

The second term of the slope, $\varphi\mathcal{B}\boldsymbol{\lambda}^\top\text{diag}\left(\boldsymbol{\Omega}_F\mathbf{1}\right)\mathcal{N}_y$, is shaped by the open economy dimension. In contrast to the one-sector economy, the closed- and open-economy slopes no longer coincide:

PROPOSITION 1. *The slope of the Phillips curve* (36) *with respect to the output gap is strictly steeper than its closed-economy counterpart.*

PROOF. See Appendix A. □

Proposition 1 shows that international production networks steepen the Phillips curve relative to a closed economy. Importantly, this result does not arise because firms' pricing decisions are intrinsically more sensitive to demand. Rather, it reflects a general-equilibrium mechanism whereby producing a given increase in domestic output requires a larger increase in domestic labor input once imported intermediate goods and sectoral trade imbalances are taken into account.

To see this, consider an increase in the output gap. In an economy with international production networks, higher domestic production mechanically raises the demand for imported intermediate inputs. As shown in equation (32), aggregate output therefore increases by more than domestic absorption, generating a wedge between output and consumption that is proportional to $\boldsymbol{\lambda}^\top\,\text{diag}(\boldsymbol{\Omega}_F\mathbf{1})\,\mathcal{N}_y$. This term captures the fact that part of domestic production "leaks abroad" through imported intermediates, so that domestic consumption does not keep pace with output.

The presence of this output–consumption wedge has direct implications for labor supply. Because consumption rises by less than output, households must supply additional labor without a commensurate increase in consumption. Through the labor-supply condition, this raises the marginal rate of substitution between consumption and leisure, requiring a larger increase in real wages to induce the necessary expansion in labor input. In this sense, international production networks amplify the labor response associated with a given output gap.

Higher real wages feed directly into firms' marginal costs. As a result, for a given increase in the output gap, marginal costs rise by more in the presence of international production networks. This amplification is made explicit in equation (36), where the slope of the Phillips curve depends on the additional term $\varphi\boldsymbol{\lambda}^{\top}\text{diag}\left(\boldsymbol{\Omega}_F\mathbf{1}\right)\mathcal{N}_y$, which reflects the network-induced amplification of the labor-supply channel: because imported inputs complement domestic production, higher output requires more domestic labor rather than allowing production to be shifted abroad.

We next focus on the coefficient governing the transmission of sectoral ToT gaps into domestic prices, $\boldsymbol{\Psi}$, which is also absent in the one-sector framework. This transmission operates through the following main channels.

First, there is a direct effect on marginal costs. This effect is given by the direct exposure to sectoral ToT gaps, captured by $\boldsymbol{\Omega}_F$, and multiplied by the component $\boldsymbol{\Delta}(\boldsymbol{I}-\boldsymbol{\Omega}\boldsymbol{\Delta})^{-1}$ of $\boldsymbol{\Psi}$. As in the case of the slope with respect to the output gap, the latter term captures the direct effect on prices, $\boldsymbol{\Delta}$, and the indirect effect through the production network, via the rigidity-adjusted Leontief inverse $(\boldsymbol{I}-\boldsymbol{\Omega}\boldsymbol{\Delta})^{-1}$. That is, this channel would be present in a multi-sector economy with heterogeneous sectors even in the absence of cross-sector input–output linkages—where Ω_F would be diagonal—but is further reinforced by the production network linkages.

Second, sectoral ToT gaps also affect marginal costs indirectly through wages, via their impact on the aggregate consumer price level. In particular, a vector of ToT gaps $\widetilde{\boldsymbol{s}}_t$ induces an increase in domestic prices given by $\boldsymbol{\Delta}(\boldsymbol{I}-\boldsymbol{\Omega}\boldsymbol{\Delta})^{-1}\boldsymbol{\Omega}_F\widetilde{\boldsymbol{s}}_t$, which raises the CPI. Holding real wages fixed therefore requires an increase in the nominal wage. This scalar wage adjustment feeds back into sectoral marginal costs and prices according to the coefficient $\mathcal{B}\boldsymbol{\beta}^{\top}\boldsymbol{\Delta}(\boldsymbol{I}-\boldsymbol{\Omega}\boldsymbol{\Delta})^{-1}\boldsymbol{\Omega}_F$, which constitutes the second term of $\boldsymbol{\Psi}$. Hence, this term should be interpreted as the matrix of coefficients translating the common nominal-wage response induced by ToT movements into sectoral inflation dynamics.

The following result states that a deterioration in the ToT, such as the one caused by a price-wedge shock, tends to increase domestic prices through the two channels discussed above.

Remark 1. *The elements in* $\boldsymbol{\Psi}$ *related to the first and second transmission channels,* $\boldsymbol{\Delta}(\boldsymbol{I}-\boldsymbol{\Omega}\boldsymbol{\Delta})^{-1}\boldsymbol{\Omega}_F$ *and* $\mathcal{B}\beta^\top\boldsymbol{\Delta}(\boldsymbol{I}-\boldsymbol{\Omega}\boldsymbol{\Delta})^{-1}\boldsymbol{\Omega}_F$*, respectively, are entrywise non-negative. Moreover, any entry corresponding to a domestic sector that is directly or indirectly exposed to foreign sectors through* $\boldsymbol{\Omega}_F$ *is strictly positive. In particular, strict positivity holds entrywise under the additional assumption that the relevant network is sufficiently connected and that the associated IO shares are strictly positive.*

The third and final channel is captured by the last term of $\boldsymbol{\Psi}$, $\mathcal{B}\boldsymbol{\lambda}^\top\left(\mathrm{diag}\left(\boldsymbol{\Omega}_F\mathbf{1}\right)\mathcal{N}_s-\boldsymbol{\Omega}_F\right)$, which summarizes how inefficient movements in sectoral ToT generate sectoral trade imbalances that affect labor supply and wages, and thereby feed into sectoral inflation. Two opposing forces are at play. On the one hand, the associated scalar real-wage response to ToT-induced expenditure switching reflects a decline in imports, which is mapped into sectoral marginal costs through the coefficient $\mathcal{B}\boldsymbol{\lambda}^\top\boldsymbol{\Omega}_F$. On the other hand, in a counteracting effect, this same expenditure switching toward domestic goods raises sectoral employment, with elasticity $\mathcal{N}_s$, and thereby increases demand for imported intermediate goods, pushing up real wages and marginal costs. The following proposition also clarifies why this term is absent in a one-sector economy, even when intermediate goods are present:

Proposition 2. *The matrix* $\mathrm{diag}\left(\boldsymbol{\Omega}_F\mathbf{1}\right)\mathcal{N}_s-\boldsymbol{\Omega}_F$ *is hollow (the elements on the diagonal are all zero).*

Proof. See Appendix A. □

The previous proposition illustrates that what matters are production linkages *across* sectors. That is, since the diagonal elements of $\mathrm{diag}\left(\boldsymbol{\Omega}_F\mathbf{1}\right)\mathcal{N}_s-\boldsymbol{\Omega}_F$ are zero, the term $\mathcal{B}\boldsymbol{\lambda}^\top\left(\mathrm{diag}\left(\boldsymbol{\Omega}_F\mathbf{1}\right)\mathcal{N}_s-\boldsymbol{\Omega}_F\right)$ would be absent not only in one-sector economies, but also in multi-sector frameworks with heterogeneous sectors and no cross-sector relationships in the production network—where both $\boldsymbol{\Omega}_F$ and $\boldsymbol{\Omega}_H$ are diagonal matrices.

The intuition is as follows. Consider the one-sector economy with intermediate goods characterized by the Phillips curve (35). In that case, a 1% increase in the ToT translates one-to-one into higher demand for the domestic intermediate good, given the unitary elasticity of substitution. The associated increase in the demand for imports arising from higher sectoral employment exactly offsets this effect, given the Cobb–Douglas production function, thereby leaving net exports—and hence wages—unchanged. With cross-sector linkages, however, other domestic sectors also reallocate demand toward the now cheaper domestic sector, further increasing overall demand and generating trade imbalances that distort real wages and inflation.

Finally, the term $\boldsymbol{\chi}_t \equiv \boldsymbol{p}^H_{t-1}-(\boldsymbol{I}-\boldsymbol{\Omega}_H)^{-1}\boldsymbol{\Omega}_F\boldsymbol{\tau}_t$ extends the result in Rubbo (2019) for domestic TFP shocks to our open economy with international price-wedge shocks. It incorporates the direct effect of the international price shock into domestic prices. This effect depends on the sectoral exposure to international prices, measured by $\mathbf{\Omega}_F$, as well as on the exposure to the induced increase in domestic prices, captured by the multiplier $(\boldsymbol{I} - \mathbf{\Omega}_H)^{-1}$. These price changes feed into inflation through two offsetting channels summarized by $\mathcal{V}$: directly, through higher marginal costs, as captured in the term $\mathbf{\Delta}(\boldsymbol{I} - \mathbf{\Omega\Delta})^{-1}$, and indirectly, through lower real wages, as captured by $\mathcal{B}[\boldsymbol{\lambda}^\top - \boldsymbol{\beta}^\top \mathbf{\Delta}(\boldsymbol{I} - \mathbf{\Omega\Delta})^{-1}]$.

The above discussion shows that accounting for both domestic and international production networks is crucial for understanding the propagation of international price-wedge shocks. In our quantitative exercise in Section 4, we also document that production networks are not only key for transmission, but also increase the persistence of price dynamics, leading to a more prolonged inflationary episode in response to a given shock. To see why this is the case, we iterate backwards on (36) to find $\boldsymbol{p}_t^H = \mathcal{B}\left(1 + \varphi + \varphi\boldsymbol{\lambda}^\top \mathrm{diag}\left(\mathbf{\Omega}_F \mathbf{1}\right)\mathcal{N}_y\right) \sum_{k=0}^t \widetilde{y}_{t-k} + \mathbf{\Psi} \sum_{k=0}^t \widetilde{\boldsymbol{s}}_{t-k} - \mathcal{V} \sum_{k=0}^t \boldsymbol{\chi}_{t-k} + \beta(\boldsymbol{I} - \mathcal{V}) \sum_{k=0}^t \mathbb{E}_{t-k} \boldsymbol{\pi}_{t+1-k}^H$, which shows the dependence of current sectoral prices on past inflation expectations, the output gap, and past sectoral prices through the ToT and the wedges $\boldsymbol{\chi}_t$. As discussed, for example, in Ghassibe (2021), Pasten *et al.* (2020), and Huang and Liu (2004), production networks introduce an additional source of price inertia and persistence, as producers inherit the price stickiness of their suppliers via marginal costs. The above equation shows that the international dimension of our model additionally shapes this persistence, through the sectoral ToT.

The Divine Coincidence Inflation Index. As we have argued, multi-sectoral frameworks generate a fundamental trade-off between stabilizing inflation and closing the aggregate output gap. Rubbo (2023) derives a Divine Coincidence Inflation (DCI) index, which weights sectoral inflation by sales shares and by the inverse of sectoral price flexibility, proving that it is the unique aggregate price index whose Phillips curve contains no inefficient-price residual. This occurs because the DCI is exactly aligned with the sales-weighted markup dynamics that determine the output gap. Consequently, the DCI yields a Phillips curve with a slope driven solely by the output gap and expected DCI inflation. In such a case, closing the output gap simultaneously closes the DCI index.

The following proposition states that this closed-economy result does not hold in the open economy:

PROPOSITION 3. *There does not exist any constant-weight linear inflation index of the form* $\pi_t^\star = \sum_{i=1}^I \omega_i \pi_{i,t}^H / \left(\sum_{j=1}^I \omega_j\right)$, *with* $\boldsymbol{\omega} \in \mathbb{R}_+^I$ *and* $\boldsymbol{\omega} \neq \mathbf{0}$, *whose Phillips curve contains no inefficient-price residual (in the sense of Rubbo 2023) and that achieves the Divine Coincidence in the open economy (i.e., closing the aggregate output gap implies* $\pi_t^\star = 0$ *for all t), unless* $\boldsymbol{\lambda}^\top \mathit{diag}(\mathbf{\Omega}_F \mathbf{1})\,\mathcal{N}_s = \mathbf{0}^\top$.

The DCI index considered by Rubbo (2023) in the closed-economy framework, $\pi_t^{\mathrm{DC}} = \left[\sum_{i=1}^I (\lambda_i/\kappa_i)\pi_{i,t}^H\right] / \left(\sum_{j=1}^I \lambda_j/\kappa_j\right)$, weights sectoral inflation by sales shares and by the inverse of sectoral price flexibility. Under such index, the sectoral Phillips curves (26) can be aggregated to a single, economy-wide Phillips curve:

$$\pi_t^{DC} = \left(\boldsymbol{\lambda}^\top \boldsymbol{\kappa}^{-1}\mathbf{1}\right)^{-1} \varphi \widetilde{y}_t + \left(\boldsymbol{\lambda}^\top \boldsymbol{\kappa}^{-1}\mathbf{1}\right)^{-1} \boldsymbol{\lambda}^\top (\boldsymbol{I} - \boldsymbol{\Omega}_H)\widetilde{\boldsymbol{s}}_t + \beta \mathbb{E}_t \pi_{t+1}^{DC}. \tag{37}$$

Similarly to the closed-economy framework, the sales-and-rigidity-adjusted inflation index eliminates the inefficient-price residual that production networks generate, $\mathcal{V}\chi_t$ in (36). However, in an open economy, this DCI index is not sufficient in order to eliminate the inefficient-price movements that arise from the ToT gap, unless $\boldsymbol{\lambda}^\top \text{diag}\left(\boldsymbol{\Omega}_F \mathbf{1}\right) \mathcal{N}_s = \mathbf{0}^\top$.[15] In the open-economy framework, the output gap is proportional to a combination of sectoral ToT:

$$\widetilde{y}_t = \frac{\boldsymbol{\beta}^\top + \boldsymbol{\lambda}^\top \left(\boldsymbol{\Omega}_F - \text{diag}\left(\boldsymbol{\Omega}_F \mathbf{1}\right) \mathcal{N}_s\right)}{1 + \varphi \boldsymbol{\lambda}^\top \text{diag}\left(\boldsymbol{\Omega}_F \mathbf{1}\right) \mathcal{N}_y} \widetilde{\boldsymbol{s}}_t \tag{38}$$

and closing the output gap does not necessarily close all ToT gaps simultaneously. For instance, setting $\widetilde{y}_t = 0$ and using the equilibrium relation (38) jointly imply that the DCI Phillips curve (37) is given by $\pi_t^{DC} = \left(\boldsymbol{\lambda}^\top \boldsymbol{\kappa}^{-1}\mathbf{1}\right)^{-1} \boldsymbol{\lambda}^\top \text{diag}\left(\boldsymbol{\Omega}_F \mathbf{1}\right) \mathcal{N}_s \widetilde{\boldsymbol{s}}_t + \beta \mathbb{E}_t \pi_{t+1}^{DC}$, which does not necessarily imply $\pi_t^{DC} = 0 \; \forall t$.

To summarize, we find that in an open economy the trade-off faced by the monetary authority is worsened, and no inflation index achieves the Divine Coincidence.

4. Quantitative Analysis

A key determinant of the recent inflation surge in the EA has been the increasing energy prices (Arce *et al.* 2024). Motivated by this, we next use our model to explore the aggregate effects of a shock to the price of imported energy paid by European firms. For this purpose, we extend the analytical framework presented in Section 3 on several dimensions.

4.1. Quantitative Model

After having derived intuition for the main transmission mechanisms and implications of foreign shocks in the previous simplified framework, we now turn to a quantitative analysis introducing features commonly present in workhorse New Keynesian open economy models. Motivated by the recent energy crisis, we focus on the effects of international energy price shocks. We highlight next the main differences relative to the simpler environment discussed

[15] Intuitively, the term measures how changes in foreign input prices—transformed into ToT gaps—ripple through the domestic input–output network and affect aggregate inflation, weighted by the macroeconomic importance of each sector.

in Section 3, and relegate to Appendix B the full characterization of the equilibrium equations of the quantitative model.

4.1.1. Assumptions

First, as specified in the general environment in Section 2, we consider a $K \geq 2$ number of countries, which are potentially different in their preferences and technologies. More in particular, we no longer restrict utility (1) to be logarithmic: $U(C_{k,t}) = C_{k,t}^{1-\sigma}/(1-\sigma)$. Furthermore, we allow for nominal wage rigidities (Erceg *et al.* 2000), heterogeneous across countries but homogeneous across sectors.

We consider more general CES production function (6), without imposing a unitary elasticity of substitution:

$$Y_{kif,t} = \left[\widetilde{\alpha}_{ki}^{\frac{1}{\psi}} N_{kif,t}^{\frac{\psi-1}{\psi}} + \widetilde{\vartheta}_{ki}^{\frac{1}{\psi}} X_{kif,t}^{\frac{\psi-1}{\psi}} \right]^{\frac{\psi}{\psi-1}}. \tag{39}$$

Similarly, we assume a CES structure for the consumption (2) and intermediate (7) good aggregators. Given the quantitative focus on energy shocks, we introduce an additional layer to distinguish between energy and non-energy goods,[16]

(40)

$$C_{k,t} = \left[\widetilde{\beta}_{k}^{\frac{1}{\gamma}} (C_{k,t}^{E})^{\frac{\gamma-1}{\gamma}} + \left(1 - \widetilde{\beta}_{k}\right)^{\frac{1}{\gamma}} (C_{k,t}^{M})^{\frac{\gamma-1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}}, \quad X_{ki,t} = \left[\widetilde{\beta}_{ki}^{\frac{1}{\phi}} (X_{ki,t}^{E})^{\frac{\phi-1}{\phi}} + (1 - \widetilde{\beta}_{ki})^{\frac{1}{\phi}} (X_{ki,t}^{M})^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}},$$

where $C_{k,t}^{E}$ and $C_{k,t}^{M}$ denote the consumption of energy and non-energy goods, respectively, and $X_{ki,t}^{E}$ and $X_{ki,t}^{M}$ sector i intermediate goods' demand for energy and non-energy goods, respectively. These are given by:

$$C_{k,t}^{E} = \left[\sum_{i \in I_E} \widetilde{\nu}_{ki}^{\frac{1}{\eta}} C_{ki,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \qquad C_{k,t}^{M} = \left[\sum_{i \in I_M} \widetilde{\upsilon}_{ki}^{\frac{1}{\iota}} C_{ki,t}^{\frac{\iota-1}{\iota}} \right]^{\frac{\iota}{\iota-1}}, \tag{41}$$

$$X_{ki,t}^{E} = \left[\sum_{j \in I_E} \widetilde{\nu}_{kij}^{\frac{1}{\chi}} X_{kij,t}^{\frac{\chi-1}{\chi}} \right]^{\frac{\chi}{\chi-1}}, \qquad X_{ki,t}^{M} = \left[\sum_{j \in I_M} \widetilde{\upsilon}_{kij}^{\frac{1}{\xi}} X_{kij,t}^{\frac{\xi-1}{\xi}} \right]^{\frac{\xi}{\xi-1}}, \tag{42}$$

where I_E and I_M denote the sets of sectors producing energy and non-energy goods, respectively. The aggregation over sectoral goods produced in different countries is given by CES

[16]This specification allows us to introduce a specific elasticity of substitution of the energy consumption that does not necessarily need to be equal to the elasticity of substitution between the rest of goods and services.

aggregators:

$$(43) \qquad C_{ki,t} = \left[\sum_{l=1}^{K} \tilde{\zeta}_{kl\,i}^{\frac{1}{\delta}} C_{kl\,i,t}^{\frac{\delta-1}{\delta}}\right]^{\frac{\delta}{\delta-1}} \quad \text{and} \quad X_{kij,t} = \left[\sum_{l=1}^{K} \tilde{\zeta}_{kl\,ij}^{\frac{1}{\mu}} X_{kl\,ij,t}^{\frac{\mu-1}{\mu}}\right]^{\frac{\mu}{\mu-1}}.$$

Second, we consider the case of incomplete financial markets, where only one international bond is traded, denominated in country K's currency. This means that an analogous risk-sharing condition of the type (24) no longer holds.

Third, regarding the price-setting structure we implement the following features. We dispense with the assumption on production sectoral subsidies τ_{ki}^{p}, allowing for positive profits at steady state. In addition, we adopt the LCP paradigm (Devereux and Engel 2003), meaning that a law of one price similar to (25) no longer holds.

We assume that the (log-)price wedge, introduced in (5), follows an AR(2) process:

$$(44) \qquad \tau_{l\,kj,t} = \rho_{1,l\,kj}^{\tau} \tau_{l\,kj,t-1} + \rho_{2,l\,kj}^{\tau} \tau_{l\,kj,t-2} + \varepsilon_{l\,kj,t}^{\tau},$$

where $\varepsilon_{l\,kj,t}^{\tau} \sim \mathcal{N}\left(0, (\sigma_{l\,kj}^{\tau})^2\right)$.

4.1.2. Monetary Authority

There is a monetary authority in each country $k \in K$. In terms of the monetary stance, we differentiate between those countries that belong to a monetary union and those countries that are member states of currency unions.

Non-members of Currency Unions. Each central bank follows a Taylor rule:

$$(45) \qquad i_{k,t} = \rho_k^r i_{k,t-1} + (1 - \rho_k^r)\left(\phi_k^{\pi} \pi_{k,t}^{\phi} + \phi_k^{y} \hat{y}_{k,t}\right) + \varepsilon_{k,t}^{r},$$

where the coefficients are allowed to vary by country; ρ_k^r denotes the degree of interest rate smoothing in the monetary instrument, coefficients $\{\phi_k^{\pi}, \phi_k^{y}\}$ modulate the elasticity of the policy rate with respect to changes in a given inflation index $\pi_{k,t}^{\phi}$, and output $\hat{y}_{k,t}$, measured as the log-deviation from its steady-state value. The Taylor rule features a monetary policy shock, which follows $\varepsilon_{k,t}^{r} \sim \mathcal{N}\left(0, (\sigma_{kr})^2\right)$.

Furthermore, we allow the monetary authority to choose the particular inflation measure (CPI, PCE, PPI, GDP deflator, etc.) that they aim to stabilize, $\pi_{k,t}^{\phi} = \boldsymbol{\Phi}^{\intercal} \boldsymbol{\pi}_{k,t} = \sum_{l=1}^{K} \sum_{i=1}^{I} \phi_{kl\,i} \pi_{kl\,i,t}$, where $\sum_{l=1}^{K} \sum_{i=1}^{I} \phi_{kl\,i} = 1$. For example, when $\phi_{kl\,i}$ is equal to the consumption share of sector i in the country k, then the central bank targets headline inflation.

Members of Currency Unions. Suppose that a subset $K^{MU} \subset K$ of countries belongs to a monetary union. Without loss of generality, we assume that the central bank of a country $k^{MU} \in K^{MU}$ sets the nominal interest rate to stabilize the *union-wide* price inflation index and output deviations, π_t^{MU} and $\widehat{y}_t^{MU}$,[17]

$$i_{\mathrm{MU},t} = \rho^r_{\mathrm{MU}} i_{\mathrm{MU},t-1} + \left(1 - \rho^r_{\mathrm{MU}}\right)\left(\phi^{\pi}_{\mathrm{MU}}\pi^{\phi}_{\mathrm{MU},t} + \phi^{y}_{\mathrm{MU}}\widehat{y}_{\mathrm{MU},t}\right) + \varepsilon^r_{\mathrm{MU},t}, \tag{46}$$

where $\pi^{\phi}_{\mathrm{MU}t} = \sum_{k=1}^{K^{\mathrm{MU}}} \phi_k^{\mathrm{MU}}\pi^{\phi}_{k,t}$ and $\widehat{y}_t^{\mathrm{MU}} = \sum_{k=1}^{K^{\mathrm{MU}}} \phi_k^{\mathrm{MU}}\widehat{y}_{k,t}$ are defined as the GDP-weighted sum of member states' price inflation and output deviations, where $\phi_k^{\mathrm{MU}} = y_k / \sum_{l=1}^{K^{\mathrm{MU}}} y_l$ is the measure of the (steady-state) relative size of country k in the monetary union in terms of nominal GDP.

The central banks in the rest of countries $l \neq k^{MU}$ that belong to the monetary union adopt a peg vis-a-vis the country k^{MU} that sets the monetary stance:

$$\mathcal{E}_{k,t}^{k^{MU}} = \mathcal{E}_{k}^{k^{MU}} \quad \forall k \in K^{MU} \tag{47}$$

where $\mathcal{E}_{k}^{k^{MU}}$ is the bilateral nominal exchange rate in steady state.

4.2. Model Calibration

We calibrate the model economy presented in Section 4.1 at the quarterly frequency to $K = 6$ countries: Spain, France, Italy, Germany, the Rest of the EA (REA), and the Rest of the World (ROW). The production structure within each country contains $I = 44$ sectors.[18] We next discuss the calibration strategy and collect in Table 1 the main parameter values and the corresponding targets or sources.

Households. We set the household's discount factor β to 0.99, to target an annual real interest rate of 4.5%. The intertemporal elasticity of substitution σ is set to 1, a common value in the literature. The inverse of the Frisch elasticity φ is set to 1, in line with the estimates presented in Chetty *et al.* (2011). Households' borrowing premium γ_* is set to 0.001 so that the evolution of net foreign assets has only a small impact on the exchange rate and trade in the short run while guaranteeing that the net foreign asset position is stabilized at zero in the long run (Schmitt-Grohe and Uribe 2003).

The elasticity of substitution in consumption between energy and non-energy goods γ is set to 0.4 following Böhringer and Rivers (2021). The elasticity of substitution in consumption

[17] The specific location of the union-wide central bank is innocuous as long as it targets union-wide variables.
[18] A detailed list of the sectors included in the analysis can be found on Table D.1, in Appendix D.

Parameter	Description	Value	Target / Source
Households			
β	Discount factor	0.99	R = 4.5% p.a.
σ	Inv. Intertemp. Elast. Subs.	1	Standard Value
φ	Inv. Frisch Elasticity	1	Chetty *et al.* (2011)
γ	Elast. Subst. E and M	0.4	Böhringer and Rivers (2021)
η	Elast. Subst. E	0.9	Atalay (2017)
ι	Elast. Subst. M	0.9	Atalay (2017)
δ	Trade Elasticity	1	Standard value
$\{\widetilde{\beta}_k, \widetilde{\nu}_{ki}, \widetilde{\upsilon}_{ki}, \widetilde{\zeta}_{kl\,i}\}$	Quasi-shares consumption		ICIO tables (OECD)
θ^w_k	Calvo wage prob.	0.75	Christoffel *et al.* (2008)
Firms			
ψ	Elast. Subst. N and X	0.5	Atalay (2017)
ϕ	Elast. Subst. E and M	0.4	Böhringer and Rivers (2021)
χ	Elast. Subst. E	0.2	Atalay (2017)
ξ	Elast. Subst. M	0.2	Atalay (2017)
μ	Trade Elasticity	1	Standard value
$\{\widetilde{\alpha}_{ki}, \widetilde{\vartheta}_{ki}, \widetilde{\beta}_{ki}, \widetilde{\nu}_{kij}, \widetilde{\upsilon}_{kij}, \widetilde{\zeta}_{kij}\}$	Quasi-shares production		ICIO tables (OECD)
$\mathcal{M}_{ki}$	Markups		Labor shares (Eurostat)
θ^p_{ki}	Calvo price prob.		Gautier *et al.* (2024)
Monetary Policy			
$\rho_{k,r}$	Interest Rate Smoothing	0.7	Standard Value
$\phi_{k,\pi}$	Reaction to Inflation	1.5	Galí (2015)
$\phi_{k,y}$	Reaction to real GDP	0.125	Galí (2015)
Exogenous Shock Process			
$\rho^\tau_{1,kl\,i}$	Persistence price wedge shock	1.17	Brent crude oil
$\rho^\tau_{2,kl\,i}$	Persistence price wedge shock	-0.2	Brent crude oil
$\sigma^\tau_{kl\,i}$	Std. Dev. price wedge shock	1	Standard Value
σ^r_k	Std. Dev. monetary shock	1	Standard Value

Notes: List of calibrated parameters. See the main text for a discussion on targets, values, and data used.

TABLE 1. Calibration

between energy sources η and between non-energy sectors ι is set to 0.9 following Atalay

(2017). Household's trade elasticity δ is set to 1.[19]

To calibrate the quasi-consumption shares $\{\widetilde{\beta}_k, \widetilde{\nu}_{ki}, \widetilde{\upsilon}_{ki}, \widetilde{\zeta}_{kli}\}$ we rely on the linearized model to target the respective consumption sectoral consumption shares in each country. More precisely, in Appendix B we show that once the model has been linearized, it is possible to read directly consumption shares from the data as long as we have as many quasi-consumption shares parameters as data targets. Implementing this strategy, we obtain consumption shares by country from Inter-country Input Output (ICIO) tables produced by the OECD, using 2019 as our baseline period. Figure D.1A in Appendix D reports a heatmap of the consumption share $\beta_{kli} = P_{ki}C_{kli}/(P^C_k C_k)$, where each element denotes the consumption share of sector i of country l in households' basket of country k.

Regarding wage rigidities, ECB (2009) report limited cross-sectoral heterogeneity in wage frequency adjustments for EA countries. Therefore, we fix the Calvo frequency wage adjustment probability θ^w_k to 0.75 for all countries, in line with the evidence presented in Christoffel *et al.* (2008) for the EA.

Production. The elasticity of substitution in production between labor and intermediate inputs ψ is set to 0.5 (Atalay 2017). The elasticity of substitution in production between energy and non-energy goods ϕ is set to 0.4 (Böhringer and Rivers 2021). The elasticity of substitution in production between energy sectors χ and between non-energy sectors ξ is set to 0.2, following the estimates of Atalay (2017). Finally, as with households, we set the trade elasticity for firms μ, equal to one.

We follow the same strategy as with households to calibrate the quasi-shares in production $\{\widetilde{\alpha}_{ki}, \widetilde{\beta}_{ki}, \widetilde{\nu}_{kij}, \widetilde{\upsilon}_{kij}, \widetilde{\zeta}_{kij}\}$. Namely, using the linearized model around the steady-state we directly read from the data shares in of each intermediate good in production as well as the shares of labor and production in total costs. Our data source here again is the 2019 ICIO tables from the OECD. Figure D.1C reports a heatmap of the home IO matrix of the EA, $\omega_{kkij} = P_{kj}X_{kkij}/(P_{ki}Y_{ki})$, where each element denotes the input share of sector j for output sector i, both sectors inside the EA. Similarly, figure D.1D reports a heatmap of the foreign IO matrix of the EA, $\omega_{klij} = P_{lj}X_{klij}/(P_{ki}Y_{ki})$ for $l \neq k$, where each element denotes the input share of sector j from ROW for output sector i inside EA. We report in Appendix D the equivalent graphs for each country separately (Figure D.2).

We complement the ICIO tables with the Figaro database by Eurostat to calibrate the labor share of each industry. Namely, once the quasi-shares in production have been used, we calibrate the sector-specific markups $\mathcal{M}_{ki}$ to target the wage-bill-over-sales observed in

[19] A growing body of literature has estimated the value of these elasticities for different time horizons, finding that the values of trade elasticities are significantly greater than one in the long term but not in the short term, with values around 1 for horizons of up to two years (Boehm *et al.* 2023). Given that the focus of our work is closer to a cyclical analysis rather than long-term, we choose the value of 1.

the data. Figure D.1B reports a heatmap of the labor share $\alpha_{ki} = W_k N_{ki}/(P_{ki} Y_{ki})$, where each element denotes the labor share of sector i in country k.

Sectoral price rigidities are obtained from Gautier *et al.* (2024). Using CPI micro-data from several EA countries, the authors report the frequency of price adjustment by COICOP categories for each country separately, and from the aggregate EA. Using the COICOP–to–NACE correspondence tables (Kouvavas *et al.* 2021), we compute the frequency of price adjustment by each NACE category in each country, and obtain the heterogeneous price rigidities θ^p_{ki} for Spain, France, Italy, Germany and REA. Finally, we assume that the ROW price rigidities coincide with the aggregate REA price rigidities.

A drawback of the evidence presented in Gautier *et al.* (2024) is that it does not contain consistent price adjustment frequency data on energy goods. Therefore, we complement this with the evidence presented in Dhyne *et al.* (2006) on price adjustments for energy goods for EA countries. In line with the data presented there, and not surprisingly, energy sectors in the model have the steepest price Phillips Curves, with nearly fully flexible prices. Figure D.1E reports a heatmap of the pricing rigidities θ^p_{ki}, where each element denotes nominal price-setting rigidity of sector i in country k.

Monetary Policy. All Taylor rule parameters are set to standard values, and are homogeneous across countries. The interest-rate smoothing coefficient ρ^r_k is set to 0.7. The coefficients for inflation and output, ϕ^π_k and ϕ^y_k, are set to their standard values of 1.5 and 0.125, respectively. Furthermore, we assume that central banks target the headline inflation index.

Exogenous Processes. We fit the persistence coefficients of the energy price shock to the time-series data of the Brent crude oil. The variance of the innovation is set to 1. Lastly, the variance of the monetary policy shock is also set to 1.[20]

4.3. Results

In this section, we first analyze the dynamics of EA variables, with a focus on how these dynamics are shaped by IO linkages. Second, we analyze the contribution of production networks to inflation dynamics through a series of counterfactuals. Third, we explore how the trade-off between the output gap and inflation stabilization is affected by the presence of the international IO production networks.

The energy price shock we analyze is structured as follows. In both the model and the

[20]Setting the innovation variances to one is a normalization choice. The quantitative experiments in the next section are defined directly in terms of a 10% increase in the energy price wedge τ_{klj}, so the scale of the stochastic innovations does not affect the impulse responses reported there.

data, the energy mining sector of the ROW extracts the main energy products.[21] These energy goods are then sold to EA firms that primarily belong to the energy sectors *Coke and refined petroleum* and *Electricity*.[22] After being processed by these sectors, energy goods are then supplied to households as consumption goods, and to the remaining sectors of the economy as energy intermediate goods used in the production process.

In line with the previous reasoning, we consider a 10% increase in the price wedge τ_{klj} between the price charged by the energy mining sector located ROW and the price paid by EA firms. Formally, we set $k = \{ES,DE,FR,IT,REA\}$, l = ROW and j = energy mining.

4.3.1. The Macroeconomic Effects of Rising International Energy Prices

Figure 1 shows the IRFs of EA GDP (Panel 1B), real consumption (Panel 1C), headline inflation (Panel 1D), core inflation (Panel 1E), wage inflation (Panel 1F), net exports (Panel 1G), MU central bank policy rate (Panel 1H), and the nominal exchange rate with respect to the ROW (Panel 1I) . The increase in the price of imported energy paid by EA firms is shown in Panel 1A.

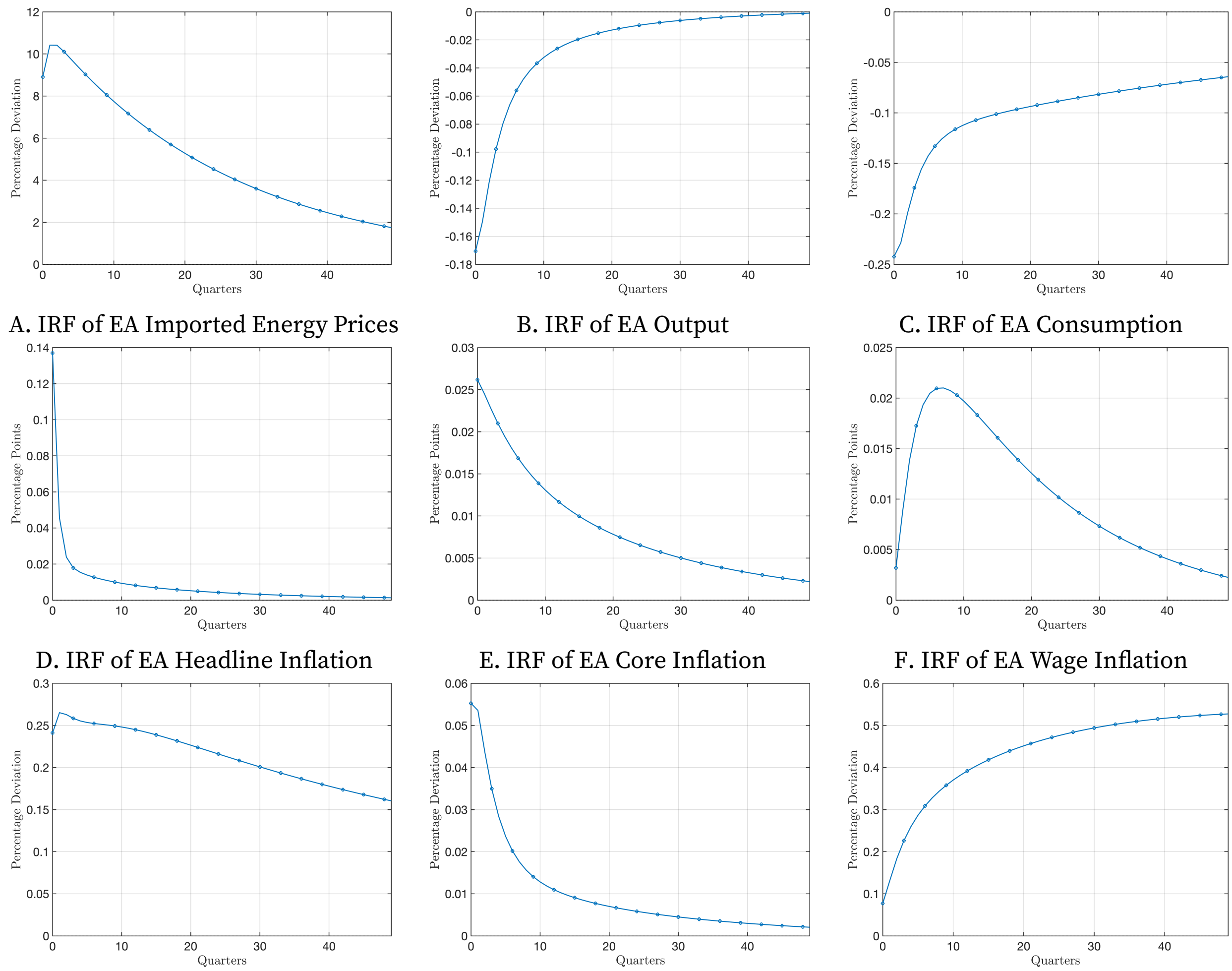
The increase of production costs for EA firms induces them to decrease labor demand and hence production, with value-added (real GDP) falling. In addition, the increase in international energy prices means a negative wealth shock for households, reducing their demand for domestic goods. Overall, we obtain a fall in real consumption larger than the fall in real GDP. Both imports and exports fall, with net exports increasing, due to the relative larger increase in the price of imported goods compared to exported goods. The systematic monetary policy stance of the central bank of the monetary union reacts by increasing the policy rate to control inflation. As a result, the currency of the monetary union appreciates, which—under our convention—corresponds to a decline in the nominal exchange rate (domestic currency per unit of foreign currency) and therefore to an increase in its reciprocal, as shown in Panel 1I.

Headline inflation responds immediately and sharply, reflecting the high price flexibility of energy sectors in the model and the non-negligible share of energy goods in households' consumption basket. The inflationary spike is followed by a more persistent rise in core inflation, which remains elevated long after the initial shock, contributing to the persistent increase of headline inflation.[23] Intuitively, the increase in energy prices energy induces production costs for firms to increase. As a result, firms respond by increasing the prices of

[21] In the data, this corresponds with the Mining and quarrying of energy products sectors, which accounts for sections B.5 and B.6 in the ISIC, Rev.4 classification.

[22] In the data, these correspond with sections C.19 and D.35 in the ISIC, Rev.4 , classification respectively.

[23] On impact, the pass-through of headline to core inflation is significant, amounting to roughly 20%, in line with the empirical evidence presented in Adolfsen *et al.* (2024).



A. IRF of EA Imported Energy Prices

B. IRF of EA Output

C. IRF of EA Consumption

D. IRF of EA Headline Inflation

E. IRF of EA Core Inflation

F. IRF of EA Wage Inflation

G. IRF of EA Net Exports

H. IRF of EA Policy Rate

I. IRF of (inverse) EA Nominal Exchange Rate vs. ROW

FIGURE 1. Effects of an International Energy Price Shock on EA Variables

Notes: IRFs of EA macroeconomic variables: import energy prices (Panel 1A), real GDP (Panel 1B), real consumption (Panel 1C), headline inflation (Panel 1D), core inflation (Panel 1E), wage inflation (Panel 1F), net exports (Panel 1G), nominal interest rate (Panel 1H), and nominal exchange rate vs. Rest of the World (Panel 1I) to a 10% peak increase in imported energy prices.

their products. Therefore, through the IO linkages, the costs of imported and domestically produced goods for firms further increase, leading to an additional rise in prices. This feedback between increasing selling prices and rising production costs results in a generalized increase in core and headline inflation. Finally, we obtain a remarkably persistent and hump-shaped response of wage inflation, which contributes to the persistent rise in core inflation.

4.3.2. Dissecting the Role of Production Networks

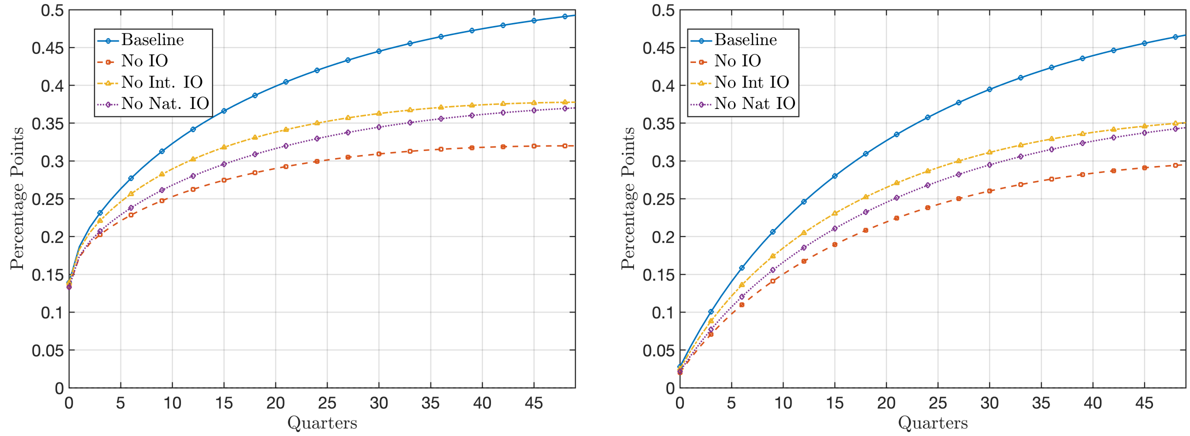
We next assess the role played by national and international production networks. Toward this end, we consider a series of counterfactual economies in which we selectively shut down *non-energy* input–output linkages, while always preserving the role of energy as both a consumption good and a production input for firms.

We first consider an economy without non-energy production networks, where all non-energy intermediate-input coefficients are set to zero, $\omega_{kl\,ij} = 0 \quad \forall k, l, i, j \in I_M$, while intermediate inputs from energy sectors ($j \in I_E$) are kept at their baseline values. We next consider an economy without domestic non-energy production networks, setting $\omega_{kl\,ij} = 0 \quad \forall k = l, i, j \in I_M$, while preserving international non-energy linkages and all energy inputs. Finally, we consider an economy without international non-energy production networks, setting $\omega_{kl\,ij} = 0 \quad \forall k \neq l, i, j \in I_M$, while preserving domestic non-energy linkages and all energy inputs.

Figure 2 shows the CIRFs of headline (Panel 2A) and core inflation (Panel 2B) for our baseline calibration and for each of the three counterfactual economies. The dashed red lines represent our counterfactual economy with national and international IO links removed altogether. On impact, headline inflation increases roughly by the same amount as in the baseline calibration (solid blue lines). This is a consequence of headline inflation being driven initially by the rise of international energy prices, which is common across counterfactuals.

However, the presence of IO linkages is key in explaining inflation dynamics beyond impact. When we conduct our first counterfactual by turning off the production networks, cumulative inflation increases only 60% of our baseline at the end of the simulation horizon (compare the blue and red lines). On the one hand, this is a consequence of the smaller increase in core inflation (Panel 2B). Without IO links, the feedback loop between increasing selling prices and rising production costs is absent, significantly dampening the increase in core inflation. On the other hand, inflation shows less persistence, dying out substantially quicker than in our baseline. More precisely, in our baseline simulation, inflation continues to rise steadily throughout the entire simulation horizon, whereas in the absence of production networks it stabilizes much earlier. This finding formalizes the intuition provided in section 3, whereby the presence of intermediate goods in the marginal costs of firms in interaction with IO linkages leads to more persistent inflation dynamics.

The next two counterfactuals dissect the contribution of national (dotted purple line) and international (dashed yellow line) production networks. We find that the international IO network is a key factor that adds significant persistence on headline and core inflation. Given the upstream position of the energy within production chains, the shock propagates particularly strongly to other productive sectors. Consequently, due to the high level of integration between industrial sectors across European economies, there are significant spillovers from the effects of the shock through the cross-country links captured in the IO tables. This finding highlights the quantitative relevance of the multi-country dimension to account for international spillover effects, which explain around 20% of the overall response



A. CIRF of Headline Inflation

B. CIRF of Core Inflation

FIGURE 2. Inflation Dynamics and Production Networks

Notes: Cumulative IRF of EA headline (Panel 2A) and core (Panel 2B) inflation for the baseline and turning off the full, international, or national IO structure. When turning off the IO structure, we always keep the use of energy as an intermediate input.

of headline inflation, and would be underestimated in the simpler small open economy framework in Section 3.[24]

The analytical framework in section 3 formalized the notion that national and international IO networks induce persistent in inflation dynamics, and the analysis in this section shows the quantitative relevance of our contribution. In other words, it is crucial to account for both the national and international dimensions of production networks simultaneously. Intuitively, higher domestic inflation leads to increased export prices, which contribute to higher inflation abroad. In turn, rising inflation abroad translates into higher import prices, feeding back into domestic inflation. This interaction between national and international production networks amplifies the inflationary episode, resulting in a larger impact than if these dimensions were considered in isolation.

4.3.3. Additional Results

In this section we summarize additional quantitative results, which are discussed in greater detail in the Appendix.

[24]Interestingly, national and international production networks interact with each other. Without IO linkages, cumulative inflation increases by 0.32 percentage points (p.p.), while with only national or international production networks, it increases by roughly 0.37 p.p. In other words, the "marginal effect" of each of them is approximately 0.05 p.p., and adding those to the counterfactual without IO explains only 85% of the total cumulative response in the baseline.

The Role of Nominal Price Rigidities. What if the foreign shock originates in a rigid-price sector? In Appendix C, we compare the energy price wedge shock with an equally-sized wedge in the foreign semiconductors sector, in order to disentangle the role of sectoral price rigidities from that of production networks. Although the EA is more directly exposed to semiconductors than to energy in terms of both final demand and input use, the semiconductor shock generates a much smaller but more persistent increase in headline inflation. To rationalize this pattern, we construct two rigidity-adjusted measures of sectoral importance. First, a downstream flexibility index, which averages the price flexibility of sectors that buy inputs from a given sector, shows that downstream users of energy are about 3.6 times more flexible than downstream users of semiconductors. Second, a rigidity-adjusted network multiplier, which weights the Leontief inverse by sectoral Domar weights and Calvo parameters, reveals that the multiplier associated with energy is roughly 65 percent larger than that of semiconductors. The combination of almost fully flexible prices in the energy sector and relatively flexible downstream buyers makes energy shocks intrinsically more inflationary than shocks in a stickier but less rigidity-central upstream sector such as semiconductors.

Cross-country Heterogeneity. In Appendix C we show that the EA aggregate masks substantial cross-country heterogeneity in the transmission of the common energy price shock, attributable to differences in production structures and consumption baskets, with nominal rigidity differences playing a secondary but non-negligible role. Spain exhibits the largest impact response of headline inflation but relatively fast stabilization, reflecting the high weight of energy in its consumption basket and a more downstream, less complex production structure. Germany, by contrast, displays a much smaller impact effect but markedly more persistent headline and core inflation, consistent with its longer and more upstream production chains and stronger industry exposure to energy inputs. Counterfactuals that equalize, first, the IO matrices and, second, both IO matrices and consumption shares across countries progressively compress the dispersion in inflation dynamics, leaving residual differences that can be traced back to heterogeneous price rigidities.

Sectoral Decomposition. Following an energy-price shock, we find that the initial spike in EA headline inflation is driven entirely by energy; as energy prices revert, non-energy sectors take over—upstream contributions stabilize after 30 quarters, while downstream sectors account for an increasingly persistent share via IO pass-through with nominal rigidities.[25] Full details are relegated to Appendix C.

[25] We classify the 44 sectors into energy (mining, refined petroleum, electricity) and non-energy groups, splitting the latter into upstream vs. downstream using the Antràs *et al.* (2012) upstreamness measure.

Systematic Monetary Policy and Production Networks. In Appendix C we study the interaction between systematic policy and network amplification in the context of foreign energy price shocks by varying the inflation coefficient in the Taylor rule. Weakening the systematic response to inflation raises inflation volatility more than twice as much for core than for headline inflation, and this effect is considerably stronger when production networks are present. This reflects the combination of stickier prices and higher import intensities in core CPI sectors, together with the amplification of energy shocks along IO chains. As a result, even though IO links muted the direct impact of monetary policy shocks on inflation (Nakamura and Steinsson 2010; Rubbo 2023), they make the stance of systematic monetary policy more consequential for inflation volatility in the face of supply-side disturbances.

4.4. Production Networks and Monetary Policy Trade-offs

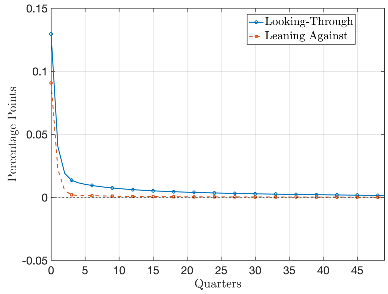
Our previous findings indicate that IO links play a central role in explaining the inflationary effects and cross-country propagation of international energy price shocks. Next, we investigate the implications of these findings for monetary policy.

In the context of rising foreign energy prices that exert upward pressure on domestic inflation, the monetary authority faces a trade-off: pursuing strict inflation targeting may succeed in stabilizing inflation, but at the cost of inducing a decline in the output gap. In this section, we quantify this trade-off—analyzed analytically in Section 3—and examine how it is shaped by the multi-sectoral structure of the economy, which constitutes our main departure from the one-sector framework in Gali and Monacelli (2005), as well as by the open-economy dimension, which extends the closed-economy setting of Rubbo (2023).

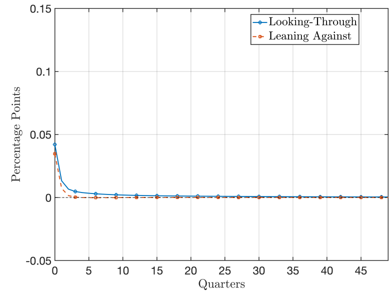
We consider two counterfactual monetary regimes that differ in the systematic component of the Taylor rule. In the *Looking-Through* (LT) regime, central banks set the policy rate according to equations (45) and (46) with coefficients $\{\phi_k^{\pi}, \phi_k^{y}\} = \{1.5, 0\}\ \forall k$. We interpret this as standard monetary policy in the face of supply-side disturbances, and we treat it as our baseline scenario. In contrast, the *Leaning Against the Wind* (LATW) regime corresponds to strict inflation targeting, modeled as $\{\phi_k^{\pi}, \phi_k^{y}\} = \{10, 0\}\ \forall k$, representing a more aggressive stance against inflation deviations.

Under both regimes, we examine the dynamics of the headline inflation and the output gap in the EA, in response to the energy price shock discussed in Section 4.3. Figure 3 presents the IRFs of headline inflation (panel 3A) and the output gap (panel 3D) for both monetary policy rules.

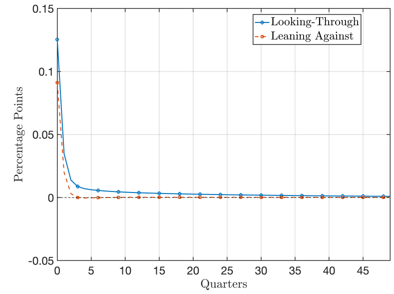
Under the LT regime (blue solid line), cumulative headline inflation increases by approximately 0.4 p.p. after the shock. The output gap initially turns negative, but then becomes positive as the energy price starts declining, and eventually returns to baseline. (Recall that



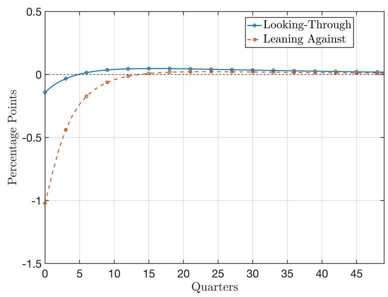
A. IRF of EA Headline Inflation



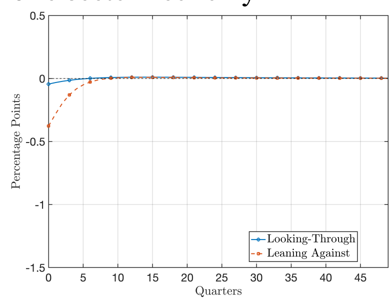
B. IRF of EA of Headline Inflation, One-sector Economy



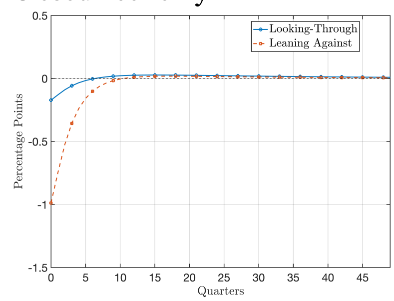
C. IRF of EA of Headline Inflation, Closed Economy



D. IRF of EA of Output Gap



E. IRF of EA of Output Gap, One-sector Economy



F. IRF of EA of Output Gap, Closed Economy

FIGURE 3. Monetary Policy Trade-Offs

Notes: IRFs of EA headline inflation (panel 3A) and output gap (panel 3D) after an increase in imported energy prices (10%) under Looking-through monetary policy $\{\phi^{\pi}_{MU}, \phi^{y}_{MU}\} = \{1.5, 0\}$ (blue and solid line) and Leaning Against the Wind $\{\phi^{\pi}_{MU}, \phi^{y}_{MU}\} = \{10, 0\}$ (red and dashed line) monetary policy. Panels 3B and 3E reproduce the analysis with common parameters across sectors and countries (the one-sector-economy limit). Panels 3C and 3F reproduce the analysis without the foreign IO network and consumption shares (the closed-economy limit).

the energy shock includes a deflationary phase after the fourth period, see Figure 1A.) In contrast, the LATW regime (red dashed line) achieves significantly lower and less persistent inflation, at the cost of a much sharper and prolonged decline in the output gap. This illustrates a clear policy trade-off: stricter inflation stabilization entails a substantially larger output gap contraction. To evaluate how this trade-off is shaped by production networks and international linkages, we conduct two counterfactual exercises.

First, we eliminate the multi-sectoral dimension by imposing full symmetry across sectors and countries.[26] These assumptions collapse the multi-sector model into a one-sector economy, akin to the small open economy setup in Gali and Monacelli (2005) discussed analytically in Section 3.2. Panels 3B and 3E show the IRFs of headline inflation and the output gap, in this one-sector economy, to foreign prices. The trade-off between inflation and output stabilization is noticeably milder. With the amplification effect of production networks

[26]Specifically, we assume homogeneous production networks, consumption shares, labor shares, and nominal rigidities: $\omega_{kl\,ij} = \overline{\omega}$, $\beta_{kl\,i} = \overline{\beta}$, $\alpha_{ki} = \overline{\alpha}$, and $\theta^{p}_{ki} = \overline{\theta}^{p}$ for all k, l, i, j, where the bar denotes the average value in the baseline calibration.

removed, stabilizing inflation becomes less costly in terms of the output gap. Quantitatively, the magnitude of the trade-off is reduced by roughly three-fourths relative to the baseline.

Second, we eliminate the international dimension.[27] This environment resembles the closed-economy setup of Rubbo (2023) discussed analytically in Section 3.2, extended to include foreign price shocks. Crucially, this framework lacks the international spillovers that amplify the domestic response to foreign shocks. Panels 3C and 3F display the corresponding IRFs. As in the one-sector case, the trade-off between inflation and output stabilization becomes less severe. With international amplification shut down, the contraction of the output gap under LATW policy is considerably smaller, and the difference with the LT regime narrows. Overall, the trade-off is reduced by about one-third compared to the baseline.

We next combine the results presented in Figures 3A-3F to uncover the sacrifice ratios. In particular, we are interested in the *differential* sacrifice ratio between the two monetary policy stances. Following the logic behind the Phillips multiplier in Barnichon and Mesters (2021), we compute the *differential* sacrifice ratio $\mathcal{P}_t$ as

$$\mathcal{P}_t = \frac{\sum_{s=0}^{t}(\partial \widetilde{y}_s/\partial \tau_0)/(s+1)\Big|_{\text{LATW}} - \sum_{s=0}^{t}(\partial \widetilde{y}_s/\partial \tau_0)/(s+1)\Big|_{\text{LT}}}{\sum_{s=0}^{t}(\partial \pi_s/\partial \tau_0)/(s+1)\Big|_{\text{LATW}} - \sum_{s=0}^{t}(\partial \pi_s/\partial \tau_0)/(s+1)\Big|_{\text{LT}}} \tag{48}$$

where $\sum_{s=0}^{t}(\partial x_s/\partial \tau_0)/(s+1)$ denotes a time-weighted aggregation of impulse responses that places relatively more weight on early horizons. This object summarizes both the magnitude and the timing of the response of variable x to an exogenous unit change in the energy price wedge τ_0. The numerator in the expression (48) measures the difference in the dynamics of the output gap, after an energy price shock, caused by the LATW and the LT monetary stances. For instance, a negative numerator indicates that the LATW monetary stance generates a larger fall in the output gap, compared to the LT stance. Similarly, the denominator measures the difference in the dynamics of headline inflation, after an energy price shock, caused by the LATW and the LT monetary stances. A negative denominator indicates that the LATW stance induces lower inflation, compared to the LT stance.

The *differential* sacrifice ratio $\mathcal{P}_t$ should be interpreted as the relative output cost per unit of inflation reduction, accounting for both the size and the timing of the responses under each policy regime. Compared to the baseline, we find that the sacrifice ratio in the one-sector economy is 21.31% lower, and 7.30% lower in the closed-economy—in order to stabilize inflation after an energy price perturbation, output gap losses are 21.31% (7.30%) lower in the one-sector (closed) economy. The analytical framework in section 3 formalized the notion that

[27] In the closed-economy counterfactual, the restrictions $\omega_{kl\,ij} = 0$ and $\beta_{kl\,i} = 0$ for $l \neq k$ apply only to non-energy goods. Energy continues to be traded internationally and enters both consumption and production as in the baseline economy, so that the energy price wedge directly affects domestic costs and prices.

both the national and international IO networks worsen the trade-off faced by the monetary authority, and this analysis shows the quantitative relevance of our contribution.

Taken together, these findings highlight the importance of modeling production structures and cross-border linkages in detail. On the one hand, for a given increase in inflation after an energy price shock, the presence of sectoral heterogeneity and international IO spillovers worsens the policy trade-off faced by the central bank. On the other hand, since these external shocks generate a significantly larger increase in inflation in the IO economy—as discussed in Section 4.3.2—the presence of sectoral heterogeneity and international IO spillovers significantly amplifies the macroeconomic effects of external shocks and exacerbates the policy trade-off overall.[28] Ignoring these dimensions would underestimate the costs of strict inflation stabilization in an open and interconnected economy.

5. Conclusions

This paper highlights the critical role of production networks in shaping the transmission of international price shocks and the trade-offs faced by monetary policy. We show analytically that production networks—particularly their international dimension— introduce a novel monetary policy trade-off through sector-level trade imbalances and ToT movements that propagate to domestic inflation. Quantitatively, we study the transmission of an increase in the international price of imported energy in a multi-country model with production networks calibrated to the main EA countries and their trading partners. We find that production networks significantly amplify the cumulative response of inflation over time, while simultaneously increasing the sacrifice ratio faced by the central bank.

More generally, our framework is well suited to analyze the effects of trade policies, such as tariffs, on inflation dynamics. In this context, it could be extended to incorporate the international dimension of investment IO networks (vom Lehn and Winberry 2021; Quintana 2024), in addition to the trade in intermediate goods that we currently consider. We view these extensions as fruitful avenues for future research.

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[28] Intuitively, the only difference is the dynamics of the nominal interest rate. Since production networks augment the degree of non-neutrality (see Appendix C.4), it is more costly to reduce each excess unit of inflation.

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Appendix

Appendix A. Inspecting the Mechanisms

In this section, we derive the equilibrium conditions under the simplified version of the model.

A.1. Production Networks and Inflation

Starting from a general Phillips curve, where $\pi^H_{i,t} = p^H_{i,t} - p^H_{i,t-1}$ denotes the inflation rate of a good in sector i in country H, $\pi^H_{i,t} = \kappa_i \left(\text{mc}_{i,t} - p^H_{i,t}\right) + \beta\mathbb{E}_t\pi^H_{i,t+1}$. The nominal marginal cost faced by sector i are given by

$$\text{mc}_{i,t} = \alpha_i w_t + (1 - \alpha_i)\, p^X_{i,t}. \tag{A.1}$$

where α_i is the steady-state ratio of labor costs to sales in sector i, and $1 - \alpha_i = P^X_i X_i/(P_i Y_i)$ denotes the steady-state ratio of input costs to sales in sector i. The price index of intermediate goods faced by sector i is given by:

$$p^X_{i,t} = \sum_{j=1}^{I} \nu_{ij} \left[(1 - \zeta_{ij})\, p^H_{j,t} + \zeta_{ij}(\tau_{j,t} + p^F_{j,t})\right] = \sum_{j=1}^{I} \nu_{ij} \left(p^H_{j,t} + \zeta_{ij} s_{j,t}\right), \tag{A.2}$$

where $\nu_{ij} = P_{ij}X_{ij}/(P^X_i X_i)$ denotes the purchases of sector i from sector j as a share of total intermediate goods' purchases of sector i, and $1 - \zeta_{ij} = (P_j X^H_{ij})/(P_{ij}X_{ij})$ denotes the purchases of sector i from Home's sector j over total intermediate goods' purchases of sector i from sector j.

Introducing condition (A.2) into the marginal cost equation (A.1), and stacking over sectors, we obtain:

$$\mathbf{mc}_t = \alpha w_t + \boldsymbol{\Omega}_H\, \boldsymbol{p}_t^H + \boldsymbol{\Omega}_F(\boldsymbol{\tau}_t + \boldsymbol{p}_t^F) = \alpha w_t + \boldsymbol{\Omega}\, \boldsymbol{p}_t^H + \boldsymbol{\Omega}_F \boldsymbol{s}_t. \tag{A.3}$$

We now seek to obtain the Phillips curve in price terms. We can write the previous Phillips curve in matrix form,

$$\begin{aligned}
\boldsymbol{\pi}_t^H &= \boldsymbol{\kappa}\left(\mathbf{mc}_t - \boldsymbol{p}_t^H\right) + \beta\mathbb{E}_t\boldsymbol{\pi}_{t+1}^H = \boldsymbol{\kappa}\left[\mathbf{mc}_t - \left(\boldsymbol{\pi}_t^H + \boldsymbol{p}_{t-1}^H\right)\right] + \beta\mathbb{E}_t\boldsymbol{\pi}_{t+1}^H \\
&= (\boldsymbol{I} + \boldsymbol{\kappa})^{-1}\boldsymbol{\kappa}\left(\mathbf{mc}_t - \boldsymbol{p}_{t-1}^H\right) + (\boldsymbol{I} + \boldsymbol{\kappa})^{-1}\beta\mathbb{E}_t\boldsymbol{\pi}_{t+1}^H \\
&= \boldsymbol{\Delta}\left[\alpha \boldsymbol{w}_t + \boldsymbol{\Omega}\, \boldsymbol{p}_t^H + \boldsymbol{\Omega}_F \boldsymbol{s}_t - \boldsymbol{p}_{t-1}^H\right] + (\boldsymbol{I} - \boldsymbol{\Delta})\beta\mathbb{E}_t\boldsymbol{\pi}_{t+1}^H \\
&= \boldsymbol{\Delta}\left[\alpha \boldsymbol{w}_t + \boldsymbol{\Omega}_F \boldsymbol{s}_t + \boldsymbol{\Omega}\boldsymbol{\pi}_t^H + \boldsymbol{\Omega}\, \boldsymbol{p}_{t-1}^H - \boldsymbol{p}_{t-1}^H\right] + (\boldsymbol{I} - \boldsymbol{\Delta})\beta\mathbb{E}_t\boldsymbol{\pi}_{t+1}^H
\end{aligned}$$

$$(\boldsymbol{I} - \boldsymbol{\Delta}\boldsymbol{\Omega})\boldsymbol{\pi}_t^H = \boldsymbol{\Delta}\left[\alpha \boldsymbol{w}_t + \boldsymbol{\Omega}_F \boldsymbol{s}_t - (\boldsymbol{I} - \boldsymbol{\Omega})\, \boldsymbol{p}_{t-1}^H\right] + (\boldsymbol{I} - \boldsymbol{\Delta})\beta\mathbb{E}_t\boldsymbol{\pi}_{t+1}^H \tag{A.4}$$

where we have introduced nominal marginal costs (A.3), and the different objects are defined in section 3.2.

A.2. Production Networks and Monetary Policy

We now seek to write the sectoral Phillips curves in gaps from the flexible-price equilibrium. Under no nominal wage rigidities, the labor supply condition is given by:

$$w_t - p_t^C = c_t + \varphi n_t \tag{A.5}$$

where p_t^C is the consumption price index given by:

$$p_t^C = \sum_{i=1}^{I} \beta_i\left[(1-\zeta_i)\, p_{i,t}^H + \zeta_i(\tau_{i,t} + p_{i,t}^F)\right] = \sum_{i=1}^{I} \beta_i\left(p_{i,t}^H + \zeta_i s_{i,t}\right) = \boldsymbol{\beta}^\top \boldsymbol{p}_t^H + \boldsymbol{\beta}_F^\top \boldsymbol{s}_t \tag{A.6}$$

where $\beta_i = P_iC_i/(P^CC)$ denotes the consumption share of households in H, and $1-\zeta_i = (P_iC_i^H)/(P_iC_i)$ denotes the consumption from sector i in H over total consumption of sector i, $\boldsymbol{\beta}^\top = [\beta_1, \dots, \beta_I]^\top$ is the vector of steady-state sectoral consumption shares, and $\boldsymbol{\beta}_F^\top = [\beta_1\zeta_1, \dots, \beta_I\zeta_I]^\top$ denotes the vector of steady-state sectoral foreign consumption shares.

The risk-sharing condition (under complete markets and constant ROW variables, i.e.

$c_t^* = 0$) is given by:

$$c_t = q_t + c_t^* = e_t - p_t^C. \tag{A.7}$$

Furthermore, the definition of nominal GDP is given by:

$$P_t^Y Y_t = P_t^C C_t + \sum_{i=1}^{I} \left(P_{i,t}^H C_{i,t}^{H,*} - (1+\tau_{i,t}) P_{i,t}^F C_{i,t}^F \right) + \sum_{i=1}^{I} \sum_{j=1}^{I} \left(P_{i,t}^H X_{ji,t}^{H,*} - (1+\tau_{i,t}) P_{i,t}^F X_{ji,t}^F \right) \tag{A.8}$$

where P_t^Y is the GDP deflator, $C_{i,t}^{H,*}$ and $X_{ij,t}^{H,*}$ denote Home's sectoral exports to households and firms in ROW, and $C_{i,t}^F$ and $X_{ij,t}^F$ are Home's sectoral imports from households and firms.

We start by finding an expression for *real* GDP. In order to do so, we first linearize the definition of nominal GDP (A.8):

$$\begin{aligned} p_t^Y + y_t = p_t^C + c_t &+ \sum_{i=1}^{I} \beta_i \zeta_i \left(p_{i,t}^H + c_{i,t}^{H,*} - \tau_{i,t} - p_{i,t}^F - c_{i,t}^F \right) \\ &+ \sum_{i=1}^{I} \sum_{j=1}^{I} \lambda_j (1-\alpha_j) \nu_{ji} \zeta_{ji} \left(p_{i,t}^H + x_{ji,t}^{H,*} - \tau_{i,t} - p_{i,t}^F - x_{ji,t}^F \right) \end{aligned} \tag{A.9}$$

Next, note that we have the following expression for the GDP deflator:

$$\begin{aligned} p_t^Y &= p_t^C + \sum_{i=1}^{I} \beta_i \zeta_i \left(p_{i,t}^H - \tau_{i,t} - p_{i,t}^F \right) + \sum_{i=1}^{I} \sum_{j=1}^{I} \lambda_j (1-\alpha_j) \nu_{ji} \zeta_{ji} \left(p_{i,t}^H - \tau_{i,t} - p_{i,t}^F \right) \\ &= p_t^C + \sum_{i=1}^{I} \left(\beta_i \zeta_i + \sum_{j=1}^{I} \lambda_j (1-\alpha_j) \nu_{ji} \zeta_{ji} \right) \left(p_{i,t}^H - \tau_{i,t} - p_{i,t}^F \right) = p_t^C - (\boldsymbol{\beta}_F^\top + \boldsymbol{\lambda}^\top \boldsymbol{\Omega}_F) \boldsymbol{s}_t \end{aligned} \tag{A.10}$$

and hence we have that real GDP is given by:

$$y_t = c_t + \sum_{i=1}^{I} \beta_i \zeta_i \left(c_{i,t}^{H,*} - c_{i,t}^F \right) + \sum_{i=1}^{I} \sum_{j=1}^{I} \lambda_j \nu_{ji} (1-\alpha_j) \zeta_{ji} \left(x_{ji,t}^{H,*} - x_{ji,t}^F \right). \tag{A.11}$$

Consider first the term $\sum_{i=1}^{I} \beta_i \zeta_i \left(c_{i,t}^{H,*} - c_{i,t}^F \right)$. Note that we have $c_{i,t}^{H,*} = e_t + p_i^{F,*} - p_{i,t}^H + c_i^* = e_t + p_i^{F,*} - p_{i,t}^H + (e_t + p_t^{C,*}) - (e_t + p_{i,t}^{F,*}) + c_t^* = -p_{i,t}^H + (e_t + p_t^{C,*}) + c_t - q_t = -p_{i,t}^H + p_t^C + c_t$, where we have made use of the risk-sharing condition (A.7) and the relation $p_{i,t}^{F,*} + c_{i,t}^* = p_t^{C,*} + c_t^*$. In addition, note that we have $c_{i,t}^F = p_{i,t}^C - \tau_{i,t} - p_{i,t}^F + c_{i,t} = p_{i,t}^C - \tau_{i,t} - p_{i,t}^F + (p_t^C - p_{i,t}^C) + c_t = p_t^C + c_t - \tau_{i,t} - p_{i,t}^F$.

Therefore, we have that:

$$\sum_{i=1}^{I}\beta_i\zeta_i\left(c_{i,t}^{H,*}-c_{i,t}^{F}\right)=\sum_{i=1}^{I}\beta_i\zeta_i\left(\tau_{i,t}+p_{i,t}^{F}-p_{i,t}^{H}\right)=\boldsymbol{\beta}_F^{\top}\boldsymbol{s}_t. \tag{A.12}$$

Next, we work with the term $x_{ji,t}^{H,*}-x_{ji,t}^{F}$. First, note that we have $x_{ji,t}^{H,*}=e_t+p_i^{F,*}-p_{i,t}^{H}+x_{ji,t}^{*}=e_t+p_i^{F,*}-p_{i,t}^{H}+(e_t+p_j^{X,*})-(e_t+p_i^{F,*})+x_j^{*}=-p_{i,t}^{H}+e_t+p_t^{C,*}+(w_t^{*}-p_t^{C,*})+n_{j,t}^{*}=-p_{i,t}^{H}+p_t^{C}+q_t+c_t^{*}+\varphi n_t^{*}+n_{j,t}^{*}=-p_{i,t}^{H}+p_t^{C}+q_t+c_t-q_t+\varphi n_t^{*}+n_{j,t}^{*}=-p_{i,t}^{H}+p_t^{C}+c_t$, where we have made use of the first-order conditions of the firms for labor demand and intermediate goods' demand, $x_{j,t}^{*}+p_{j,t}^{X,*}=n_{j,t}^{*}+w_t^{*}$, and of the fact that foreign variables are constant, in addition to the risk-sharing condition (A.7) and the relation $p_{i,t}^{F,*}+x_{ji,t}^{*}=p_{j,t}^{X,*}+x_{j,t}^{*}$. Next, note that we have $x_{ji,t}^{F}=p_{ji,t}-\tau_{i,t}-p_{i,t}^{F}+x_{ji,t}=p_{ji,t}-\tau_{i,t}-p_{i,t}^{F}+(p_{j,t}^{X}-p_{ji,t})+x_{j,t}=-\tau_{i,t}-p_{i,t}^{F}+p_t^{C}+(w_t-p_t^{C})+n_{j,t}=-\tau_{i,t}-p_{i,t}^{F}+p_t^{C}+c_t+\varphi n_t+n_{j,t}$. Therefore, we have that:

(A.13)

$$\begin{aligned}\sum_{i=1}^{I}\sum_{j=1}^{I}\lambda_j\nu_{ji}(1-\alpha_j)\zeta_{ji}\left(x_{ji,t}^{H,*}-x_{ji,t}^{F}\right)&=\sum_{i=1}^{I}\sum_{j=1}^{I}\lambda_j\nu_{ji}(1-\alpha_j)\zeta_{ji}\left(\tau_{i,t}+p_{i,t}^{F}-p_{i,t}^{H}-\varphi n_t-n_{j,t}\right)\\&=\sum_{i=1}^{I}\sum_{j=1}^{I}\lambda_j\nu_{ji}(1-\alpha_j)\zeta_{ji}\left(s_{i,t}-\varphi n_t-n_{j,t}\right)=\boldsymbol{\lambda}^{\top}\boldsymbol{\Omega}_F\boldsymbol{s}_t-\boldsymbol{\lambda}^{\top}\mathrm{diag}\left(\boldsymbol{\Omega}_F\mathbf{1}\right)\boldsymbol{n}_t-\omega_X\varphi\,y_t.\end{aligned}$$

In addition, we have made use of the fact that aggregate GDP equals the sales-weighted sum of sectoral value-added, and hence aggregate employment (using the labor market clearing condition $n_t=\sum_{i=1}^{I}\lambda_i\alpha_i n_{i,t}$), $y_t=\sum_{i=1}^{I}\lambda_i\alpha_i n_{i,t}=n_t$. Furthermore, $\omega_X=\sum_{i=1}^{I}\sum_{j=1}^{I}\lambda_j\nu_{ji}(1-\alpha_j)\zeta_{ji}$.

Next, plugging in (A.12) and (A.13) into (A.11) we obtain:

$$(1+\varphi\omega_X)\,y_t=c_t+(\boldsymbol{\beta}_F^{\top}+\boldsymbol{\lambda}^{\top}\boldsymbol{\Omega}_F)\boldsymbol{s}_t-\boldsymbol{\lambda}^{\top}\mathrm{diag}\left(\boldsymbol{\Omega}_F\mathbf{1}\right)\boldsymbol{n}_t. \tag{A.14}$$

Natural Equilibrium. In the natural allocation with flexible prices (denoted by superscript n) where prices equal marginal costs and nominal wages remain constant (Rubbo 2023), we have that $\boldsymbol{p}_t^{H,n}=\boldsymbol{\alpha}w_t^{n}+\boldsymbol{\Omega}\,\boldsymbol{p}_t^{H,n}+\boldsymbol{\Omega}_F\boldsymbol{s}_t^{n}$. Therefore, we can write:

$$\boldsymbol{p}_t^{H,n}=(\boldsymbol{I}-\boldsymbol{\Omega})^{-1}\boldsymbol{\Omega}_F\boldsymbol{s}_t^{n}=(\boldsymbol{I}-\boldsymbol{\Omega}_H)^{-1}\boldsymbol{\Omega}_F\left(\boldsymbol{\tau}_t+\boldsymbol{p}_t^{F,n}\right). \tag{A.15}$$

That is, in response to an increase in import prices, domestic prices adjust through a direct effect via its exposure to foreign markets (measured by $\boldsymbol{\Omega}_F$) and an indirect effect via IO linkages mediated by the domestic Leontief inverse $(\boldsymbol{I}-\boldsymbol{\Omega}_H)^{-1}$.

Note also that making use of this results and of the relationship between consumption shares and Domar weights in steady state, $\boldsymbol{\lambda}^\top = \boldsymbol{\beta}^\top (\boldsymbol{I}-\boldsymbol{\Omega})^{-1}$, we have that the consumer price level in the flexible price equilibrium is given by:

(A.16)
$$p_t^{C,n} = \boldsymbol{\beta}^\top \boldsymbol{p}_t^{H,n} + \boldsymbol{\beta}_F^\top \boldsymbol{s}_t^n = \left[\boldsymbol{\beta}_F^\top + \boldsymbol{\beta}^\top (\boldsymbol{I}-\boldsymbol{\Omega})^{-1}\boldsymbol{\Omega}_F\right] \boldsymbol{s}_t^n = \left[\boldsymbol{\beta}_F^\top + \boldsymbol{\lambda}^\top (\boldsymbol{I}-\boldsymbol{\Omega})(\boldsymbol{I}-\boldsymbol{\Omega})^{-1}\boldsymbol{\Omega}_F\right] \boldsymbol{s}_t^n = (\boldsymbol{\beta}_F^\top + \boldsymbol{\lambda}^\top\boldsymbol{\Omega}_F)\boldsymbol{s}_t^n.$$

Next, writing the expression for real GDP (A.14) in natural terms and using (A.16) we obtain $(1+\varphi\omega_X)\,y_t^n = c_t^n + p_t^{C,n} - \boldsymbol{\lambda}^\top \mathrm{diag}\,(\boldsymbol{\Omega}_F\mathbf{1})\,\boldsymbol{n}_t^n$. In addition, make use of the labor supply condition with flexible prices and constant nominal wages, together with $y_t^n = n_t^n$ and $-\varphi\, y_t^n = c_t^n + p_t^{C,n}$ to obtain:

$$[1+\varphi(\omega_X+1)]\; y_t^n = -\boldsymbol{\lambda}^\top \mathrm{diag}\,(\boldsymbol{\Omega}_F\mathbf{1})\,\boldsymbol{n}_t^n \tag{A.17}$$

Phillips Curve in Gaps from Natural Equilibrium. First, we start by writing the labor supply condition (A.5), using $y_t = n_t$ together with (A.14) to substitute out c_t:

$$w_t - p_t^C = [1+\varphi(\omega_X+1)]\; y_t - \left(\boldsymbol{\beta}_F^\top + \boldsymbol{\lambda}^\top\boldsymbol{\Omega}_F\right)\boldsymbol{s}_t + \boldsymbol{\lambda}^\top \mathrm{diag}\,(\boldsymbol{\Omega}_F\mathbf{1})\,\boldsymbol{n}_t \tag{A.18}$$

Next, we substract $p_t^{C,n}$ from both sides and use (A.16) to obtain $w_t - \widetilde{p}_t^C = [1+\varphi(\omega_X+1)]\; y_t - \left(\boldsymbol{\beta}_F^\top + \boldsymbol{\lambda}^\top\boldsymbol{\Omega}_F\right)\widetilde{\boldsymbol{s}}_t + \boldsymbol{\lambda}^\top \mathrm{diag}\,(\boldsymbol{\Omega}_F\mathbf{1})\,\boldsymbol{n}_t$, where tildes denote deviations from the flexible price equilibrium: $\widetilde{x}_t = x_t - x_t^n$ for a variable x. Finally, make use of (A.17) and add and subtract $[1+\varphi(\omega_X+1)]\; y_t^n$ on the right-hand side to obtain:

$$w_t - \widetilde{p}_t^C = [1+\varphi(\omega_X+1)]\; \widetilde{y}_t - \left(\boldsymbol{\beta}_F^\top + \boldsymbol{\lambda}^\top\boldsymbol{\Omega}_F\right)\widetilde{\boldsymbol{s}}_t + \boldsymbol{\lambda}^\top \mathrm{diag}\,(\boldsymbol{\Omega}_F\mathbf{1})\,\widetilde{\boldsymbol{n}}_t \tag{A.19}$$

We next work with the stacked sectoral Phillips curves (A.4). Next, we subtract $\boldsymbol{\Delta\alpha}\, p_t^C$ on both sides, and use (A.6) to write $p_t^C = \boldsymbol{\beta}^\top \boldsymbol{p}_t^H + \boldsymbol{\beta}_F^\top(\boldsymbol{s}_t - \boldsymbol{\tau}_t) = \boldsymbol{\beta}^\top \boldsymbol{\pi}_t^H + \boldsymbol{\beta}^\top \boldsymbol{p}_{t-1}^H + \boldsymbol{\beta}_F^\top(\boldsymbol{s}_t - \boldsymbol{\tau}_t)$, obtaining:

(A.20)
$$(\boldsymbol{I} - \boldsymbol{\Delta\Omega} - \boldsymbol{\Delta\alpha\beta}^\top)\boldsymbol{\pi}_t^H = \boldsymbol{\Delta}\left[\boldsymbol{\alpha}(w_t - p_t^C) - (\boldsymbol{I} - \boldsymbol{\Omega} - \boldsymbol{\alpha\beta}^\top)\,\boldsymbol{p}_{t-1}^H + (\boldsymbol{\Omega}_F + \boldsymbol{\alpha\beta}_F^\top)\boldsymbol{s}_t\right] + \beta(\boldsymbol{I}-\boldsymbol{\Delta})\mathbb{E}_t\boldsymbol{\pi}_{t+1}^H$$

Adding and subtracting $p_t^{C,n}$ on the right-hand side, using (A.16), we obtain $(\boldsymbol{I}-\boldsymbol{\Delta\Omega}-\boldsymbol{\Delta\alpha\beta}^\top)\boldsymbol{\pi}_t^H = \boldsymbol{\Delta}\left[\boldsymbol{\alpha}(w_t - \widetilde{p}_t^C) - (\boldsymbol{I} - \boldsymbol{\Omega} - \boldsymbol{\alpha\beta}^\top)\,\boldsymbol{p}_{t-1}^H + (\boldsymbol{\Omega}_F + \boldsymbol{\alpha\beta}_F^\top)\boldsymbol{s}_t - \boldsymbol{\alpha\beta}^\top \boldsymbol{p}_t^{H,n} - \boldsymbol{\alpha\beta}_F^\top\boldsymbol{s}_t^n\right] + \beta(\boldsymbol{I}-\boldsymbol{\Delta})\mathbb{E}_t\boldsymbol{\pi}_{t+1}^H$. Next, adding and subtracting $(\boldsymbol{I}-\boldsymbol{\Omega})\,\boldsymbol{p}_t^{H,n}$ on the right-hand side and using the fact that $(\boldsymbol{I}-\boldsymbol{\Omega})\,\boldsymbol{p}_t^{H,n} = \boldsymbol{\Omega}_F\boldsymbol{s}_t^n$ to obtain $(\boldsymbol{I} - \boldsymbol{\Delta\Omega} - \boldsymbol{\Delta\alpha\beta}^\top)\boldsymbol{\pi}_t^H = \boldsymbol{\Delta}\left[\boldsymbol{\alpha}(w_t - \widetilde{p}_t^C) - (\boldsymbol{I} - \boldsymbol{\Omega} - \boldsymbol{\alpha\beta}^\top)(\boldsymbol{p}_{t-1}^H - \boldsymbol{p}_t^{H,n}) + (\boldsymbol{\Omega}_F + \boldsymbol{\alpha\beta}_F^\top)\widetilde{\boldsymbol{s}}_t\right] + \beta(\boldsymbol{I}-\boldsymbol{\Delta})\mathbb{E}_t\boldsymbol{\pi}_{t+1}^H$. Finally, using $\boldsymbol{p}_t^{H,n} = (\boldsymbol{I}-\boldsymbol{\Omega}_H)^{-1}\boldsymbol{\Omega}_F(\boldsymbol{\tau}_t+\boldsymbol{p}_t^{F,n})$ we obtain $(\boldsymbol{I}-\boldsymbol{\Delta\Omega}-\boldsymbol{\Delta\alpha\beta}^\top)\boldsymbol{\pi}_t^H = \boldsymbol{\Delta}\left\{\boldsymbol{\alpha}(w_t-\widetilde{p}_t^C)-(\boldsymbol{I}-\boldsymbol{\Omega}-\boldsymbol{\alpha\beta}^\top)\left[\boldsymbol{p}_{t-1}^H-(\boldsymbol{I}-\boldsymbol{\Omega}_H)^{-1}\boldsymbol{\Omega}_F(\boldsymbol{\tau}_t+\boldsymbol{p}_t^{F,n})\right]+(\boldsymbol{\Omega}_F+\boldsymbol{\alpha\beta}_F^\top)\widetilde{\boldsymbol{s}}_t\right\} + \beta(\boldsymbol{I}-\boldsymbol{\Delta})\mathbb{E}_t\boldsymbol{\pi}_{t+1}^H$. Next, plugging in the expression for the real wage gap (A.19) into (A.20) to obtain:

$$(\boldsymbol{I}-\boldsymbol{\Delta\Omega}-\boldsymbol{\Delta\alpha\beta}^\top)\boldsymbol{\pi}_t^H = \boldsymbol{\Delta\alpha}\left[1+\varphi(\omega_X+1)\right]\widetilde{y}_t-\boldsymbol{\Delta}(\boldsymbol{I}-\boldsymbol{\Omega}-\boldsymbol{\alpha\beta}^\top)\left[\boldsymbol{p}_{t-1}^H-(\boldsymbol{I}-\boldsymbol{\Omega}_H)^{-1}\boldsymbol{\Omega}_F(\boldsymbol{\tau}_t+\boldsymbol{p}_t^{F,n})\right]$$
$$+\boldsymbol{\Delta\alpha}\boldsymbol{\lambda}^\top\mathrm{diag}\left(\boldsymbol{\Omega}_F\mathbf{1}\right)\widetilde{\boldsymbol{n}}_t+\boldsymbol{\Delta}(\boldsymbol{I}-\boldsymbol{\alpha}\boldsymbol{\lambda}^\top)\boldsymbol{\Omega}_F\widetilde{\boldsymbol{s}}_t+\beta(\boldsymbol{I}-\boldsymbol{\Delta})\mathbb{E}_t\boldsymbol{\pi}_{t+1}^H \tag{A.21}$$

Social Planner's Problem. We now seek to obtain the natural equilibrium variables as a function of exogenous variables. To do so, we first solve the social planner problem. The planner faces the following problem:

$$\max_{\{C_{i,t}^{H,n},C_{i,t}^{F,n},N_{i,t}^n,X_{ji,t}^{H,n},X_{ji,t}^{F,n},\mathcal{E}_t^n,D_{t+1}^n\}_{i=1,j=1}^I}\mathbb{E}_t\sum_{t=0}^{\infty}\beta^t\left\{\log C_t^n-\left(\sum_{i=1}^I N_{i,t}^n\right)^{1+\varphi}/(1+\varphi)\right\}$$

subject to the set of market clearing conditions and the aggregate resource constraint:

$$(N_{i,t}^n)^{\alpha_i}(X_{i,t}^n)^{1-\alpha_i}\geq C_{i,t}^{H,n}+\zeta_i\frac{\mathcal{E}_t^nP_{i,t}^{F,*,n}}{\mathcal{E}_t^nP_{i,t}^{H,*,n}}C_{i,t}^{*,n}+\sum_{j=1}^I\left(\zeta_{ji}\frac{\mathcal{E}_t^nP_{i,t}^{F,*,n}}{\mathcal{E}_t^nP_{i,t}^{H,*,n}}X_{ji,t}^{*,n}+X_{ji,t}^{H,n}\right)\quad\forall i\in I,t \tag{A.22}$$

$$\sum_{i=1}^I\mathcal{E}_t^nP_{i,t}^{F,*,n}\left[\zeta_iC_{i,t}^{*,n}-(1+\tau_{i,t})C_{i,t}^{F,n}+\sum_{j=1}^I\left(\zeta_{ji}X_{ji,t}^{*,n}-(1+\tau_{i,t})X_{ji,t}^{F,n}\right)\right]\geq\mathbb{E}_tQ_{t,t+1}^nD_{t+1}^n-D_t^n\quad\forall t \tag{A.23}$$

together with definitions of the consumption and intermediate goods' aggregators.

Let $\xi_{i,t}$ and ξ_t denote the lagrange multipliers on the constraints (A.22) and (A.23), respectively. The first-order conditions of the planner's problem for $\{C_{i,t}^{H,n},C_{i,t}^{F,n},N_{i,t}^n,X_{ji,t}^{H,n},X_{ji,t}^{F,n}\}_{i=1,j=1}^I$ are, respectively, given by:

$$\left(\sum_{j=1}^I N_{j,t}^n\right)^{\varphi}=\xi_{i,t}\alpha_i\frac{Y_{i,t}^n}{N_{i,t}^n} \tag{A.24}$$

$$\frac{1}{C_t^n}\frac{\partial C_t^n}{\partial C_{i,t}^n}\frac{\partial C_{i,t}^n}{\partial C_{i,t}^{H,n}}=\xi_{i,t} \tag{A.25}$$

(A.26) $$\frac{1}{C_t^n}\frac{\partial C_t^n}{\partial C_{i,t}^n}\frac{\partial C_{i,t}^n}{\partial C_{i,t}^{F,n}} = \xi_t(1+\tau_{i,t})\mathcal{E}_t^n P_{i,t}^{F,*,n}$$

(A.27) $$\xi_{j,t}(1-\alpha_j)\frac{Y_{j,t}^n}{X_{j,t}^n}\frac{\partial X_{j,t}^n}{\partial X_{ji,t}^n}\frac{\partial X_{ji,t}^n}{\partial X_{ji,t}^{H,n}} = \xi_{i,t}$$

(A.28) $$\xi_{j,t}(1-\alpha_j)\frac{Y_{j,t}^n}{X_{j,t}^n}\frac{\partial X_{j,t}^n}{\partial X_{ji,t}^n}\frac{\partial X_{ji,t}^n}{\partial X_{ji,t}^{F,n}} = \xi_t(1+\tau_{i,t})\mathcal{E}_t^n P_{i,t}^{F,*,n}$$

(A.29) $$\xi_t\mathbb{E}_t Q_{t,t+1}^n = \mathbb{E}_t\xi_{t+1}$$

Employing the definitions of the intermediate goods' and consumption aggregators, we can rewrite the first-order (A.25) - (A.28) conditions as:

(A.30) $$\beta_i(1-\zeta_i) = \xi_{i,t}C_{i,t}^{H,n}$$

(A.31) $$\beta_i\zeta_i = \xi_t(1+\tau_{i,t})\mathcal{E}_t^n P_{F,t}^{*,n}C_{i,t}^{F,n}$$

(A.32) $$\xi_{j,t}(1-\alpha_j)Y_{j,t}^n\nu_{ji}(1-\zeta_{ji}) = \xi_{i,t}X_{ji,t}^{H,n}$$

(A.33) $$\xi_{j,t}(1-\alpha_j)Y_{j,t}^n\nu_{ji}\zeta_{ji} = \xi_t(1+\tau_{i,t})\mathcal{E}_t^n P_{F,t}^{*,n}X_{ji,t}^{F,n}$$

Combining (A.24) and (A.25) we have that:

(A.34) $$(N_{i,t}^n)^{\varphi} = \beta_i(1-\zeta_i)\alpha_i Y_{i,t}^n/(N_{i,t}^n C_{i,t}^{H,n}).$$

Noting that in the flexible price equilibrium we have that $\beta_i(1-\zeta_i) = P_{i,t}^{H,n}C_{i,t}^{H,n}/(P_t^{C,n}C_t^n)$ and using the labor supply condition, we obtain that (A.34) is equivalent to the labor demand condition of firms.

Next, combining (A.24) and (A.27) we have that:

(A.35) $$\beta_j(1-\zeta_j)(1-\alpha_j)\nu_{ji}(1-\zeta_{ji})Y_{j,t}^n/(X_{ji,t}^{H,n}C_{j,t}^{H,n}) = \beta_i(1-\zeta_i)/C_{i,t}^{H,n}.$$

Again, noting that in the flexible price equilibrium we have that $\beta_j(1-\zeta_j) = P_{j,t}^{H,n}C_{j,t}^{H,n}/(P_t^{C,n}C_t^n)$ and $\beta_i(1-\zeta_i) = P_{i,t}^{H,n}C_{i,t}^{H,n}/(P_t^{C,n}C_t^n)$ we observe that (A.35) is equivalent to the intermediate goods demand of domestic goods condition of firms.

Next, inserting (A.25) and (A.26) in (A.28), we obtain:

(A.36) $$\beta_j(1-\zeta_j)\frac{1}{C_{j,t}^{H,n}}(1-\alpha_j)\frac{Y_{j,t}^n}{X_{ji,t}^{F,n}}\nu_{ji}\zeta_{ji} = \beta_i\zeta_i\frac{1}{C_{i,t}^{F,n}}$$

and again noting that in the flexible price equilibrium we have that $\beta_j(1-\zeta_j)\frac{1}{C^{H,n}_{j,t}} = \frac{P^{H,n}_{j,t}}{P^{C,n}_t C^n_t}$ and $\beta_i\zeta_i\frac{1}{C^{F,n}_{i,t}} = \frac{P^{F,n}_{i,t}}{P^{C,n}_t C^n_t}$ we observe that (A.36) is equivalent to the intermediate goods demand of foreign goods condition of firms.

Finally, using (A.25) into (A.29) to obtain:

$$\mathbb{E}_t Q^n_{t,t+1} = \mathbb{E}_t \frac{C^{F,n}_{i,t}}{C^{F,n}_{i,t+1}} \frac{(1+\tau_{i,t})\mathcal{E}^n_t P^{F,*,n}_{i,t}}{(1+\tau_{i,t+1})\mathcal{E}^n_t P^{F,*,n}_{i,t+1}} \tag{A.37}$$

and again noting that in the flexible price equilibrium we have that $C^{F,n}_{i,t}(1+\tau_{i,t})\mathcal{E}^n_t P^{F,*,n}_{i,t} = \beta_i\zeta_i P^{C,n}_t C^n_t$ we observe that (A.37) is equivalent to Euler Equation in the flexible price equilibrium.

Next, note that under the following definitions of the Lagrange multipliers:

$$P^{C,n}_t C^n_t \xi_{i,t} = P^{H,n}_{i,t} \tag{A.38}$$

$$P^{C,n}_t C^n_t \xi_t = 1 \tag{A.39}$$

and using the labor supply condition $W_t/P^C_t = C^n_t N^{\varphi}_t$, we have that the first-order conditions coincide with the first-order conditions of households and firms in the flexible price equilibrium:

$$N^n_{i,t} W^n_t = P^{H,n}_{i,t}\alpha_i Y^n_{i,t} \tag{A.40}$$

$$\beta_i(1-\zeta_i) = \frac{P^{H,n}_{i,t} C^{H,n}_{i,t}}{P^{C,n}_t C^n_t} \tag{A.41}$$

$$\beta_i\zeta_i = \frac{P^F_{i,t} C^{F,n}_{i,t}}{P^{C,n}_t C^n_t} \tag{A.42}$$

$$P^{H,n}_{j,t}(1-\alpha_j)Y^n_{j,t}\nu_{ji}(1-\zeta_{ji}) = P^{H,n}_{i,t} X^{H,n}_{ji,t} \tag{A.43}$$

$$P^{H,n}_{j,t}(1-\alpha_j)Y^n_{j,t}\nu_{ji}\zeta_{ji} = P^{F,n}_{i,t} X^{F,n}_{ji,t} \tag{A.44}$$

$$\mathbb{E}_t Q^n_{t,t+1} = \mathbb{E}_t\left[P^n_{C,t+1} C^n_{t+1}/(P^{C,n}_t C^n_t)\right] \tag{A.45}$$

Finally, the first-order condition of the planner with respect the exchange rate is given by:

$$\xi_t \sum_i P^{F,*,n}_{i,t}\left\{\zeta_i C^{*,n}_{i,t} - (1+\tau_{i,t})C^{F,n}_{i,t} + \sum_{j=1}^{I}\left[\zeta_{ji}X^{*,n}_{ji,t} - (1+\tau_{i,t})X^{F,n}_{ji,t}\right]\right\} = 0$$

Note that since $\xi_t = 1/(P_t^{C,n} C_t^n) > 0$ we have that nominal exports are equal to zero at all times under the planner's allocation:

$$\text{(A.46)} \qquad \sum_i \mathcal{E}_t^n P_{i,t}^{F,*,n} \left\{ \zeta_i C_{i,t}^{*,n} - (1+\tau_{i,t}) C_{i,t}^{F,n} + \sum_{j=1}^{I} \left[\zeta_{ji} X_{ji,t}^{*,n} - (1+\tau_{i,t}) X_{ji,t}^{F,n} \right] \right\} = 0.$$

To see the implications of this condition for the flexible price allocation, we linearize equation (A.46) to obtain:

$$\text{(A.47)} \quad -(\beta_F^\top + \lambda^\top \Omega_F) s_t^n + \sum_{i=1}^{I} \beta_i \zeta_i \left(c_{i,t}^{H,*,n} - c_{i,t}^{F,n} \right) + \sum_{i=1}^{I} \sum_{j=1}^{I} (1-\alpha_j) \nu_{ji} \zeta_{ji} \left(x_{ji,t}^{H,*,n} - x_{ji,t}^{F,n} \right) = 0.$$

Using our derivation from the expressions (A.12)-(A.13), we can write (A.47) as $-(\beta_F^\top + \lambda^\top \Omega_F) s_t^n + (\beta_F^\top + \lambda^\top \Omega_F) s_t^n - \omega_X \varphi n_t^n - \lambda^\top \text{diag}(\Omega_F \mathbf{1})\, \boldsymbol{n}_t^n = 0$, which implies that:

$$\text{(A.48)} \qquad -\lambda^\top \text{diag}(\Omega_F \mathbf{1})\, \boldsymbol{n}_t^n = \omega_X \varphi n_t^n$$

Next, using (A.48) into the expression for real GDP (A.14) we have that $y_t^n = c_t^n + (\beta_F^\top + \lambda^\top \Omega_F) s_t^n$, and using that in the flexible price equilibrium $p_t^{C,n} = (\beta_F^\top + \lambda^\top \Omega_F) s_t^n$ (equation A.16), together with the labor supply condition, we obtain $y_t^n = -\varphi\, y_t^n$, which only holds if (34) is satisfied.

In addition, using the risk-sharing condition $c_t^n = e_t^n - p_t^{C,n}$ we have that $e_t^n = 0$. Under these conditions, we obtain that the response of domestic prices in the flexible price equilibrium is given by $\boldsymbol{p}_t^{H,n} = (\boldsymbol{I} - \Omega_H)^{-1} \Omega_F \boldsymbol{\tau}_t$, and that the response of the ToT is given by:

$$\text{(A.49)} \qquad \boldsymbol{s}_t^n = \left[\boldsymbol{I} - (\boldsymbol{I} - \Omega_H)^{-1} \Omega_F \right] \boldsymbol{\tau}_t.$$

We next show that $y_t^n = 0$—and hence, from the labor supply condition, $p_t^{C,n} + c_t^n = 0$—implies that sectoral-level employment remains constant (that is, $\boldsymbol{n}_t^n = \mathbf{0}$). To see this, we start from the market clearing condition for good i, $Y_{i,t}^n = C_{i,t}^{H,n} + C_{i,t}^{H,*,n} + \sum_j \left(X_{ji,t}^{H,n} + X_{ji,t}^{H,*,n} \right)$. Substituting out the demands for good i of domestic and foreign households and firms, we have that: $Y_{i,t}^n = \beta_i \frac{P_{C,t}^n C_t^n}{P_{i,t}^{H,n}} + \sum_{j=1}^{I} \left(\nu_{ji}(1-\zeta_{ji}) \frac{1-\alpha_j}{\alpha_j} \frac{P_{C,t}^n C_t^n}{P_{i,t}^{H,n} C_t^n} N_t^\varphi N_{j,t} + \nu_{ji} \zeta_{ji} \frac{1-\alpha_j}{\alpha_j} \frac{P_{C,t}^n C_t^n}{P_{i,t}^{H,n} C_t^n} (N_t^{*,n})^\varphi N_{j,t}^{*,n} \right)$. Linearizing this expression, and dividing by steady state GDP, we have that:

(A.50)

$$\lambda_i y_{i,t}^n = \beta_i \left(p_t^{C,n} + c_t^n - p_{i,t}^{H,n} \right) +$$

$$\sum_{j=1}^{I}\left[\lambda_j\nu_{ji}(1-\zeta_{ji})(1-\alpha_j)(p_t^{C,n}+c_t^n-p_{i,t}^{H,n}+\varphi n_t^n+n_{j,t}^n)+\lambda_j\nu_{ji}\zeta_{ji}(1-\alpha_j)(p_t^{C,n}+c_t^n-p_{i,t}^{H,n})\right]$$

Next, using that in the flexible price equilibrium $p_t^{C,n}+c_t^n=-\varphi n_t^n=0$, and collecting terms, the above expression simplifies to $\lambda_i y_{i,t}^n=-\left[\beta_i+\sum_{j=1}^I\lambda_j\nu_{ji}(1-\alpha_j)\right]p_{i,t}^{H,n}+\sum_{j=1}^I\lambda_j\nu_{ji}(1-\zeta_{ji})(1-\alpha_j)n_{j,t}^n$. In addition, note that at the steady state we have $\lambda_i=[\beta_i+\sum_{j=1}^I\lambda_j\nu_{ji}(1-\alpha_j)]$, leading to $\lambda_i(p_{i,t}^{H,n}+y_{i,t}^n)=\sum_{j=1}^I\lambda_j\nu_{ji}(1-\zeta_{ji})(1-\alpha_j)n_{j,t}^n$. Using the result that with flexible prices we have that $p_{i,t}^{H,n}+y_{i,t}^n=n_{i,t}^n$ from the labor demand condition of firms and stacking over i we have that:

$$\boldsymbol{\Lambda}\boldsymbol{n}_t^n=\boldsymbol{\Omega}_H^\top\boldsymbol{\Lambda}\boldsymbol{n}_t^n \tag{A.51}$$

where $\boldsymbol{\Lambda}=\text{diag}(\lambda)$. Since both $\boldsymbol{\Lambda}$ and $(\boldsymbol{I}-\boldsymbol{\Omega_H})$ are invertible, the only solution for (A.51) is $\boldsymbol{n}_t^n=\boldsymbol{0}$.

Finally, using $\boldsymbol{e}_t^n=\boldsymbol{0}$ in the flexible price equilibrium we can write $\boldsymbol{p}_t^{F,n}=\boldsymbol{0}$. Introducing this expression into the Phillips curve (A.21), $(\boldsymbol{I}-\boldsymbol{\Delta\Omega}-\boldsymbol{\Delta\alpha}\beta^\top)\boldsymbol{\pi}_t^H=\boldsymbol{\Delta\alpha}\left[1+\varphi(\omega_X+1)\right]\widetilde{y}_t+\boldsymbol{\Delta\alpha}\lambda^\top\text{diag}\,(\boldsymbol{\Omega}_F\boldsymbol{1})\,\widetilde{\boldsymbol{n}}_t+\boldsymbol{\Delta}(\boldsymbol{I}-\boldsymbol{\alpha}\lambda^\top)\boldsymbol{\Omega}_F\widetilde{\boldsymbol{s}}_t+\beta(\boldsymbol{I}-\boldsymbol{\Delta})\mathbb{E}_t\boldsymbol{\pi}_{t+1}^H-\boldsymbol{\Delta}(\boldsymbol{I}-\boldsymbol{\Omega}-\boldsymbol{\alpha}\beta^\top)\,\boldsymbol{p}_{t-1}^H+\boldsymbol{\Delta}(\boldsymbol{I}-\boldsymbol{\Omega}-\boldsymbol{\alpha}\beta^\top)\,(\boldsymbol{I}-\boldsymbol{\Omega}_H)^{-1}\,\boldsymbol{\Omega}_F\boldsymbol{\tau}_t$. Following the same transformations as in Rubbo (2023) to invert the term $(\boldsymbol{I}-\boldsymbol{\Delta\Omega}-\boldsymbol{\Delta\alpha}\beta^\top)$, we obtain

$$\boldsymbol{\pi}_t^H=\mathcal{B}\,(1+\varphi+\varphi\lambda^\top\boldsymbol{\Omega}_F\boldsymbol{1})\,\widetilde{y}_t+\mathcal{B}\lambda^\top\text{diag}\,(\boldsymbol{\Omega}_F\boldsymbol{1})\,\widetilde{\boldsymbol{n}}_t+\mathcal{V}(\boldsymbol{I}-\boldsymbol{\Omega})^{-1}\boldsymbol{\Omega}_F\widetilde{\boldsymbol{s}}_t-\mathcal{V}\chi_t+\beta(\boldsymbol{I}-\mathcal{V})\mathbb{E}_t\boldsymbol{\pi}_{t+1}^H. \tag{A.52}$$

Consider now the market clearing condition (A.50) in the rigid-price equilibrium, $\lambda_i y_{i,t}=\left[\beta_i+\sum_{j=1}^I\lambda_j\nu_{ji}(1-\alpha_j)\right]\left(p_t^C+c_t-p_{i,t}^H\right)+\sum_{j=1}^I\lambda_j\nu_{ji}(1-\zeta_{ji})(1-\alpha_j)(\varphi n_t+n_{j,t})=\lambda_i\left(p_t^C+c_t-p_{i,t}^H\right)+\sum_{j=1}^I\lambda_j\nu_{ji}(1-\zeta_{ji})(1-\alpha_j)(\varphi n_t+n_{j,t})$, where we have used $\lambda_i=\beta_i+\sum_{j=1}^I\lambda_j\nu_{ji}(1-\alpha_j)$. Furthermore, we can write $p_t^C+c_t-p_{i,t}^H=e_t-p_{i,t}^H=p_{i,t}^F-p_{i,t}^H=s_{i,t}-\tau_{i,t}$, where we have used (A.7). Thus, we can write the previous expression as

$$\lambda_i y_{i,t}=\lambda_i\left(s_{i,t}-\tau_{i,t}\right)+\sum_{j=1}^I\lambda_j\nu_{ji}(1-\zeta_{ji})(1-\alpha_j)(\varphi n_t+n_{j,t}) \tag{A.53}$$

Log-linearizing the production function (22)-(23), $y_{i,t}=\alpha_i n_{i,t}+\sum_{j=1}^I(1-\alpha_i)\nu_{ij}(1-\zeta_{ij})x_{ij,t}^H+\sum_{j=1}^I(1-\alpha_i)\nu_{ij}\zeta_{ij}x_{ij,t}^F=\alpha_i n_{i,t}+\sum_{j=1}^I(1-\alpha_i)\nu_{ij}x_{ij,t}^H-\sum_{j=1}^I(1-\alpha_i)\nu_{ij}\zeta_{ij}s_{j,t}=\alpha_i n_{i,t}+\sum_{j=1}^I(1-\alpha_i)\nu_{ij}(s_{j,t}-\tau_{j,t}+\varphi n_t+n_{i,t})-\sum_{j=1}^I(1-\alpha_i)\nu_{ij}\zeta_{ij}s_{j,t}=n_{i,t}+(1-\alpha_i)\varphi n_t+(1-\alpha_i)\sum_{j=1}^I\nu_{ij}(1-\zeta_{ij})s_{j,t}-(1-\alpha_i)\sum_{j=1}^I\nu_{ij}\tau_{j,t}$, where the second equality made use of the international input demand curve $x_{ij,t}^F-x_{ij,t}^H=$

$p^H_{j,t} - (p^F_{j,t} + \tau_{j,t}) = -s_{j,t}$, and the third equality made use of the domestic input demand curve $x^H_{ij,t} = x_{ij,t} + p_{ij,t} - p^H_{j,t} = p^X_{i,t} + x_{i,t} - p^H_{j,t} = w_t + n_{i,t} - p^H_{j,t} = p^C_t + c_t + \varphi n_t + n_{i,t} - p^H_{j,t} = p^F_{j,t} + \varphi n_t + n_{i,t} - p^H_{j,t} = \varphi n_t + n_{i,t} + s_{j,t} - \tau_{j,t}$.

Pre-multiplying this last expression by λ_i, and combining it with (A.53), we can write $\lambda_i[n_{i,t} + (1-\alpha_i)\varphi n_t] - \sum_{j=1}^I \lambda_j \nu_{ji}(1-\zeta_{ji})(1-\alpha_j)(\varphi n_t + n_{j,t}) = \lambda_i \left[s_{i,t} - (1-\alpha_i)\sum_{j=1}^I \nu_{ij}(1-\zeta_{ij})s_{j,t} - \tau_{i,t} + (1-\alpha_i)\sum_{j=1}^I \nu_{ij}\tau_{j,t}\right]$, which can be written in stacked form as $(\boldsymbol{I} - \boldsymbol{\Omega}_H^\top)\boldsymbol{\Lambda}\boldsymbol{n}_t = [\boldsymbol{\Lambda}\boldsymbol{\alpha} - (\boldsymbol{I} - \boldsymbol{\Omega}_H^\top)\boldsymbol{\lambda}]\varphi y_t + \boldsymbol{\Lambda}(\boldsymbol{I} - \boldsymbol{\Omega}_H)\boldsymbol{s}_t - \boldsymbol{\Lambda}(\boldsymbol{I} - \boldsymbol{\Omega})\boldsymbol{\tau}_t$. Rearranging, $\boldsymbol{n}_t = [\boldsymbol{\Lambda}^{-1}(\boldsymbol{I} - \boldsymbol{\Omega}_H^\top)^{-1}\boldsymbol{\Lambda}\boldsymbol{\alpha} - \mathbf{1}]\varphi y_t + \boldsymbol{\Lambda}^{-1}(\boldsymbol{I} - \boldsymbol{\Omega}_H^\top)^{-1}\boldsymbol{\Lambda}(\boldsymbol{I} - \boldsymbol{\Omega}_H)\boldsymbol{s}_t - \boldsymbol{\Lambda}^{-1}(\boldsymbol{I} - \boldsymbol{\Omega}_H^\top)^{-1}\boldsymbol{\Lambda}(\boldsymbol{I} - \boldsymbol{\Omega})\boldsymbol{\tau}_t = (\mathcal{N}_y - \mathbf{1})\varphi y_t + \mathcal{N}_s \boldsymbol{s}_t - \mathcal{N}_\tau \boldsymbol{\tau}_t$, where $\mathcal{N}_y = \left(\boldsymbol{I} - \boldsymbol{\Lambda}^{-1}\boldsymbol{\Omega}_H^\top\boldsymbol{\Lambda}\right)^{-1}\boldsymbol{\alpha}$, $\mathcal{N}_s = \left(\boldsymbol{I} - \boldsymbol{\Lambda}^{-1}\boldsymbol{\Omega}_H^\top\boldsymbol{\Lambda}\right)^{-1}(\boldsymbol{I} - \boldsymbol{\Omega}_H)$, $\mathcal{N}_\tau = \left(\boldsymbol{I} - \boldsymbol{\Lambda}^{-1}\boldsymbol{\Omega}_H^\top\boldsymbol{\Lambda}\right)^{-1}(\boldsymbol{I} - \boldsymbol{\Omega})$; and where the second equality uses the fact that, since $\boldsymbol{\Lambda}$ is diagonal and invertible, we have $\boldsymbol{\Lambda}^{-1}(\boldsymbol{I} - \boldsymbol{\Omega}_H^\top)^{-1}\boldsymbol{\Lambda} = \left(\boldsymbol{I} - \boldsymbol{\Lambda}^{-1}\boldsymbol{\Omega}_H^\top\boldsymbol{\Lambda}\right)^{-1}$. Rewriting this expression in terms of gaps from the flexible-price equilibrium, $\tilde{\boldsymbol{n}}_t = (\mathcal{N}_y - \mathbf{1})\varphi\tilde{y}_t + \mathcal{N}_s\tilde{\boldsymbol{s}}_t$. Inserting this expression into (A.52), we obtain (36).

Proof of Proposition 1. The Neumann series $(\mathbf{I} - \boldsymbol{\Omega}_H^\top)^{-1} = \sum_{n=0}^\infty (\boldsymbol{\Omega}_H^\top)^n$ converges, and each power $(\boldsymbol{\Omega}_H^\top)^n$ is entrywise non-negative because $\boldsymbol{\Omega}_H^\top \geq 0$ elementwise. Hence, $(\mathbf{I} - \boldsymbol{\Omega}_H^\top)^{-1} \geq 0$ entrywise. Premultiplying and postmultiplying by the diagonal matrices $\boldsymbol{\Lambda}^{-1}$ and $\boldsymbol{\Lambda}$, which have strictly positive diagonal elements, preserves entrywise non-negativity. It follows that $\boldsymbol{B} \equiv \boldsymbol{\Lambda}^{-1}(\mathbf{I} - \boldsymbol{\Omega}_H^\top)^{-1}\boldsymbol{\Lambda}$ is entrywise non-negative.

Consider now $\boldsymbol{\Theta} = \boldsymbol{\lambda}^\top \operatorname{diag}(\boldsymbol{\Omega}_F\mathbf{1})\,\boldsymbol{B}\,\boldsymbol{\alpha} = \sum_{j=1}^I \sum_{k=1}^I \lambda_j \left(\sum_{i=1}^I (\boldsymbol{\Omega}_F)_{ji}\right) B_{jk}\,\alpha_k$. Under weakly positive labor shares, each term $\lambda_j\left(\sum_i (\boldsymbol{\Omega}_F)_{ji}\right) B_{jk}\alpha_k$ in the double sum is therefore non-negative, and we obtain $\Theta \geq 0$.

To show strict positivity, note first that by the definition of $\mathbf{B}$, $B_{jk} = \left[\boldsymbol{\Lambda}^{-1}(\mathbf{I} - \boldsymbol{\Omega}_H^\top)^{-1}\boldsymbol{\Lambda}\right]_{jk} = \frac{\lambda_k}{\lambda_j}\left[(\mathbf{I} - \boldsymbol{\Omega}_H^\top)^{-1}\right]_{jk}$. Under actual openness, there exists an index $j^\star$ such that $\sum_{i=1}^I (\boldsymbol{\Omega}_F)_{j^\star i} > 0$ (sector $j^\star$ uses imported intermediate inputs), $\lambda_{j^\star} > 0$, and there exists an index $k^\star$ with $\alpha_{k^\star} > 0$ such that the $(j^\star, k^\star)$ entry of $(\mathbf{I} - \boldsymbol{\Omega}_H^\top)^{-1}$ is strictly positive. The latter implies $B_{j^\star k^\star} > 0$.

Combining these conditions, there exist indices $(j^\star, k^\star)$ such that $\lambda_{j^\star} > 0$, $\sum_{i=1}^I (\boldsymbol{\Omega}_F)_{j^\star i} > 0$, $B_{j^\star k^\star} > 0$, $\alpha_{k^\star} > 0$. The corresponding term in the sum for Θ, $\lambda_{j^\star}\left(\sum_{i=1}^I (\boldsymbol{\Omega}_F)_{j^\star i}\right) B_{j^\star k^\star}\alpha_{k^\star}$ is therefore strictly positive, while all other terms are non-negative. Consequently, $\Theta > 0$ under openness.

□

Proof of Proposition 2. Define $\boldsymbol{D} \equiv \operatorname{diag}(\boldsymbol{\Omega}_F\mathbf{1})$ and $\mathcal{N}_s \equiv \boldsymbol{D}^{-1}\boldsymbol{\Omega}_F$, so that $\boldsymbol{\Xi} \equiv \boldsymbol{D}\mathcal{N}_s - \boldsymbol{\Omega}_F$. We show that $\boldsymbol{\Xi}$ is hollow, i.e., $\Xi_{ii} = 0$ for all i.

Let i be arbitrary. Expanding the (i,i) entry of the product gives $\Xi_{ii} = \left[\boldsymbol{D}\mathcal{N}_s\right]_{ii} - (\Omega_F)_{ii} = \sum_{j=1}^I D_{ij}(\mathcal{N}_s)_{ji} - (\Omega_F)_{ii}$. Since $\boldsymbol{D}$ is diagonal, $D_{ij} = 0$ for $j \neq i$, and therefore $\Xi_{ii} = D_{ii}(\mathcal{N}_s)_{ii} - (\Omega_F)_{ii}$.

Next, note that the definition $\mathcal{N}_s = \boldsymbol{D}^{-1}\boldsymbol{\Omega}_F$ implies $\boldsymbol{D}\mathcal{N}_s = \boldsymbol{\Omega}_F$, or equivalently $\boldsymbol{D}^{-1}\boldsymbol{D}\mathcal{N}_s = \boldsymbol{D}^{-1}\boldsymbol{\Omega}_F$. Taking the (i,i) entry of $\mathcal{N}_s = \boldsymbol{D}^{-1}\boldsymbol{\Omega}_F$ yields $(\mathcal{N}_s)_{ii} = \sum_{j=1}^{I}(D^{-1})_{ij}(\Omega_F)_{ji}$. Again, $\boldsymbol{D}^{-1}$ is diagonal, so $(D^{-1})_{ij} = 0$ for $j \neq i$, hence $(\mathcal{N}_s)_{ii} = (D^{-1})_{ii}(\Omega_F)_{ii}$. Substituting into the expression for Ξ_{ii} gives $\Xi_{ii} = D_{ii}(D^{-1})_{ii}(\Omega_F)_{ii} - (\Omega_F)_{ii} = (\Omega_F)_{ii} - (\Omega_F)_{ii} = 0$. Since i was arbitrary, $\Xi_{ii} = 0$ for all i, and thus $\boldsymbol{D}\mathcal{N}_s - \boldsymbol{\Omega}_F$ is hollow.

□

Proof of Proposition 3. Let $\boldsymbol{\omega} \in \mathbb{R}_+^I$, $\boldsymbol{\omega} \neq \boldsymbol{0}$, and define a constant-weight linear inflation index $\pi_t^\star(\boldsymbol{\omega}) \equiv \boldsymbol{\omega}^\top \boldsymbol{\pi}_t^H / (\boldsymbol{\omega}^\top \boldsymbol{1})$. Premultiplying the sectoral Phillips curve (36) by $\boldsymbol{\omega}^\top$ and dividing by $\boldsymbol{\omega}^\top\boldsymbol{1}$ yields the index-level relation

(A.54)
$$\pi_t^\star(\boldsymbol{\omega}) = \frac{\boldsymbol{\omega}^\top \mathcal{B}}{\boldsymbol{\omega}^\top\boldsymbol{1}}\left(1 + \varphi + \varphi\boldsymbol{\lambda}^\top \mathrm{diag}\left(\boldsymbol{\Omega}_F\boldsymbol{1}\right)\mathcal{N}_y\right)\widetilde{y}_t \;+\; \frac{\boldsymbol{\omega}^\top\boldsymbol{\Psi}}{\boldsymbol{\omega}^\top\boldsymbol{1}}\widetilde{\boldsymbol{s}}_t \;-\; \frac{\boldsymbol{\omega}^\top\mathcal{V}}{\boldsymbol{\omega}^\top\boldsymbol{1}}\chi_t \;+\; \beta\frac{\boldsymbol{\omega}^\top(\boldsymbol{I}-\mathcal{V})}{\boldsymbol{\omega}^\top\boldsymbol{1}}\mathbb{E}_t\boldsymbol{\pi}_{t+1}^H.$$

Equation (A.54) shows that, generically, any constant-weight index inherits the inefficient-price residual term $-\boldsymbol{\omega}^\top\mathcal{V}/(\boldsymbol{\omega}^\top\boldsymbol{1})\,\chi_t$ from (36). Hence, a necessary condition for an index to be a "DCI" in the sense of Rubbo (2023) (i.e., to have a Phillips curve with no inefficient-price residual) is that its weights eliminate the residual contribution of χ_t.

In a production-network economy, Rubbo (2023) shows that the unique constant-weight linear inflation index whose Phillips curve contains no inefficient-price residual is the sales-and-rigidity-adjusted index with weights proportional to λ_i/κ_i.[29] Therefore, any constant-weight index that qualifies as a DCI must coincide (up to normalization) with $\pi_t^{\mathrm{DC}} \equiv \sum_{i=1}^{I}(\lambda_i/\kappa_i)\pi_{i,t}^H / \left(\sum_{j=1}^{I}\lambda_j/\kappa_j\right)$. Consequently, to establish non-existence it is sufficient to show that π_t^{DC} fails to deliver the Divine Coincidence in the open economy, except under $\boldsymbol{\lambda}^\top\mathrm{diag}(\boldsymbol{\Omega}_F\boldsymbol{1})\,\mathcal{N}_s = \boldsymbol{0}^\top$.

Define the mark-up gap as

(A.55)
$$\boldsymbol{\mu}_t = \widetilde{\boldsymbol{p}}_t^H - \widetilde{\mathrm{mc}}_t.$$

Rewriting the marginal cost curve (27) in gap terms, $\widetilde{\mathrm{mc}}_t = \boldsymbol{\alpha}\widetilde{w}_t + \boldsymbol{\Omega}\,\widetilde{\boldsymbol{p}}_t^H + \boldsymbol{\Omega}_F\widetilde{\boldsymbol{s}}_t$, and combining both equations,

(A.56)
$$\widetilde{\mathrm{mc}}_t = \boldsymbol{\alpha}\widetilde{w}_t + \boldsymbol{\Omega}(\boldsymbol{\mu}_t + \widetilde{\mathrm{mc}}_t) + \boldsymbol{\Omega}_F\widetilde{\boldsymbol{s}}_t = \boldsymbol{1}\widetilde{w}_t + (\boldsymbol{I}-\boldsymbol{\Omega})^{-1}\boldsymbol{\Omega}\boldsymbol{\mu}_t + (\boldsymbol{I}-\boldsymbol{\Omega})^{-1}\boldsymbol{\Omega}_F\widetilde{\boldsymbol{s}}_t$$

where the second equality used $(\boldsymbol{I}-\boldsymbol{\Omega})^{-1}\boldsymbol{\alpha} = \boldsymbol{1}$ by CRS.

In the open economy, the CPI is given by (A.6), which in gap terms can be written as $\widetilde{p}_t^C = \boldsymbol{\beta}^\top\widetilde{\boldsymbol{p}}_t^H + \boldsymbol{\beta}_F^\top\widetilde{\boldsymbol{s}}_t$. Inserting (A.55)-(A.56) into the CPI gap, $\widetilde{p}_t^C = \boldsymbol{\beta}^\top(\boldsymbol{\mu}_t + \widetilde{\mathrm{mc}}_t) + \boldsymbol{\beta}_F^\top\widetilde{\boldsymbol{s}}_t =$

[29] Formally, see Proposition 1 in Rubbo (2023).

$\boldsymbol{\beta}^\intercal\left[\boldsymbol{\mu}_t + \mathbf{1}\widetilde{w}_t + (\boldsymbol{I}-\boldsymbol{\Omega})^{-1}\boldsymbol{\Omega}\boldsymbol{\mu}_t + (\boldsymbol{I}-\boldsymbol{\Omega})^{-1}\boldsymbol{\Omega}_F\widetilde{\boldsymbol{s}}_t\right] + \boldsymbol{\beta}_F^\intercal\widetilde{\boldsymbol{s}}_t = \boldsymbol{\lambda}^\intercal\boldsymbol{\mu}_t + \widetilde{w}_t + (\boldsymbol{\beta}_F^\intercal + \boldsymbol{\lambda}^\intercal\boldsymbol{\Omega}_F)\widetilde{\boldsymbol{s}}_t$, where we have used $\boldsymbol{\beta}^\intercal\mathbf{1} = 1$ and $\boldsymbol{\beta}^\intercal + \boldsymbol{\lambda}^\intercal\boldsymbol{\Omega} = \boldsymbol{\lambda}^\intercal$. Using the above expression, one can write

$$\widetilde{w}_t - \widetilde{p}_t^C = -\boldsymbol{\lambda}^\intercal\boldsymbol{\mu}_t - (\boldsymbol{\beta}_F^\intercal + \boldsymbol{\lambda}^\intercal\boldsymbol{\Omega}_F)\widetilde{\boldsymbol{s}}_t \tag{A.57}$$

Consider now the labor supply condition (29), which can be written in gap terms as

$$\widetilde{w}_t - \widetilde{p}_t^C = \widetilde{c}_t + \varphi\widetilde{y}_t. \tag{A.58}$$

Similarly, (32) can be written in gap terms as

$$(1 + \varphi\boldsymbol{\lambda}^\intercal\mathrm{diag}\left(\boldsymbol{\Omega}_F\mathbf{1}\right)\mathcal{N}_y)\widetilde{y}_t = \widetilde{c}_t + \left[\boldsymbol{\beta}_F^\intercal + \boldsymbol{\lambda}^\intercal\left(\boldsymbol{\Omega}_F - \mathrm{diag}\left(\boldsymbol{\Omega}_F\mathbf{1}\right)\mathcal{N}_s\right)\right]\widetilde{\boldsymbol{s}}_t. \tag{A.59}$$

Introducing this last condition into the labor supply condition (A.58), $\widetilde{w}_t - \widetilde{p}_t^C = (1 + \varphi + \varphi\boldsymbol{\lambda}^\intercal\mathrm{diag}\left(\boldsymbol{\Omega}_F\mathbf{1}\right)\mathcal{N}_y)\widetilde{y}_t - \left[\boldsymbol{\beta}_F^\intercal + \boldsymbol{\lambda}^\intercal\left(\boldsymbol{\Omega}_F - \mathrm{diag}\left(\boldsymbol{\Omega}_F\mathbf{1}\right)\mathcal{N}_s\right)\right]\widetilde{\boldsymbol{s}}_t$. Inserting this last condition into (A.57), we can write

$$-\boldsymbol{\lambda}^\intercal\boldsymbol{\mu}_t = (1 + \varphi + \varphi\boldsymbol{\lambda}^\intercal\mathrm{diag}\left(\boldsymbol{\Omega}_F\mathbf{1}\right)\mathcal{N}_y)\widetilde{y}_t + \boldsymbol{\lambda}^\intercal\mathrm{diag}\left(\boldsymbol{\Omega}_F\mathbf{1}\right)\mathcal{N}_s\widetilde{\boldsymbol{s}}_t \tag{A.60}$$

Combining the risk-sharing condition (A.7) with the CPI definition (A.6), we can write $c_t = e_t - \boldsymbol{\beta}^\intercal\boldsymbol{p}_t^H - \boldsymbol{\beta}_F^\intercal\boldsymbol{s}_t = \boldsymbol{\beta}^\intercal\boldsymbol{p}_{F,t} - \boldsymbol{\beta}^\intercal\boldsymbol{p}_t^H - \boldsymbol{\beta}_F^\intercal\boldsymbol{s}_t = \boldsymbol{\beta}_H^\intercal\boldsymbol{s}_t - \boldsymbol{\beta}^\intercal\boldsymbol{\tau}_t$, where we have used $\boldsymbol{\beta}^\intercal\boldsymbol{p}_{F,t} = \sum_{i=1}^I \beta_i p_{Fi,t} = \sum_{i=1}^I \beta_i e_t = e_t\sum_{i=1}^I \beta_i = e_t$ and the definition of ToT (28). In gap terms, we can write $\widetilde{c}_t = \boldsymbol{\beta}_H^\intercal\widetilde{\boldsymbol{s}}_t$. Introducing this expression into (A.59), we can write (38), and thus (A.60) can be simplified to

$$-\boldsymbol{\lambda}^\intercal\boldsymbol{\mu}_t = \varphi\widetilde{y}_t + \boldsymbol{\lambda}^\intercal(\boldsymbol{I} - \boldsymbol{\Omega}_H)\widetilde{\boldsymbol{s}}_t \tag{A.61}$$

where we have used the steady-state identity $\boldsymbol{\beta}^\intercal + \boldsymbol{\lambda}^\intercal\boldsymbol{\Omega}_F = \boldsymbol{\lambda}^\intercal(\boldsymbol{I} - \boldsymbol{\Omega}_H)$.

Finally, consider the sectoral Phillips curves (26), which for industry i is given by $\pi_{i,t}^H = -\kappa_i\mu_{i,t} + \beta\mathbb{E}_t\pi_{i,t+1}^H$. Reorganizing terms, pre-multiplying by the Domar weight λ_i and summing across sectors, one can write $-\sum_{i=1}^I \lambda_i\mu_{i,t} = \sum_{i=1}^I \lambda_i/\kappa_i\left(\pi_{i,t}^H - \beta\mathbb{E}_t\pi_{i,t+1}^H\right)$. Making use of the definition of π_t^{DC}, we can thus write $-\boldsymbol{\lambda}^\intercal\boldsymbol{\mu}_t = \left(\sum_{j=1}^I \lambda_j/\kappa_j\right)\left(\pi_t^{\mathrm{DC}} - \beta\mathbb{E}_t\pi_{t+1}^{\mathrm{DC}}\right)$. Inserting this expression into (A.61), we obtain (37). Using (38), setting $\widetilde{y}_t = 0$ generally does not imply $\widetilde{\boldsymbol{s}}_t = \mathbf{0}$. Hence, $\pi_t^{\mathrm{DC}} = \left(\boldsymbol{\lambda}^\intercal\boldsymbol{\kappa}^{-1}\mathbf{1}\right)^{-1}\boldsymbol{\lambda}^\intercal\mathrm{diag}(\boldsymbol{\Omega}_F\mathbf{1})\mathcal{N}_s\widetilde{\boldsymbol{s}}_t + \beta\mathbb{E}_t\pi_{t+1}^{\mathrm{DC}}$, which does not imply $\pi_t^{\mathrm{DC}} = 0$ for all t unless $\boldsymbol{\lambda}^\intercal\mathrm{diag}(\boldsymbol{\Omega}_F\mathbf{1})\mathcal{N}_s = \mathbf{0}^\intercal$.

Since any constant-weight DCI must coincide with π_t^{DC} up to normalization, it follows that no constant-weight DCI index can simultaneously close the output gap and stabilize the index in the open economy unless $\boldsymbol{\lambda}^\intercal\mathrm{diag}(\boldsymbol{\Omega}_F\mathbf{1})\mathcal{N}_s = \mathbf{0}^\intercal$.

□

The One-Sector Phillips Curve. In the one-sector economy, we can write the Phillips curve as $\pi_{H,t} = \kappa(\mathrm{mc}_t - p_{H,t}) + \beta\mathbb{E}_t\pi_{H,t+1}$. Inserting the expression for nominal marginal costs (A.3) in the one-sector economy, $\mathrm{mc}_t = \alpha w_t + (1-\alpha)\,p_{H,t} + \omega_F s_t$—where we have made use of the CRS assumption $\omega_H = 1 - \alpha - \omega_F$—we can rewrite the Phillips curve as $\pi_{H,t} = \kappa[\alpha(w_t - p_{H,t}) + \omega_F s_t] + \beta\mathbb{E}_t\pi_{H,t+1}$. Inserting the one-sector expression for (A.6), $p_t^C = p_{H,t} + \beta_F s_t$, we can write $\pi_{H,t} = \kappa[\alpha(w_t - p_t^C) + (\alpha\beta_F + \omega_F)s_t] + \beta\mathbb{E}_t\pi_{H,t+1}$. Rewriting $p_t^C = \widetilde{p}_t^C + p_t^{C,n}$, and making use of (A.15) in the one-sector economy ($p_{H,t}^n = \omega_F/\alpha s_t^n$) and (A.16), which in the one-sector limit is given by $p_t^{C,n} = (\beta_F + \omega_F/\alpha)s_t^n$, we can write[30]

$$\pi_{H,t} = \kappa[\alpha(w_t - \widetilde{p}_t^C) + (\alpha\beta_F + \omega_F)\widetilde{s}_t] + \beta\mathbb{E}_t\pi_{H,t+1}.$$

Introducing (A.19), $w_t - \widetilde{p}_t^C = \left[1 + \varphi(\omega_X + 1)\right]\widetilde{y}_t - (\beta_F + \omega_F/\alpha)\,\widetilde{s}_t + \omega_F/\alpha\widetilde{n}_t$, where $\widetilde{y}_t = \widetilde{n}_t$ and having made use of the identity in footnote 30, we can write the Phillips curve as (35) (where we have used the identity $\omega_X = \boldsymbol{\lambda}^\top\boldsymbol{\Omega}_F\mathbf{1} = \omega_F/\alpha$).

Appendix B. Model Derivation and Log-linearization

In this section, we derive the model equilibrium conditions, outlining the final set of log-linearized equations. We further enlarge the theoretical model presented in the main text—which only contains exogenous perturbations to the foreign price wedges and the monetary policy shock—to accommodate the standard perturbations explored in the literature: an internal demand shock, sectoral TFP, sectoral price cost-push shocks, and wage cost-push shocks.

B.1. Households

Each household is made up of a continuum of members, each specialized in a different labor service, indexed by $g \in [0,1]$. Income is pooled within each household, acting as a risk-sharing mechanism. The per-period utility function (1) is modified to accommodate nominal wage rigidities and a discount factor disturbance. Namely,

$$U_t = \left[C_{k,t}^{1-\sigma}/(1-\sigma) - \int_0^1 \mathcal{N}_{gk,t}^{1+\varphi}/(1+\varphi)\,dg\right] Z_{k,t}, \tag{B.1}$$

[30] In the one-sector limit, we can thus write $\boldsymbol{\lambda}^\top\boldsymbol{\Omega}_F$ as ω_F/α.

where $\mathcal{N}_{gk,t}$ denotes the labor supply of labor service g, $Z_{k,t}$ is an exogenous preference shifter, and σ denotes the inverse of the intertemporal elasticity of substitution.[31]

Consumption Demand Curves. The optimal allocation between energy and non-energy goods is the result of a cost minimization programme $\min P^E_{k,t}C^E_{k,t} + P^M_{k,t}C^M_{k,t}$ subject to (40). Similarly, the optimal allocation between energy (non-energy) consumption is the result of a cost minimization programme $\min \sum_{i\in I_E} P^C_{ki,t}C_{ki,t}$ ($\min \sum_{i\in I_M} P^C_{ki,t}C_{ki,t}$) subject to (41). Finally, the optimal allocation between the different consumption goods is the result of a cost minimization programme $\min \sum_{l=1}^{K}(1+\tau_{kl\,i,t})P^k_{kl\,i,t}C_{kl\,i,t}$ subject to (43). The implied demand curves are given by

$$\text{(B.2)} \qquad P^E_{k,t} = P^C_{k,t}\left(\widetilde{\beta}_k C_{k,t}/C^E_{k,t}\right)^{\frac{1}{\gamma}}, \quad P^M_{k,t} = P^C_{k,t}\left[(1-\widetilde{\beta}_k)C_{k,t}/C^M_{k,t}\right]^{\frac{1}{\gamma}},$$

$$\text{(B.3)} \qquad P^C_{ki,t} = P^E_{k,t}\left(\widetilde{\nu}_{ki}C^E_{k,t}/C_{ki,t}\right)^{\frac{1}{\eta}} \quad \forall \quad i \in I_E, \quad P^C_{ki,t} = P^M_{k,t}\left(\widetilde{\upsilon}_{ki}C^M_{k,t}/C_{ki,t}\right)^{\frac{1}{\iota}} \quad \forall \quad i \in I_M,$$

$$\text{(B.4)} \qquad (1+\tau_{kl\,i,t})P^k_{kl\,i,t} = P^C_{ki,t}\left(\widetilde{\zeta}_{kl\,i}C_{ki,t}/C_{kl\,i,t}\right)^{\frac{1}{\delta}} \quad \forall \quad l \in K.$$

The log-linearized versions of the consumption demand curves (B.2)-(B.4) are given by

$$\text{(B.5)} \qquad \widehat{p}^E_{k,t} = \gamma^{-1}(\widehat{c}_{k,t} - \widehat{c}^E_{k,t}), \quad \widehat{p}^M_{k,t} = \gamma^{-1}(\widehat{c}_{k,t} - \widehat{c}^M_{k,t}),$$

$$\text{(B.6)} \qquad \widehat{p}^C_{ki,t} - \widehat{p}^E_{k,t} = \eta^{-1}(\widehat{c}^E_{k,t} - \widehat{c}_{ki,t}), \quad \widehat{p}^C_{ki,t} - \widehat{p}^M_{k,t} = \iota^{-1}(\widehat{c}^M_{k,t} - \widehat{c}_{ki,t}),$$

$$\text{(B.7)} \qquad \tau_{kl\,i,t} + \widehat{p}^k_{kl\,i,t} - \widehat{p}^C_{ki,t} = \delta^{-1}\left(\widehat{c}_{ki,t} - \widehat{c}_{kl\,i,t}\right),$$

where $\widehat{p}^E_{k,t} = p^E_{k,t} - p^C_{k,t}$, $\widehat{p}^M_{k,t} = p^M_{k,t} - p^C_{k,t}$, $\widehat{p}^C_{ki,t} = p^C_{ki,t} - p^C_{k,t}$, and $\widehat{p}^k_{kl\,i,t} = p^k_{kl\,i,t} - p^C_{k,t}$ are well-defined as a ratio of prices.[32]

Currency Pricing Paradigm. Under PCP, using the log-linearized version of (15), we obtain

$$\text{(B.8)} \quad \widehat{p}^k_{kl\,i,t} = p^k_{kl\,i,t} - p^C_{k,t} = e^l_{k,t} + p_{kl\,i,t} - p^C_{k,t} = e^l_{k,t} + (p_{l\,i,t} - p^C_{l,t}) + (p^C_{l,t} - p^C_{k,t}) = \widehat{p}_{l\,i,t} + q^l_{k,t},$$

$$\text{(B.9)} \quad \widehat{p}^k_{l\,ki,t} = p^k_{l\,ki,t} - p^C_{k,t} = p_{l\,ki,t} - p^C_{k,t} = p_{ki,t} - p^C_{k,t} = \widehat{p}_{ki,t},$$

where we have used that, under PCP, $p_{kl\,i,t} = p_{l\,i,t}$ since the law of one price (LOP) is satisfied; the transformation $\widehat{p}_{l\,i,t} = p_{l\,i,t} - p^C_{l,t}$; and the definition of the bilateral real exchange rate

[31] Each household takes as given labor income since wages are set by labor unions and employment is decided by firms. Thus, the only decisions made by the household are the optimal allocation of consumption expenditures among different good varieties across different countries, and the optimal intertemporal allocation of consumption.

[32] The price levels are not well-defined in the steady state, but their ratio is.

between country k and country m: $\mathcal{Q}^m_{k,t} = P^C_{m,t}\mathcal{E}^m_{k,t}/P^C_{k,t}$, which log-linearized takes the form

$$q^m_{k,t} = e^m_{k,t} + p^C_{m,t} - p^C_{k,t}. \tag{B.10}$$

Under LCP, using the log-linearized version of (16), we can write

(B.11)
$$\widehat{p}^k_{kl\,i,t} = p^k_{kl\,i,t} - p^C_{k,t} = \widehat{p}_{kl\,i,t},$$
(B.12)
$$\widehat{p}^k_{l\,ki,t} = p^k_{l\,ki,t} - p^C_{k,t} = e^l_{k,t} + p_{l\,ki,t} - p^C_{k,t} = (p_{l\,ki,t} - p^C_{l,t}) + e^l_{k,t} + p^C_{l,t} - p^C_{k,t} = q^l_{k,t} + \widehat{p}_{l\,ki,t},$$

where we have made use of (B.10), and of the definition $\widehat{p}_{kl\,i,t} = p_{kl\,i,t} - p^C_{k,t}$.

To summarize pricing, using (B.8), (B.11), (B.9), and (B.12), we have that

$$\widehat{p}^k_{kl\,i,t} = \begin{cases} q^l_{k,t} + \widehat{p}_{l\,i,t}, & \text{if PCP,} \\ \widehat{p}_{kl\,i,t}, & \text{if LCP.} \end{cases}, \qquad \widehat{p}^k_{l\,ki,t} = \begin{cases} \widehat{p}_{ki,t}, & \text{if PCP,} \\ q^l_{k,t} + \widehat{p}_{l\,ki,t}, & \text{if LCP.} \end{cases} \tag{B.13}$$

Consumption Baskets. The log-linearized consumption aggregator (40) is given by

$$\widehat{c}_{k,t} = \beta_k \widehat{c}^E_{k,t} + (1-\beta_k)\widehat{c}^M_{k,t}, \tag{B.14}$$

where $\beta_k = P^E_k C^E_k/(P^C_k C_k) = \widetilde{\beta}_k^{\frac{1}{\gamma}} \left(C^E_k/C_k\right)^{\frac{\gamma-1}{\gamma}}$ and $(1-\beta_k) = P^M_k C^M_k/(P^C_k C_k) = (1-\widetilde{\beta}_k)^{\frac{1}{\gamma}} \left(C^M_k/C_k\right)^{\frac{\gamma-1}{\gamma}}$ can be verified using the steady-state consumption aggregator (40) and the demand curves (B.2).

The log-linearized versions of the energy and non-energy consumption aggregators (41) are given by

$$\widehat{c}^E_{k,t} = \sum_{i\in I_E} \nu_{ki}\widehat{c}_{ki,t}, \quad \widehat{c}^M_{k,t} = \sum_{i\in I_M} \upsilon_{ki}\widehat{c}_{ki,t}, \tag{B.15}$$

where $\nu_{ki} = P^C_{ki}C_{ki}/(P^E_k C^E_k) = \widetilde{\nu}_{ki}^{\frac{1}{\eta}} \left(C_{ki}/C^E_k\right)^{\frac{\eta-1}{\eta}}$ and $\upsilon_{ki} = P^C_{ki}C_{ki}/(P^M_k C^M_k) = \widetilde{\upsilon}_{ki}^{\frac{1}{\iota}} \left(C_{ki}/C^M_k\right)^{\frac{\iota-1}{\iota}}$ can be verified using the steady-state energy and non-energy consumption aggregators (41) and the demand curves (B.3).

The log-linearized version of the final layer of the consumption aggregator, (43), is given by

$$\widehat{c}_{ki,t} = \sum_{l=1}^{K} \zeta_{kl\,i}\widehat{c}_{kl\,i,t}, \tag{B.16}$$

where $\zeta_{kl\,i} = P_{kl\,i}C_{kl\,i}/(P^C_{ki}C_{ki}) = \widetilde{\zeta}_{kl\,i}^{\frac{1}{\delta}}\left(C_{kl\,i}/C_{ki}\right)^{\frac{\delta-1}{\delta}}$ can be verified using the steady-state international consumption aggregator (43) and the consumption demand curves (B.4).

Price Indices. The different price indices can be derived by combining the consumption demand curves previously derived with the different consumption aggregators. The consumption price index, the energy and non-energy price index, and the consumption import price index are given by $P^C_{k,t} = \left[\widetilde{\beta}_k(P^E_{k,t})^{1-\gamma} + (1-\widetilde{\beta}_k)(P^M_{k,t})^{1-\gamma}\right]^{\frac{1}{1-\gamma}}$, $P^E_{k,t} = \left[\sum_{i\in I_E}\widetilde{\nu}_{ki}(P^C_{ki,t})^{1-\eta}\right]^{\frac{1}{1-\eta}}$, $P^M_{k,t} = \left[\sum_{i\in I_M}\widetilde{\upsilon}_{ki}(P^C_{ki,t})^{1-\iota}\right]^{\frac{1}{1-\iota}}$, and $P^C_{ki,t} = \left\{\sum_{l=1}^{K}\widetilde{\zeta}_{kl\,i}[(1+\tau_{kl\,i,t})P^k_{kl\,i,t}]^{1-\delta}\right\}^{\frac{1}{1-\delta}}$. Their log-linearized counterparts are given by

$$0 = \beta_k\widehat{p}^E_{k,t} + (1-\beta_k)\,\widehat{p}^M_{k,t} \tag{B.17}$$

$$\widehat{p}^E_{k,t} = \sum_{i\in I_E}\nu_{ki}\widehat{p}^C_{ki,t}, \quad \widehat{p}^M_{k,t} = \sum_{i\in I_M}\upsilon_{ki}\widehat{p}^C_{ki,t} \tag{B.18}$$

$$\widehat{p}^C_{ki,t} = \sum_{l=1}^{K}\zeta_{kl\,i}(\widehat{p}^k_{kl\,i,t} + \tau_{kl\,i,t}) \tag{B.19}$$

Intertemporal Household Problem. International financial markets are incomplete, $\mathcal{H} = 1$ in (4), with households in each country only having access to two risk-free bonds. More precisely, the household in country k has access to a domestic bond, and an internationally traded bond, $B^K_{k,t}$, issued, without loss of generality, by country K and denominated in country K's currency. The agent maximizes the present discounted value of per-period utility flows (B.1), with discount factor β, subject to her budget constraint,

$$\begin{aligned} &P^C_{k,t}C_{k,t} + B_{k,t} + B^K_{k,t}\left[1-\Gamma(\mathrm{NFA}^K_{k,t})\right]^{-1}\mathcal{E}^K_{k,t} + \Xi_{k,t} \leq \\ &B_{k,t-1}(1+i_{k,t-1}) + B^K_{k,t-1}\mathcal{E}^K_{k,t}(1+i_{K,t-1}) + \int_0^1 W_{gk,t}\mathcal{N}_{gk,t}dg + \Pi_{k,t} - T_{k,t} \end{aligned} \tag{B.20}$$

where $\int_0^1 W_{gk,t}\mathcal{N}_{gk,t}dg$ is the nominal labor income received by the representative household, $\mathrm{NFA}^K_{k,t} = B^K_{k,t}\mathcal{E}^K_{k,t}$ is the net foreign asset position of households in the country k, and where $\Gamma(x) = \gamma_*\left(\exp\left\{x/\mathcal{Y}_{k,t}\right\}-1\right)$ is an external financial intermediary premium that depends on the economy-wide net holdings of internationally traded foreign bonds as a ratio to the national nominal GDP $\mathcal{Y}_{k,t}$, with $\gamma_* > 0$.[33] The incurred intermediation premium is rebated to households in a lump-sum manner through the fiscal instrument $\Xi_{k,t}$. Finally, $T_{k,t}$ denotes government transfers, also rebated to households in lump sum.

The above program delivers two sets of different Euler conditions,

$$C_{k,t}^{-\sigma} = \mathbb{E}_t \beta C_{k,t+1}^{-\sigma} \frac{1+i_{k,t}}{1+\pi_{k,t+1}^C} \frac{Z_{k,t+1}}{Z_{k,t}} \tag{B.21}$$

$$C_{k,t}^{-\sigma} = \mathbb{E}_t \beta C_{k,t+1}^{-\sigma} \frac{1+i_{K,t}}{1+\pi_{k,t+1}^C} \left[1-\Gamma(\mathrm{NFA}_{k,t}^K)\right] \frac{\mathcal{E}_{k,t+1}^K}{\mathcal{E}_{k,t}^K} \frac{Z_{k,t+1}}{Z_{k,t}} \quad \forall k \neq K \tag{B.22}$$

where we assume that the (log-)demand shock follows an AR(1) process:

$$z_{k,t} = \rho_k^z z_{k,t-1} + \varepsilon_{k,t}^z \tag{B.23}$$

where $z_{k,t} := \log Z_{k,t}$, and $\varepsilon_{k,t}^z \sim \mathcal{N}\left(0, \sigma_{kz}^2\right)$.

The log-linearized version of the household's first-order conditions (B.21)-(B.22) are given by

$$\widehat{c}_{k,t} = -\frac{1}{\sigma}(i_{k,t} - \mathbb{E}_t \pi_{k,t+1}^C) + \mathbb{E}_t \widehat{c}_{k,t+1} + \frac{1}{\sigma}(1-\rho_k^z) z_{k,t}, \tag{B.24}$$

$$\widehat{c}_{k,t} = -\frac{1}{\sigma}(i_{K,t} - \mathbb{E}_t \pi_{k,t+1}^C) + \mathbb{E}_t \widehat{c}_{k,t+1} + \frac{1}{\sigma}(1-\rho_k^z) z_{k,t} - \frac{1}{\sigma}\mathbb{E}_t \Delta e_{k,t+1}^K + \frac{1}{\sigma}\gamma_* \mathrm{nfa}_{k,t}^K \quad \forall k \neq K, \tag{B.25}$$

where we define the different log-linear NFA positions as $\mathrm{nfa}_{k,t}^K = B_{k,t}^K \mathcal{E}_{k,t}^K / \mathcal{Y}_k$ and $\mathrm{nfa}_{k,t}^{MU} = B_{k,t}^{MU} \mathcal{E}_{k,t}^{MU} / \mathcal{Y}_k$ since $B_{k,t}^K = 0$ and $B_{k,t}^{MU} = 0$ in the steady state.

Combining the log-linearized first-order conditions for the holdings of domestic and internationally traded bonds (B.24)-(B.25), yields a risk-adjusted Uncovered Interest Parity (UIP) condition

$$i_{k,t} - i_{K,t} = \mathbb{E}_t \Delta e_{k,t+1}^K - \gamma_* \mathrm{nfa}_{k,t}^K. \tag{B.26}$$

[33] The role of this intermediation premium is to stabilize the net foreign asset position in response to transitory shocks, a common practice in open economies with incomplete financial markets (Schmitt-Grohe and Uribe 2003). Furthermore, this specification guarantees that, in the non-stochastic steady state, households have no incentive to hold foreign bonds and the economy's net foreign asset position is zero.

B.2. Firms

We augment the production function (39) to include a sectoral TFP shock $A_{ki,t}$,

$$Y_{kif,t} = A_{ki,t}\left[\widetilde{\alpha}_{ki}^{\frac{1}{\psi}} N_{kif,t}^{\frac{\psi-1}{\psi}} + \widetilde{\vartheta}_{ki}^{\frac{1}{\psi}} X_{kif,t}^{\frac{\psi-1}{\psi}}\right]^{\frac{\psi}{\psi-1}}, \tag{B.27}$$

where the (log-)TFP shock follows an AR(1) process:

$$a_{ki,t} = \rho_{ki}^{a} a_{ki,t-1} + \varepsilon_{ki,t}^{a}, \tag{B.28}$$

where $a_{ki,t} := \log A_{ki,t}$, and $\varepsilon_{ki,t}^{a} \sim \mathcal{N}\left(0, \sigma_{kia}^{2}\right)$.

Intermediate Input Demand Curves. The optimal allocation between labor and intermediate inputs is the result of a cost minimization programme min $W_{k,t}N_{kif,t} + P_{ki,t}^{X}X_{kif,t}$ subject to (B.27). The optimal allocation between energy and non-energy intermediate goods is the result of a cost minimization programme min $P_{ki,t}^{X,E}X_{ki,t}^{E} + P_{ki,t}^{X,M}X_{ki,t}^{M}$ subject to (40). Similarly, the optimal allocation between energy (non-energy) intermediate inputs is the result of a cost minization programme min $\sum_{j\in I_E} P_{kij,t}^{X}X_{kij,t}$ (min $\sum_{j\in I_M} P_{kij,t}^{X}X_{kij,t}$) subject to (42). Finally, the optimal allocation between the different consumption goods is the result of a cost minimization programme min $\sum_{l=1}^{K}(1+\tau_{klj,t})P_{klj,t}^{k}X_{kl\,ij,t}$ subject to (43). The implied intermediate input demand curves are given by

$$W_{k,t} = \mathrm{MC}_{kif,t}A_{ki,t}^{\frac{\psi-1}{\psi}}\left(\widetilde{\alpha}_{ki}Y_{kif,t}/N_{kif,t}\right)^{\frac{1}{\psi}}, \tag{B.29}$$

$$P_{ki,t}^{X} = \mathrm{MC}_{ki,t}A_{ki,t}^{\frac{\psi-1}{\psi}}\left(\widetilde{\vartheta}_{ki}Y_{kif,t}/X_{kif,t}\right)^{\frac{1}{\psi}}, \tag{B.30}$$

$$P_{ki,t}^{X,E} = P_{ki,t}^{X}\left(\widetilde{\beta}_{ki}X_{ki,t}/X_{ki,t}^{E}\right)^{\frac{1}{\phi}}, \quad P_{ki,t}^{X,M} = P_{ki,t}^{X}\left((1-\widetilde{\beta}_{ki})X_{ki,t}/X_{ki,t}^{M}\right)^{\frac{1}{\phi}}, \tag{B.31}$$

(B.32)

$$P_{kij,t}^{X} = P_{ki,t}^{X,E}\left(\widetilde{\nu}_{kij}X_{ki,t}^{E}/X_{kij,t}\right)^{\frac{1}{\chi}} \quad \forall \quad j\in I_E, \quad P_{kij,t}^{X} = P_{ki,t}^{X,M}\left(\widetilde{\upsilon}_{kij}X_{ki,t}^{M}/X_{kij,t}\right)^{\frac{1}{\xi}} \quad \forall \quad j\in I_M,$$

$$(1+\tau_{klj,t})P_{klj,t}^{k} = P_{kij,t}^{X}\left(\zeta_{kl\,ij}X_{kij,t}/X_{kl\,ij,t}\right)^{\frac{1}{\mu}} \quad \forall \quad l\in K. \tag{B.33}$$

The log-linearized versions of the labor and intermediate inputs demand curves (B.29)-(B.33) are given by

$$\widehat{w}_{k,t} - \widehat{\mathrm{mc}}_{ki,t} = \frac{\psi-1}{\psi}a_{ki,t} + \psi^{-1}\left(\widehat{y}_{kif,t} - \widehat{n}_{kif,t}\right), \tag{B.34}$$

(B.35) $$\widehat{p}^{X}_{ki,t} - \widehat{\mathrm{mc}}_{ki,t} = \frac{\psi - 1}{\psi} a_{ki,t} + \psi^{-1}\left(\widehat{y}_{kif,t} - \widehat{x}_{kif,t}\right),$$

(B.36) $$\widehat{p}^{X,E}_{ki,t} - \widehat{p}^{X}_{ki,t} = \phi^{-1}\left(\widehat{x}_{ki,t} - \widehat{x}^{E}_{ki,t}\right), \quad \widehat{p}^{X,M}_{ki,t} - \widehat{p}^{X}_{ki,t} = \phi^{-1}\left(\widehat{x}_{ki,t} - \widehat{x}^{M}_{ki,t}\right),$$

(B.37)
$$\widehat{p}^{X}_{kij,t} - \widehat{p}^{X,E}_{ki,t} = \chi^{-1}\left(\widehat{x}^{E}_{ki,t} - \widehat{x}_{kij,t}\right) \quad \forall \quad j \in I_E, \quad \widehat{p}^{X}_{kij,t} - \widehat{p}^{X,M}_{ki,t} = \xi^{-1}\left(\widehat{x}^{M}_{ki,t} - \widehat{x}_{kij,t}\right) \quad \forall \quad j \in I_M$$

(B.38) $$\tau_{klj,t} + \widehat{p}^{k}_{klj,t} - \widehat{p}^{X}_{kij,t} = \mu^{-1}\left(\widehat{x}_{kij,t} - \widehat{x}_{klij,t}\right) \quad \forall \quad l \in K,$$

where $\widehat{w}_{k,t} = w_{k,t} - p^{C}_{k,t}$, $\widehat{\mathrm{mc}}_{ki,t} = \mathrm{mc}_{ki,t} - p^{C}_{k,t}$, $\widehat{p}^{X}_{ki,t} = p^{X}_{ki,t} - p^{C}_{k,t}$, $\widehat{p}^{X,E}_{ki,t} = p^{X,E}_{ki,t} - p^{C}_{k,t}$, $\widehat{p}^{X,M}_{ki,t} = p^{X,M}_{ki,t} - p^{C}_{k,t}$, and $\widehat{p}^{X}_{kij,t} = p^{X}_{kij,t} - p^{C}_{k,t}$.

Intermediate Inputs Baskets. The log-linearized intermediary input aggregator (40) is given by

(B.39) $$\widehat{x}_{ki,t} = \beta_{ki}\widehat{x}^{E}_{ki,t} + (1 - \beta_{ki})\widehat{x}^{M}_{ki,t},$$

where $\beta_{ki} = P^{X,E}_{ki}X^{E}_{ki}/(P^{X}_{ki}X_{ki}) = \widetilde{\beta}^{\frac{1}{\phi}}_{ki}\left(X^{E}_{ki}/X_{ki}\right)^{\frac{\phi-1}{\phi}}$ and $(1 - \beta_{ki}) = P^{X,M}_{ki}X^{M}_{ki}/(P^{X}_{ki}X_{ki}) = (1 - \widetilde{\beta}_{ki})^{\frac{1}{\phi}}\left(X^{M}_{ki}/X_{ki}\right)^{\frac{\phi-1}{\phi}}$ can be verified using the steady-state intermediate input aggregator (40) and the input demand curves (B.31).

The log-linearized versions of the energy and non-energy intermediate input aggregators (42) are given by

(B.40) $$\widehat{x}^{E}_{ki,t} = \sum_{j\in I_E} \nu_{kij}\widehat{x}_{kij,t}, \quad \widehat{x}^{M}_{ki,t} = \sum_{j\in I_M} \upsilon_{kij}\widehat{x}_{kij,t},$$

where $\nu_{kij} = P^{X}_{kij}X_{kij}/(P^{X,E}_{ki}X^{E}_{ki}) = \widetilde{\nu}^{\frac{1}{\chi}}_{kij}\left(X_{kij}/X^{E}_{ki}\right)^{\frac{\chi-1}{\chi}}$ and $\upsilon_{kij} = P^{X}_{kij}X_{kij}/(P^{X,M}_{ki}X^{M}_{ki}) = \widetilde{\upsilon}^{\frac{1}{\xi}}_{kij}\left(X_{kij}/X^{M}_{ki}\right)^{\frac{\xi-1}{\xi}}$ can be verified using the steady-state energy and non-energy intermediate input aggregators (42) and the demand curves (B.32).

The log-linearized version of the final layer of the intermediate input aggregator, (43), is given by

(B.41) $$\widehat{x}_{kij,t} = \sum_{l=1}^{K} \zeta_{klij}\widehat{x}_{klij,t},$$

where $\zeta_{klij} = P_{klj}X_{klij}/(P^{X}_{kij}X_{kij}) = \widetilde{\zeta}^{\frac{1}{\mu}}_{klij}\left(X_{klij}/X_{kij}\right)^{\frac{\mu-1}{\mu}}$ can be verified using the steady-state international intermediate input aggregators (43) and the demand curve (B.33).

Price Indices. The different price indices can be derived by combining the intermediate input demand curves previously derived with the other intermediate input aggregators. The marginal cost of production, the intermediate input price index, the energy and non-energy input price index, and the input import price index are given by $MC_{ki,t} = A_{ki,t}^{-1}\left[\widetilde{\alpha}_{ki}W_{k,t}^{1-\psi} + \widetilde{\vartheta}_{ki}(P_{ki,t}^{X})^{1-\psi}\right]^{\frac{1}{1-\psi}}$, $P_{ki,t}^{X} = \left[\widetilde{\beta}_{ki}(P_{ki,t}^{X,E})^{1-\phi} + (1-\widetilde{\beta}_{ki})(P_{ki,t}^{X,M})^{1-\phi}\right]^{\frac{1}{1-\phi}}$, $P_{ki,t}^{X,E} = \left[\sum_{j\in I_E}\widetilde{\nu}_{kij}(P_{kij,t}^{X})^{1-\chi}\right]^{\frac{1}{1-\chi}}$, $P_{ki,t}^{X,M} = \left[\sum_{j\in I_M}\widetilde{\upsilon}_{kij}(P_{kij,t}^{X})^{1-\xi}\right]^{\frac{1}{1-\xi}}$, and $P_{kij,t}^{X} = \left\{\sum_{l=1}^{K}\widetilde{\zeta}_{kl\,ij}[(1+\tau_{klj,t})P_{klj,t}]^{1-\mu}\right\}^{\frac{1}{1-\mu}}$. Their log-linearized counterparts are given by

$$\widehat{mc}_{ki,t} = -a_{ki,t} + \mathcal{M}_{ki}\frac{W_kN_{ki}}{P_{ki}Y_{ki}}\widehat{w}_{k,t} + \mathcal{M}_{ki}\frac{P_{ki}^{X}X_{ki}}{P_{ki}Y_{ki}}\widehat{p}_{ki,t}^{X}, \tag{B.42}$$

$$\widehat{p}_{ki,t}^{X} = \beta_{ki}\widehat{p}_{ki,t}^{X,E} + (1-\beta_{ki})\,\widehat{p}_{ki,t}^{X,M}, \tag{B.43}$$

$$\widehat{p}_{ki,t}^{X,E} = \sum_{j\in I_E}\nu_{kij}\widehat{p}_{kij,t}^{X}, \quad \widehat{p}_{ki,t}^{X,M} = \sum_{j\in I_M}\upsilon_{kij}\widehat{p}_{kij,t}^{X}, \tag{B.44}$$

$$\widehat{p}_{kij,t}^{X} = \sum_{l=1}^{K}\zeta_{kl\,ij}(\widehat{p}_{klj,t}^{k} + \tau_{klj,t}), \tag{B.45}$$

where $\mathcal{M}_{ki}W_kN_{ki}/(P_{ki}Y_{ki}) = \left(W_k/\mathrm{MC}_{ki}\right)^{1-\psi}\widetilde{\alpha}_{ki}$ and $\mathcal{M}_{ki}P_{ki}^{X}X_{ki}/(P_{ki}Y_{ki}) = \left(P_{ki}^{X}/\mathrm{MC}_{ki}\right)^{1-\psi}\widetilde{\vartheta}_{ki}$ can be derived using (B.29)-(B.30) in steady-state.

Production Structure. The log-linearized version of the production function (B.27) is given by

$$\widehat{y}_{kif,t} = a_{ki,t} + \mathcal{M}_{ki}\alpha_{ki}\widehat{n}_{kif,t} + \mathcal{M}_{ki}\vartheta_{ki}\widehat{x}_{kif,t}, \tag{B.46}$$

where $\alpha_{ki} = W_kN_{ki}/(P_{ki}Y_{ki}) = W_kN_{ki}/(\mathcal{M}_{ki}\mathrm{MC}_{ki}Y_{ki})$ denotes the (steady-state) labor income share of total sales of firm i, and the identities $\mathcal{M}_{ki}\alpha_{ki} = \mathcal{M}_{ki}W_kN_{ki}/(P_{ki}Y_{ki}) = N_{ki}/Y_{ki}\left(\widetilde{\alpha}_{ki}Y_{ki}/N_{ki}\right)^{\frac{1}{\psi}}$ and $\mathcal{M}_{ki}\vartheta_{ki} = \mathcal{M}_{ki}P_{ki}^{X}X_{ki}/(P_{ki}Y_{ki}) = \frac{X_{ki}}{Y_{ki}}\left(\widetilde{\vartheta}_{ki}Y_{ki}/X_{ki}\right)^{\frac{1}{\psi}}$ can be verified using the first-order conditions from the firms' problem (B.29) and (B.30) in steady-state and the standard monopolistic competition pricing condition in steady-state, $P_{ki} = \mathcal{M}_{ki}\mathrm{MC}_{ki}$. Using the previous identities, together with the production function (B.27) in steady-state, one can verify that

$$\mathcal{M}_{ki}\frac{W_kN_{ki}}{P_{ki}Y_{ki}} + \mathcal{M}_{ki}\frac{P_{ki}^{X}X_{ki}}{P_{ki}Y_{ki}} = \frac{W_kN_{ki}}{\mathrm{MC}_{ki}Y_{ki}} + \frac{P_{ki}^{X}X_{ki}}{\mathrm{MC}_{ki}Y_{ki}} = 1. \tag{B.47}$$

B.3. Price–Setting

We extend our framework to allow for a time-varying elasticity of substitution between different good varieties, in order to micro-found price cost-push shocks. We extend (3) to

$$Y^m_{kl\,i,t} = \left(\int_0^1 (Y^m_{fkl\,i,t})^{(\epsilon^p_{ki,t}-1)/\epsilon^p_{ki,t}} df\right)^{\epsilon^p_{ki,t}/(\epsilon^p_{ki,t}-1)}, \tag{B.48}$$

where $Y^m_{kl\,i,t}$ denotes the demand for domestic sector i from country k invoiced in currency m in country l. The implied sectoral price index is

$$P^m_{kl\,i,t} = \left(\int_0^1 (P^m_{fkl\,i,t})^{1-\epsilon^p_{ki,t}} df\right)^{\frac{1}{1-\epsilon^p_{ki,t}}}. \tag{B.49}$$

Producers of each differentiated variety face the demand function

$$Y^m_{kl\,i,t+s|t} = \left(P^m_{fkl\,i,t}/P^m_{kl\,i,t+s}\right)^{-\epsilon^p_{ki,t}} Y^m_{kl\,i,t+s} \tag{B.50}$$

Firms set prices à la Calvo, which implies that the aggregate price dynamics are described by the equation

$$(\Pi^m_{l\,ki,t})^{1-\epsilon^p_{ki,t}} = \theta^p_{ki} + (1-\theta^p_{ki})\left(\overline{P}^m_{l\,ki,t}/P^m_{l\,ki,t-1}\right)^{1-\epsilon^p_{ki,t}}, \tag{B.51}$$

which can be written in log-linearized terms as

$$\pi^m_{l\,ki,t} = (1-\theta^p_{ki})(\overline{p}^m_{l\,ki,t} - p^m_{l\,ki,t-1}). \tag{B.52}$$

Denote by $P^m_{l\,ki,t}$ the price of a good from sector i, originating in country k, sold in country l, and invoiced in the currency of country m. The per-period nominal profits of the domestic firm producing good from sector i are then given by

$$\Pi_{ki,t} = \sum_{l=1}^{K}\sum_{m=1}^{K} \mathcal{E}^m_{k,t}(1+\tau_{l\,ki,t}P^m_{l\,ki,t}Y^m_{l\,ki,t} - \mathcal{C}_{ki,t}(Y_{ki,t}) \tag{B.53}$$

where $Y_{ki,t} = \sum_{l=1}^K \sum_{m=1}^K Y^m_{l\,ki,t}$ denotes the total demand across destination markets l and invoicing currencies k. $\mathcal{C}_{ki,t}(Y_{ki,t})$ denotes total costs of country k firms in their home currency.

Consider the pricing problem of a firm from country k selling in country l and invoicing in currency m, and denote $\overline{P}^{m}_{l\,ki,t}$ its optimally-chosen reset price, resulting from the problem

(B.54)

$$\max_{\overline{P}^{m}_{l\,ki,t}} \sum_{s=0}^{\infty} (\theta^{p}_{ki})^{s} \mathbb{E}_t \left\{ \frac{\Lambda_{t,t+s}}{P_{ki,t+s}} \left[\sum_{l=1}^{K} \sum_{m=1}^{K} \mathcal{E}^{m}_{k,t+s} (1+\tau_{l\,ki,t}) \overline{P}^{m}_{l\,ki,t} Y^{m}_{l\,ki,t+s|t} - \mathcal{C}_{ki,t+s} \left(\sum_{l=1}^{K} \sum_{m=1}^{K} Y^{m}_{l\,ki,t+s|t} \right) \right] \right\}$$

subject to the sequence of demand constraints (B.50), with $P_{ki,t} = P^{k}_{kki,t}$. The optimality condition associated with the problem takes the form

$$\sum_{s=0}^{\infty} (\theta^{p}_{ki})^{s} \mathbb{E}_t \left\{ \frac{\Lambda_{t,t+s} Y^{m}_{l\,ki,t+s|t}}{P_{ki,t+s}} [\mathcal{E}^{m}_{k,t+s} (1+\tau_{l\,ki,t}) \overline{P}^{m}_{l\,ki,t} - \mathcal{M}_{ki,t+s} \mathrm{MC}_{ki,t+s|t}] \right\} = 0. \tag{B.55}$$

A first-order Taylor expansion of (B.55) around the zero inflation steady state yields

$$\tau_{l\,ki,t} + \overline{p}^{m}_{l\,ki,t} = (1-\beta\theta^{p}_{ki}) \sum_{s=0}^{\infty} (\beta\theta^{p}_{ki})^{s} \mathbb{E}_t \left(\mathrm{mc}_{ki,t+s|t} - e^{m}_{k,t+s} + \mu_{ki,t+s} \right). \tag{B.56}$$

where $\mathrm{mc}_{ki,t+s|t} \equiv \log \mathrm{MC}_{ki,t+s|t}$ is the log marginal cost, and $\mu_{ki,t} := \log \mathcal{M}_{ki,t}$ is the log of the desired gross markup.

The log marginal cost for an individual firm that last set its price in period t is given by $\mathrm{mc}_{ki,t+s|t} = w_{k,t+s} - (\psi-1)/\psi a_{ki,t+s} - \psi^{-1} \left(\log \widetilde{\alpha}_{ki} + y_{ki,t+s|t} - n_{ki,t+s|t} \right)$, where $n_{ki,t+s|t}$ denotes the log employment in period $t+s$ for a firm that last reset its price in period t, and where we have made use of (B.34). Letting $\mathrm{mc}_{ki,t} = \int_0^1 \mathrm{mc}_{kif,t}\, df = (1-\theta^{p}_{ki}) \sum_{s=0}^{\infty} (\theta^{p}_{ki})^{s} \mathrm{mc}_{ki,t|t-s}$ represent the log average marginal cost, it follows that $\mathrm{mc}_{ki,t} = w_{k,t} - (\psi-1)/\psi a_{ki,t} - \psi^{-1} \left(\log \widetilde{\alpha}_{ki} + y_{ki,t} - n_{ki,t} \right)$. Thus, the following relation holds between firm-specific and economy-wide marginal costs $\mathrm{mc}_{ki,t+s|t} = \mathrm{mc}_{ki,t+s} - \psi^{-1} \left[\left(y_{ki,t+s|t} - y_{ki,t+s} \right) - \left(n_{ki,t+s|t} - n_{ki,t+s} \right) \right]$. Notice that, making use of both marginal cost expressions (B.29)-(B.30), the identity $x_{ki,t+s|t} - x_{ki,t+s} = n_{ki,t+s|t} - n_{ki,t+s}$ is satisfied. Hence, we can write $y_{ki,t+s|t} - y_{ki,t+s} = \left(\mathcal{M}_{ki}\alpha_{ki} + \mathcal{M}_{ki}\vartheta_{ki} \right) (n_{ki,t+s|t} - n_{ki,t+s}) = n_{ki,t+s|t} - n_{ki,t+s}$, where we have used the identity (B.47), and where we have used the linearized production function (B.46). Hence, we can finally write the relation between marginal costs as $\mathrm{mc}_{ki,t+s|t} = \mathrm{mc}_{ki,t+s}$. Introducing this last expression into the log-linearized firms' FOC (B.56), we can write

$$\tau_{l\,ki,t} + \overline{p}^{m}_{l\,ki,t} = (1-\beta\theta^{p}_{ki}) \sum_{s=0}^{\infty} (\beta\theta^{p}_{ki}) \mathbb{E}_t \left(\mathrm{mc}_{ki,t+s} - e^{m}_{k,t+s} + \mu_{ki,t+s} \right). \tag{B.57}$$

Rewriting the (linearized) inflation dynamics (B.52) as

$$\overline{p}^{m}_{l\,ki,t} - p^{m}_{l\,ki,t} = \frac{\theta^{p}_{ki}}{1-\theta^{p}_{ki}}\pi^{m}_{l\,ki,t}. \tag{B.58}$$

Subtracting $p^{m}_{l\,ki,t}$ on both sides of (B.57), adding and subtracting $p^{m}_{l\,ki,t+s}$ on the RHS, and inserting (B.58) on the LHS,

$$\begin{aligned}\frac{\theta^{p}_{ki}}{1-\theta^{p}_{ki}}(\Delta\tau_{l\,ki,t}+\pi^{m}_{l\,ki,t}) &= (1-\beta\theta^{p}_{ki})\sum_{s=0}^{\infty}(\beta\theta^{p}_{ki})^{s}\mathbb{E}_t\left(\mathrm{mc}_{ki,t+s}-p^{m}_{l\,ki,t+s}-\tau_{l\,ki,t+s}-e^{m}_{k,t+s}+\mu_{ki,t+s}\right)\\ &\quad+(1-\beta\theta^{p}_{ki})\sum_{s=1}^{\infty}(\beta\theta^{p}_{ki})^{s}\mathbb{E}_t(\Delta\tau_{l\,ki,t+s}+\pi^{m}_{l\,ki,t+s}),\end{aligned} \tag{B.59}$$

where $\Delta\tau_{l\,ki,t} = \tau_{l\,ki,t} - \tau_{l\,ki,t-1}$ Taking conditional expectations at time $t+1$, multiplying by $\beta\theta^{p}_{ki}$, and rearranging terms using (B.52), we can write

$$(\Delta\tau_{l\,ki,t}+\pi^{m}_{l\,ki,t}) = \kappa_{ki}\left(\mathrm{mc}_{ki,t}-p^{m}_{l\,ki,t}-\tau_{l\,ki,t}-e^{m}_{k,t}\right)+\beta\mathbb{E}_t(\Delta\tau_{l\,ki,t+1}+\pi^{m}_{l\,ki,t+1})+u^{p}_{ki,t}, \tag{B.60}$$

where $\kappa_{ki} = (1-\theta^{p}_{ki})(1-\beta\theta^{p}_{ki})/\theta^{p}_{ki}$.

We assume that the sectoral price cost-push shocks $u^{p}_{ki,t} = \kappa_{ki}\widehat{\mu}_{ki,t} = \kappa_{ki}(\mu_{ki,t}-\mu_{ki})$, micro-founded through a time-varying elasticity of substitution $\epsilon^{p}_{ki,t}$ in (3), follow independent AR(1) processes:

$$u^{p}_{ki,t} = \rho^{p}_{ki}u^{p}_{ki,t-1}+\varepsilon^{p}_{ki,t}, \tag{B.61}$$

where $u^{p}_{ki,t} \sim \mathcal{N}\left(0, \sigma^{2}_{ki\,p}\right)$.

Producer Currency Pricing. Under PCP ($m = k$), we have $\pi^{k}_{l\,ki,t} = \pi_{ki,t}$, and we can write (B.60) as

$$(\Delta\tau_{l\,ki,t}+\pi_{ki,t}) = \kappa_{ki}\left(\widehat{\mathrm{mc}}_{ki,t}-\widehat{p}_{ki,t}-\tau_{l\,ki,t}\right)+\beta\mathbb{E}_t(\Delta\tau_{l\,ki,t+1}+\pi_{ki,t+1})+u^{p}_{ki,t}, \tag{B.62}$$

where the sectoral-inflation rates and sectoral-level real prices ($\widehat{p}_{ki,t}$) are related through the identity

$$\pi_{ki,t} = p_{ki,t}-p_{ki,t-1} = \widehat{p}_{ki,t}-\widehat{p}_{ki,t-1}+\pi^{C}_{k,t}, \tag{B.63}$$

and $\widehat{p}_{ki,t} = p_{ki,t}-p^{C}_{k,t}$.

Local Currency Pricing. Under LCP ($m = l$), we have $\pi^{l}_{lki,t} = \pi_{lki,t}$, and we can write (B.60) as

$$(\Delta\tau_{lki,t} + \pi_{lki,t}) = \kappa_{ki}\left(\widehat{\mathrm{mc}}_{ki,t} - \widehat{p}_{lki,t} - \tau_{lki,t} - q^{l}_{k,t}\right) + \beta\mathbb{E}_t(\Delta\tau_{lki,t+1} + \pi_{lki,t+1}) + u^{p}_{ki,t}, \tag{B.64}$$

which is the export Phillips curve, where we have used (B.10), and where

$$\pi_{lki,t} = p_{lki,t} - p_{lki,t-1} = \widehat{p}_{lki,t} - \widehat{p}_{lki,t-1} + \pi^{C}_{l,t} \tag{B.65}$$

denotes price inflation in sector i exported to country l from country k. For domestic sales, $k = l$, we have that $\pi_{kki,t} = \pi_{ki,t}$ and $\widehat{p}_{kki,t} = \widehat{p}_{ki,t}$, and both the Phillips curve and the sectoral inflation rates are given by (B.62) and (B.63), respectively.

Marginal costs and aggregate inflation. In this open IO economy, the price Phillips curves depend on the international supply network through the real marginal costs faced by firm i in country k, $\widehat{\mathrm{mc}}_{ki,t}$. Combining the log-linearized intermediate input prices indices (B.42)-(B.45), we obtain the marginal cost equation,

$$\widehat{\mathrm{mc}}_{ki,t} = -a_{ki,t} + \mathcal{M}_{ki}\alpha_{ki}\widehat{w}_{k,t} + \sum_{l=1}^{K}\sum_{j=1}^{I}\mathcal{M}_{ki}\omega_{klij}(\widehat{p}^{k}_{klj,t} + \tau_{klj,t}) \tag{B.66}$$

where, in the absence of a production subsidy, $\omega_{klij} = P_{klj}X_{klij}/(P_{ki}Y_{ki}) = P_{klj}X_{klij}/(\mathcal{M}_{ki}\mathrm{MC}_{ki}Y_{ki})$ denotes the (steady-state) IO expenditure share of total sales of firm i, and $\mathcal{M}_{ki} = \epsilon^{p}_{ki}/(\epsilon^{p}_{ki} - 1)$ denotes the steady-state markup charged by firm i.

Writing (B.17)-(B.19) in first-differences, we can obtain consumer price inflation,

$$\pi^{C}_{k,t} = \sum_{i=1}^{I}\sum_{l=1}^{K}\beta_{kli}(\pi^{k}_{kli,t} + \Delta\tau_{kli,t}) \tag{B.67}$$

where $\beta_{kli} = P_{kli}C_{kli}/(P^{C}_{k}C_{k}) = \zeta_{kli}\left[\nu_{ki}\beta_{k}\mathbb{1}_{\{i\in I_E\}} + \upsilon_{ki}(1-\beta_{k})\left(1 - \mathbb{1}_{\{i\in I_E\}}\right)\right]$ and

$$\pi^{k}_{kli,t} = p^{k}_{kli,t} - p^{k}_{kli,t-1} = \widehat{p}^{k}_{kli,t} - \widehat{p}^{k}_{kli,t-1} + \pi^{C}_{k,t}. \tag{B.68}$$

B.4. Wage–Setting

Following Erceg *et al.* (2000), wage stickiness is introduced in a way analogous to price stickiness. Labor unions specialized in any given labor type can reset their nominal wage only with probability $1 - \theta^{w}_{k}$ each period, independently of the time elapsed since they last adjusted their wage. We assume that firms employ a continuum of differentiated labor services. In particular, $N_{kif,t}$ is an index of labor input used by firm f, and defined by

$$N_{kif,t} = \left(\int_0^1 N_{fgki,t}^{(\epsilon_k^w - 1)/\epsilon_k^w} dg \right)^{\epsilon_k^w/(\epsilon_k^w - 1)}, \tag{B.69}$$

where $N_{fgki,t}$ denotes the quantity of type-g labor employed by firm f in period t. Note that $\epsilon_{k,t}^w$ represents the elasticity of substitution among labor varieties. Note also the assumption of a continuum of labor types, indexed by $g \in [0, 1]$.

Let $W_{gk,t}$ denote the nominal wage for type-g labor prevailing in period t. Nominal wages are set by workers of each type (or a union representing them) and taken as given by firms. Given the wages effective at any point in time for the different types of labor services, cost minimization yields a corresponding set of demand schedules for each firm f and labor type g, given the firm's total employment $N_{fk,t}$,

$$N_{fgki,t} = \left(W_{gk,t}/W_{k,t} \right)^{-\epsilon_{k,t}^w} N_{kif,t}, \tag{B.70}$$

where

$$W_{k,t} \equiv \left(\int_0^1 W_{gk,t}^{1-\epsilon_{k,t}^w} dg \right)^{\frac{1}{1-\epsilon_{k,t}^w}} \tag{B.71}$$

is an aggregate wage index. Combining the previous conditions, one can obtain a convenient aggregation result, $\int_0^1 W_{gk,t} N_{fgki,t} dg = W_{k,t} N_{kif,t}$. That is, the wage bill of any given firm can be expressed as the product of the wage index and the firm's employment index.

Consider a union resetting its members' wage in period t, and let $W_{k,t}^*$ denote the newly set wage. The union chooses $W_{k,t}^*$ in a way consistent with utility maximization of its members' households, taking as given the decisions of other unions as well as the paths of aggregate consumption and prices. Specifically, the union seeks to maximize

$$\max_{W_{k,t}^*} \mathbb{E}_t \sum_{s=0}^{\infty} (\beta\theta_k^w)^l \left[C_{k,t+s} W_{k,t}^* N_{k,t+s|t} / P_{k,t+s}^C - N_{k,t+s|t}^{1+\varphi}/(1+\varphi) \right]$$

subject to the sequence of labor demand schedules

$$N_{k,t+s|t} = \left(W_{k,t}^*/W_{k,t+s} \right)^{-\epsilon_{k,t}^w} \int_0^1 N_{gk,t} dg,$$

where $N_{k,t+s|t}$ denotes the level of employment in period $t + s$ among workers that last reset their wage in period t. The first-order condition is given by

$$\sum_{s=0}^{\infty}(\beta\theta_k^w)^l\,\mathbb{E}_t\left[N_{k,t+s|t}C_{t+s}^{-\sigma}\left(W_{k,t}^*/P_{k,t+s}^C-\mathcal{M}_{k,t}^w\mathrm{MRS}_{k,t+s|t}\right)\right]=0,$$

where $\mathcal{M}_{k,t}^w=\epsilon_{k,t}^w/(\epsilon_{k,t}^w-1)$, and $\mathrm{MRS}_{k,t+s|t}=C_{t+s}^{-\sigma}N_{k,t+s|t}^{\varphi}$ denotes the marginal rate of substitution between household consumption and employment in period $t+s$ relevant to the workers resetting their wage in period t. Log-linearizing the above expression around a zero inflation steady-state yields the wage setting rule

$$w_{k,t}^*=(1-\beta\theta_k^w)\sum_{s=0}^{\infty}(\beta\theta_k^w)^s\mathbb{E}_t\left(\mathrm{mrs}_{t+s|t}+\mu_{k,t}^w+p_{k,t+s}^C\right), \tag{B.72}$$

where $\mu_{k,t}^w=\log\mathcal{M}_{k,t}^w$ and $\mathrm{mrs}_{t+s|t}=\sigma c_{k,t+s}+\varphi n_{k,t+s|t}$.

Letting $\mathrm{mrs}_{t+s}=\sigma c_{k,t+s}+\varphi n_{k,t+s}$ define the economy's average marginal rate of substitution, where $n_{k,t+s}=\log\int_0^1\int_0^1 N_{fgk}\,df\,dg$ denotes the log aggregate employment. Up to a first-order approximation, $\mathrm{mrs}_{t+s|t}=\mathrm{mrs}_{t+s}+\varphi(n_{k,t+s}-n_{k,t+s|t})=\mathrm{mrs}_{t+s}-\epsilon_k^w\varphi(w_{k,t}^*-w_{k,t+s})$. Hence, we can write (B.72) as

$$w_{k,t}^*=\frac{1-\beta\theta_k^w}{1+\epsilon_k^w\varphi}\sum_{s=0}^{\infty}(\beta\theta_k^w)^s\mathbb{E}_t\left[(1+\epsilon_k^w\varphi)w_{k,t+s}-\left(w_{k,t+s}-p_{k,t+s}^C-\mathrm{mrs}_{k,t+s}-\mu_{k,t+s}^w\right)\right]. \tag{B.73}$$

Given the assumed wage setting structure, the evolution of the aggregate wage index is given by $W_{k,t}=\left(\theta_k^w W_{k,t-1}^{1-\epsilon_k^w}+(1-\theta_k^w)(W_{k,t}^*)^{1-\epsilon_k^w}\right)^{1/(1-\epsilon_k^w)}$. Log-linearized, $w_{k,t}=\theta_k^w w_{k,t-1}+(1-\theta_k^w)w_{k,t}^*$. Combining the last expression with (B.73), and letting $\pi_{k,t}^w=w_{k,t}-w_{k,t-1}$, we obtain the wage inflation equation:

$$\pi_{k,t}^w=\kappa_k^w\left(\sigma\hat{c}_{k,t}+\varphi\hat{n}_{k,t}-\hat{w}_{k,t}\right)+\beta\mathbb{E}_t\pi_{k,t+1}^w+u_{k,t}^w, \tag{B.74}$$

where

$$\pi_{k,t}^w=w_{k,t}-w_{k,t-1}=\hat{w}_{k,t}-\hat{w}_{k,t-1}+\pi_{k,t}^C \tag{B.75}$$

denotes wage inflation, where both aggregate consumption $\hat{c}_{k,t}$ and employment $\hat{n}_{k,t}$ appear in log-deviations from their steady-state values, and $u_{k,t}^w=\kappa_k^w\hat{\mu}_{k,t}^w=\kappa_k^w(\mu_{k,t}^w-\mu_k^w)$ with $\mu_k^w=\log\mathcal{M}_k^w$. We assume that the wage cost-push shock, micro-founded through a time-varying elasticity of substitution in the labor demand aggregator, follows an AR(1) processes:

$$u_{k,t}^w=\rho_k^w u_{k,t-1}^w+\varepsilon_{k,t}^w \tag{B.76}$$

where $u^{w}_{k,t} \sim \mathcal{N}\left(0, \sigma^2_{kw}\right)$.

B.5. Monetary Authority

The log-linearized bilateral nominal exchange rate (47) is given by $e^{k^{MU}}_{k,t} = e^{k^{MU}}_{k}$, $\forall k \in K^{MU}$. In stationary terms, taking first differences, this can be written as

$$\Delta e^{k^{MU}}_{k,t} = 0 \quad \forall k \in K^{MU} \tag{B.77}$$

Log-linearizing the expression for the real exchange rate (B.10) and first-differencing, we obtain

$$\Delta q^{l}_{k,t} = \Delta e^{l}_{k,t} + \pi^{C}_{l,t} - \pi^{C}_{k,t}. \tag{B.78}$$

Similarly, log-linearizing and first-differencing the symmetry of nominal exchange rates condition $\mathcal{E}^{l}_{k,t} = (\mathcal{E}^{k}_{l,t})^{-1}$ yields

$$\Delta e^{l}_{k,t} = -\Delta e^{k}_{l,t} \tag{B.79}$$

B.6. Market Clearing, GDP, and Trade Balance

Market Clearing. We first consider the goods market clearing condition (11). Pre-multiplying by $P_{ki,t}/(P_{k,t}C_{k,t}) = P_{ki,t}/E_{k,t}$, and making use of (B.15),

$$\begin{aligned}
\frac{P_{ki,t}Y_{ki,t}}{E_{k,t}} &= \sum_{l=1}^{K} \frac{P_{ki,t}C_{lki,t}}{E_{k,t}} + \sum_{l=1}^{K}\sum_{j=1}^{I} \frac{P_{ki,t}X_{lkji,t}}{E_{k,t}} \\
&= \sum_{l=1}^{K} \frac{P_{ki,t}}{P_{lki,t}} \frac{P_{lki,t}C_{lki,t}}{E_{k,t}} + \sum_{l=1}^{K}\sum_{j=1}^{I} \frac{P_{ki,t}}{P_{lki,t}} \frac{P_{lj,t}Y_{lj,t}}{E_{k,t}} \frac{P_{lki,t}X_{lkji,t}}{P_{lj,t}Y_{lj,t}} \\
&= \sum_{l=1}^{K} \frac{P_{ki,t}}{P_{lki,t}} \frac{E_{l,t}}{E_{k,t}} \frac{P_{lki,t}C_{lki,t}}{E_{l,t}} + \sum_{l=1}^{K}\sum_{j=1}^{I} \frac{P_{ki,t}}{P_{lki,t}} \frac{E_{l,t}}{E_{k,t}} \frac{P_{lj,t}Y_{lj,t}}{E_{l,t}} \frac{P_{lki,t}X_{lkji,t}}{P_{lj,t}Y_{lj,t}} \quad \forall i \in I
\end{aligned} \tag{B.80}$$

which we can write in steady-state as

$$\lambda_{ki} = \sum_{l=1}^{K} \mathcal{y}_{lk}\beta_{lki} + \sum_{l=1}^{K}\sum_{j=1}^{I} \mathcal{y}_{lk}\lambda_{lj}\omega_{lkji} \quad \forall i \in I \tag{B.81}$$

where the Domar weight for sector i in country k is $\lambda_{ki} = P_{ki}Y_{ki}/\mathcal{Y}_k$, the nominal GDP ratio between countries l and k is defined as $\mathcal{Y}_{lk} = P_{lC}C_l/(P^C_k C_k)$, and the IO share is given by $\omega_{lkji} = P_{lki}X_{lkji}/(P_{lj}Y_{lj}) = P_{ki}X_{lkji}/(P_{lj}Y_{lj}) = \zeta_{lkji}\vartheta_{lj}\left[\nu_{lji}\beta_{lj}\mathbb{1}_{\{i\in I_E\}} + \upsilon_{lji}(1-\beta_{lj})\left(1-\mathbb{1}_{\{i\in I_E\}}\right)\right]$, where we have made use of the law of one price in steady-state, $P_{klj} = P_{lj}$.[34]

Hence, we can write the log-linearized version of the goods market clearing condition (11), $Y_{ki}\widehat{y}_{ki,t} = \sum_{l=1}^{K}\left(C_{lki}\widehat{c}_{lki,t} + \sum_{j=1}^{I} X_{lkji}\widehat{x}_{lkji}\right)$. Pre-multiplying the expression by P_{ki}/E_k, we can write

$$\lambda_{ki}\widehat{y}_{ki,t} = \sum_{l=1}^{K}\mathcal{Y}_{lk}\left(\beta_{lki}\widehat{c}_{lki,t} + \sum_{j=1}^{I}\lambda_{lj}\omega_{lkji}\widehat{x}_{lkji}\right) \tag{B.82}$$

where we have pre-multiplied the first expression by $\frac{P_{ki}}{E_k}$.

The NFA from the "global" country K (13) can be log-linearized to

$$\frac{1}{\beta}\sum_{k=1}^{K-1}\text{nfa}^K_{k,t-1} - \sum_{k=1}^{K-1}\text{nfa}^K_{k,t} = \Upsilon_K\left(\widehat{\text{exp}}_{K,t} - \widehat{\text{imp}}_{K,t} + \widehat{p}^{\text{EXP}}_{K,t} - \widehat{p}^{\text{IMP}}_{K,t}\right), \tag{B.83}$$

the NFA from country $k \neq K$ $k \notin$ MU, (13) can be log-linearized to

$$\text{nfa}^K_{k,t} - \beta^{-1}\text{nfa}^K_{k,t-1} = \Upsilon_k\left(\widehat{\text{exp}}_{k,t} - \widehat{\text{imp}}_{k,t} + \widehat{p}^{\text{EXP}}_{k,t} - \widehat{p}^{\text{IMP}}_{k,t}\right) \tag{B.84}$$

where $\widehat{p}^{\text{IMP}}_{k,t} = p^{\text{IMP}}_{k,t} - p^C_{k,t}$ and $\widehat{p}^{\text{EXP}}_{k,t} = p^{\text{EXP}}_{k,t} - p^C_{k,t}$. The linearized export and import price deflators are given by:

$$\begin{aligned}\widehat{p}^{\text{IMP}}_{k,t} &= \sum_{l\neq k}\sum_{i=1}^{I}\frac{P_{kli}C_{kli} + \sum_{j=1}^{I}P_{kli}X_{klji}}{P^{\text{IMP}}_k\text{IMP}_k}\left(\widehat{p}^k_{kli,t} + \tau_{kli,t}\right)\\ &= \sum_{l\neq k}\sum_{i=1}^{I}\Upsilon^{-1}_k\left(\beta_{kli} + \sum_{j=1}^{I}\lambda_{kj}\omega_{klji}\right)\left(\widehat{p}^k_{kli,t} + \tau_{kli,t}\right)\end{aligned} \tag{B.85}$$

$$\begin{aligned}\widehat{p}^{\text{EXP}}_{k,t} &= \sum_{l\neq k}\sum_{i=1}^{I}\frac{P_{ki}C_{lki} + \sum_{j=1}^{I}P_{ki}X_{lkji}}{P^{\text{EXP}}_k\text{EXP}_k}\left(\widehat{p}^k_{lki,t} + \tau_{lki,t}\right)\\ &= \sum_{l\neq k}\sum_{i=1}^{I}\frac{\mathcal{Y}_{lk}}{\Upsilon_k}\left(\beta_{lki} + \sum_{j=1}^{I}\lambda_{lj}\omega_{lkji}\right)\left(\widehat{p}^k_{lki,t} + \tau_{lki,t}\right).\end{aligned} \tag{B.86}$$

[34]Both β_{lki} and ω_{lkji} can be extracted directly from the data.

Gross Domestic Product and Net Exports. Let us now move to nominal GDP (18). In steady state, assuming zero net exports, $P_k^{\text{EXP}}\text{EXP}_k - P_k^{\text{IMP}}\text{IMP}_k = 0$, we can write $\mathcal{Y}_k = P_k^C C_k$. Using the household's budget constraint (B.20) in steady state, we can write

$$\mathcal{Y}_k = P_k^C C_k = W_k N_k + \Pi_k = W_k N_k + \sum_{i=1}^{I}\left(1-\mathcal{M}_{ki}^{-1}\right) P_{ki} Y_{ki} \tag{B.87}$$

where the last equality makes use of (B.47).

The log-linearized version of the labor market clearing condition (12) is given by

$$\widehat{n}_{k,t} = \sum_{i=1}^{I} \delta_{ki}\widehat{n}_{ki,t} \tag{B.88}$$

where

$$\delta_{ki} = \frac{N_{ki}}{N_k} = \frac{W_k N_{ki}}{P_{ki}Y_{ki}}\frac{P_{ki}Y_{ki}}{P_k^C C_k}\frac{P_k^C C_k}{W_k N_k} = \frac{\alpha_{ki}\lambda_{ki}}{1-\sum_{j=1}^{I}\left(1-\mathcal{M}_{kj}^{-1}\right)\lambda_{kj}}$$

can be derived using (B.87).

Log-linearizing the real GDP (19) definition,

$$\widehat{y}_{k,t} = P_k^C C_k/\mathcal{Y}_k \widehat{c}_{k,t} + P_k^{\text{EXP}}\text{EXP}_k/\mathcal{Y}_k\widehat{\text{exp}}_{k,t} - P_k^{\text{IMP}}\text{IMP}_k/\mathcal{Y}_k\widehat{\text{imp}}_{k,t} = \widehat{c}_{k,t} + \Upsilon_k\left(\widehat{\text{exp}}_{k,t} - \widehat{\text{imp}}_{k,t}\right)$$

where second equality uses that nominal consumption expenditures will be equal nominal GDP in steady state, and $\Upsilon_k = P_k^{\text{EXP}}\text{EXP}_k/\mathcal{Y}_k = P_k^{\text{IMP}}\text{IMP}_k/\mathcal{Y}_k$ is the export (or import) share of nominal GDP.

The nominal exports expression (14) can be log-linearized to:

$$\widehat{\text{exp}}_{k,t} = \sum_{l\neq k}\sum_{i\in I}\left(\frac{P_{ki}C_{lki}}{P_k^{\text{EXP}}\text{EXP}_k}\widehat{c}_{lki,t} + \sum_{j=1}^{I}\frac{P_{ki}X_{lki}}{P_k^{\text{EXP}}\text{EXP}_k}\widehat{x}_{lkji,t}\right) = \sum_{l\neq k}\sum_{i\in I}\frac{\mathcal{Y}_{lk}}{\Upsilon_k}\left(\beta_{lki}\widehat{c}_{lki,t} + \sum_{j=1}^{I}\lambda_{lj}\omega_{lkji}\widehat{x}_{lkji,t}\right), \tag{B.89}$$

where the export share of nominal GDP is given by

$$\Upsilon_k = \frac{P_k^{\text{EXP}}\text{EXP}_k}{\mathcal{Y}_k} = \sum_{l\neq k}\sum_{i=1}^{I}\mathcal{Y}_{lk}\left[\beta_{lki} + \sum_{j=1}^{I}\lambda_{lj}\omega_{lkji}\right] = \left(\sum_{i=1}^{I}\lambda_{ki}\right) - \left(\beta_{kki} + \sum_{j=1}^{I}\lambda_{kj}\omega_{kkji}\right) \tag{B.90}$$

$$= \frac{P_k^{\text{IMP}}\text{IMP}_k}{\mathcal{Y}_k} = \sum_{l\neq k}\sum_{i=1}^{I}\left[\beta_{kli} + \sum_{j=1}^{I}\lambda_{kj}\omega_{klji}\right] \tag{B.91}$$

Similarly, the nominal imports expression (17) can be log-linearized to

(B.92)

$$\widehat{\text{imp}}_{k,t} = \sum_{l \neq k} \sum_{i \in I} \left(\frac{P_{kl\,i} C_{kl\,i}}{P_k^{\text{IMP}} \text{IMP}_k} \widehat{c}_{kl\,i,t} + \sum_{j=1}^{I} \frac{P_{kl\,i} X_{kl\,ji}}{P_k^{\text{IMP}} \text{IMP}_k} \widehat{x}_{kl\,ji,t} \right) = \sum_{l \neq k} \sum_{i \in I} \Upsilon_k^{-1} \left(\beta_{kl\,i} \widehat{c}_{kl\,i,t} + \sum_{j=1}^{I} \lambda_{kj} \omega_{kl\,ji} \widehat{x}_{kl\,ji,t} \right).$$

Now we can combine the linearized expression for real GDP, the expressions for real imports and exports:

$$\widehat{y}_{k,t} = \widehat{c}_{k,t} + \sum_{l \neq k} \sum_{i \in I} \left(y_{l\,k} \beta_{l\,ki} \widehat{c}_{l\,ki,t} + \sum_{j=1}^{I} y_{l\,k} \lambda_{lj} \omega_{l\,kji} \widehat{x}_{l\,kji,t} - \beta_{kl\,i} \widehat{c}_{kl\,i,t} - \sum_{j=1}^{I} \lambda_{kj} \omega_{kl\,ji} \widehat{x}_{kl\,ji,t} \right).$$

Appendix C. Additional Results

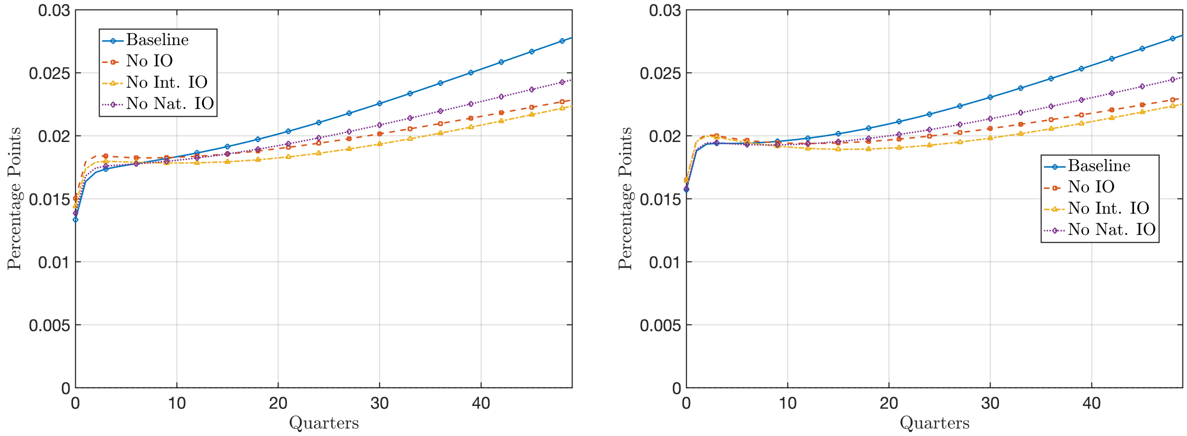
In this section, we introduce the different analyses discussed in section 4.3.3.

C.1. Dissecting the Role of Nominal Price Rigidities

A growing literature shows that nominal rigidities and production networks jointly shape which sectors matter for aggregate dynamics. Pastén *et al.* (2024); Rubbo (2023) find that heterogeneity in price stickiness distorts the "granular" effect of large sectors and can strongly amplify the capacity of idiosyncratic shocks to drive aggregate fluctuations (Pastén *et al.* 2024; Rubbo 2023). In these environments, the key propagation object is a rigidity-adjusted Leontief inverse that embeds the input–output structure and the pattern of nominal stickiness.

Our open-economy framework provides a natural counterpart to study this mechanism when granular perturbations originate from abroad. Foreign sectoral price wedges feed into domestic marginal costs through the direct exposure encoded in $\boldsymbol{\Omega}^F$, and this direct effect is propagated by the matrix $(\boldsymbol{I} - \boldsymbol{\Delta\Omega})^{-1}$ in the open-economy Phillips curve (36), where the diagonal matrix $\boldsymbol{\Delta}$ collects a transformation of the sectoral Calvo parameters. In this sense, our Phillips curve implements in an open-economy setting the rigidity-adjusted network logic emphasized by Pastén *et al.* (2024) and Rubbo (2023): sectoral shocks are amplified or dampened depending not only on where they occur in the network and on consumption weights, but also on the pattern of nominal rigidities along the production chain.

Within this framework, we compare two foreign price-wedge shocks that hit upstream sectors with markedly different pricing behaviour. The first is the international energy price shock studied above, with essentially flexible prices. The second is a price-wedge shock in sector C.26 (*Manufacture of computer, electronic and optical products*), which contains the



A. CIRF of Headline Inflation

B. CIRF of Core Inflation

FIGURE C.1. Inflation Dynamics and Price Rigidities

Notes: Cumulative IRF of EA headline (Panel C.1A) and core (Panel C.1B) inflation after a price-wedge shock to the semiconductors sector, for the baseline and turning off the full, international, or national IO structure. When turning off the IO structure, we always keep the use of semiconductors as an intermediate input.

semiconductor industry and is calibrated with highly rigid prices, $\theta^{p}_{ROW,C.26} = 0.72$.[35] In both cases, we assume that the underlying exogenous wedge follows the same stochastic process (44); thus, any differences in the response of EA inflation are driven by sectoral price rigidities, the respective positions of energy and semiconductors in the IO network, and their weights in the EA consumption basket.

Figure C.1 replicates Figure 2 for the foreign semiconductor price-wedge shock.[36] Two features stand out when comparing the two figures. First, the impact effect of the semiconductor shock on cumulative EA headline inflation is an order of magnitude smaller than effect of the energy shock. Second, the semiconductor-induced inflation is even more persistent: with the IO structure operative, the cumulative IRF keeps increasing over the entire horizon and displays an accelerating slope towards the end of the sample. In other words, the semiconductor shock generates a lower but much more drawn-out inflationary episode than the energy shock.

At first sight, this result is surprising from the perspective of direct exposure. The EA is

[35] The choice of a semiconductor-sector shock is also empirically motivated. During the COVID-19 pandemic, global semiconductor bottlenecks affected a broad range of durable goods, from consumer electronics to machinery. Furthermore, the semiconductor sector remains highly concentrated: Taiwan accounts for over 60% of world semiconductor output (Jones *et al.* 2023). Any large disruption to production in this hub—arising from natural disasters or geopolitical tensions—would constitute a salient upstream supply shock with potentially important macroeconomic effects.

[36] We report results for equal-sized wedge shocks. This choice makes transparent how sectoral price rigidities and network exposure map into different impact elasticities and persistence profiles. An alternative would be to rescale the semiconductor shock so that the impact response of EA headline inflation matches that of the energy shock, but it would obscure the fact that, for a given primitive disturbance, a shock in a sticky upstream sector is intrinsically less inflationary on impact.

more directly affected to semiconductors, both through the domestic and foreign consumption ($\beta_{kl\,i}$) and IO ($\omega_{kl\,ij}$) shares. We therefore explore the role of nominal price rigidities, and their distribution across the network.

To gain intuition, we first focus on the immediate downstream users of each shocked sector. We compute the cross-country average EA sectoral price rigidities, $\theta^{p}_{\text{MU},i}$, and the average domestic IO exposure to energy and semiconductors $\omega^{\text{s}}_{\text{MU},i} = \omega_{\text{MU},\text{MU},i,\text{s}}$ with $s \in$ {energy, semiconductors}; and we define for each shocked sector s a downstream flexibility index $D_s \equiv \sum_{i=1}^{I} \omega^{\text{s}}_{\text{MU},i} \left(1 - \theta^{p}_{\text{MU},i}\right) / \left(\sum_{j=1}^{I} \omega^{\text{s}}_{\text{MU},j}\right)$. This index measures the average price flexibility of the sectors that use inputs from s, weighting each buyer by its exposure to s. We find that downstream users of energy are 3.6 times more flexible than downstream users of semiconductors. That is, not only is the energy sector itself almost fully flexible; the sectors that buy energy and pass the shock on to the rest of the economy are, on average, much more flexible than those that buy semiconductors.

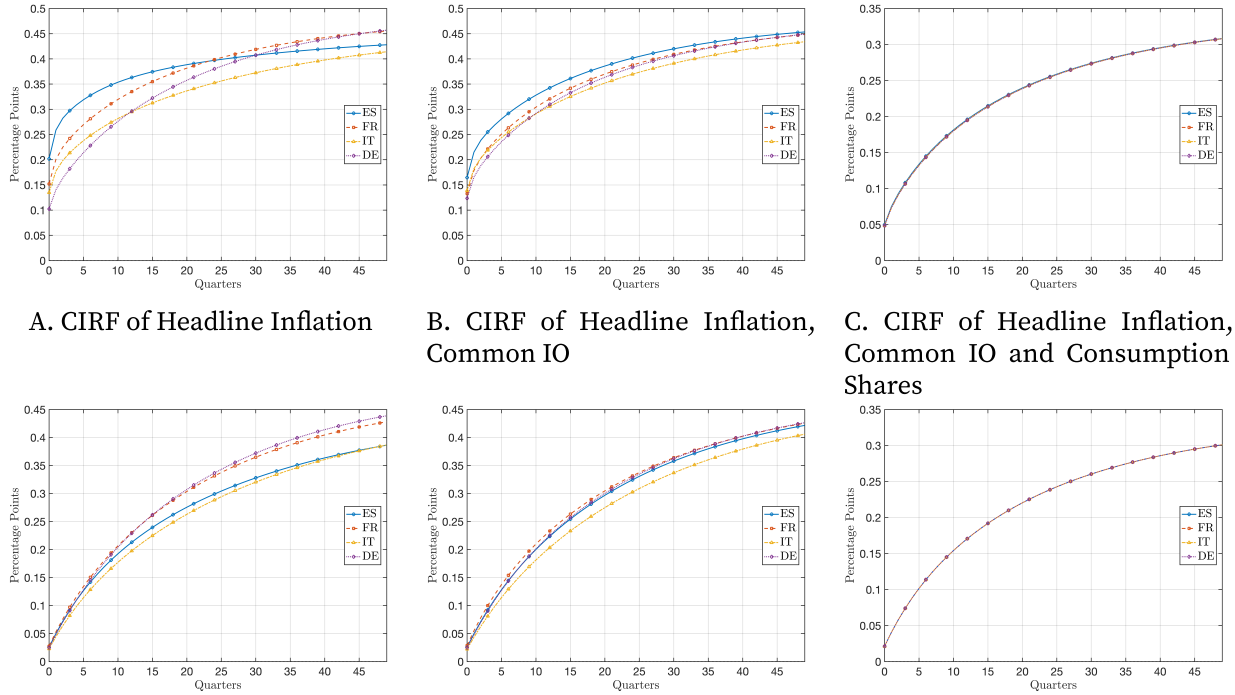
The index D_s captures only the first step in the propagation chain. To incorporate the entire IO structure, let $\boldsymbol{A}$ denote the MU-aggregated IO matrix across sectors, with elements a_{ij} equal to the MU-weighted cost share of inputs from sector j in buyer sector i, obtained from $\omega_{kl\,ij}$ by summing over trading partners. We measure the rigidity-adjusted centrality of sector s by $M_s \equiv \boldsymbol{\lambda}^{\top}(\boldsymbol{I} - \boldsymbol{\Delta A})^{-1}\boldsymbol{e}_s$, where $\boldsymbol{e}_s$ is the indicator vector with a one in position s and zeros elsewhere. We find that, once nominal rigidities are taken into account, the relevant network multiplier of energy is around 65 percent larger than that of semiconductors. This reverses the ranking implied by raw exposure in $\beta_{kl\,i}$ and $\omega_{kl\,ij}$: while the MU is more directly exposed to semiconductors than to energy, the rigidity-adjusted network makes energy more important for inflation.

C.2. Heterogeneous Production Structures and Cross-country Heterogeneity

The previous sections have analyzed the effects of an international energy price shock on EA variables and the role played by production networks in its transmission. In this section, we instead show that such a common shock propagates differently across countries that differ in their production structures.

Figure C.2 shows the CIRFs of headline (Panel C.2A) and core inflation (Panel C.2D) for the main EA countries: Spain (blue line), Germany (purple line), Italy (yellow line), and France (red line).

We find that the (common) shock results in significantly heterogeneous inflation dynamics across countries, despite all European countries facing the same increase in imported energy prices. More precisely, we see that Spain suffers the largest spike in headline inflation in the first periods. However, note that this is also the country where inflation also stabilizes



A. CIRF of Headline Inflation

B. CIRF of Headline Inflation, Common IO

C. CIRF of Headline Inflation, Common IO and Consumption Shares

D. CIRF of Core Inflation

E. CIRF of Core Inflation, Common IO

F. CIRF of Core Inflation, Common IO and Consumption Shares

FIGURE C.2. Cross-country Heterogeneity

Notes: Cumulative IRF of headline (Panel C.2A) and core (Panel C.2D) inflation for Spain (ES), France (FR), Italy (IT), and Germany (DE). Panels C.2B and C.2E reproduce the analysis with an homogeneous IO network, and C.2C and C.2F reproduce the analysis under both homogeneous IO production network and consumption shares.

the fastest. In contrast, we observe the inflation dynamics in Germany. In response to the increase in energy prices, headline inflation increases the least in the German economy. In sharp contrast to Spain, headline inflation in Germany shows substantial persistence, increasing steadily over time. The dynamics of France and Italy sit somewhere between these two extremes.

The dynamics of headline inflation can be better understood by looking at core inflation, shown in Panel C.2D. Germany's core inflation rises gradually and remains elevated for a prolonged period. Spain, on the other hand, experiences a more transient rise in core inflation. This differential evolution in core inflation rates helps explain the varying persistence of headline inflation dynamics between the two countries, since most of the headline inflation at longer horizons is explained by core inflation dynamics.

The heterogeneity in production structures and households' consumption baskets across countries can rationalize these differentials in the inflation dynamics. In the data, the consumption share of energy goods in Spain is the largest (4.49%). Therefore, headline inflation in Spain is the one that is most directly affected by the rise in energy prices. In contrast, Germany has a smaller share of energy consumption (4.09%), naturally leading to a smaller response of headline inflation on impact. However, Germany's production structure is characterized by a stronger industry exposure to the use of energy goods and long production chains. The longer production network structure of the German economy explains the persistent rise of inflation, as the feedback loops described in the previous sections apply more strongly here.[37] On the other side, Spain has a more downstream-oriented production structure, with less complex IO linkages, resulting in lower amplification associated with production networks in this case.

To isolate the role of production networks, we consider the case in which the IO matrix is homogeneous across countries.[38] Panels C.2B and C.2E present the resulting CIRFs of headline and core inflation. We find that equalization of the production network between countries reduces the gap in inflation dynamics between the different countries. The persistence induced by the network is the same across countries, and no CIRF crosses the others. Spain, which has a higher energy share in the CPI basket, reacts more initially and ends up with the highest cumulative inflation.

Finally, to eliminate the gap coming from the heterogeneous consumption shares, we consider the case in which the consumption share matrix is homogeneous across countries.[39] Panels C.2C and C.2F present the resulting CIRFs of headline and core inflation. We find that equalizing the consumption shares across countries, on top of production network, further reduces the gap in inflation between the different countries. The remaining distance can be explained by the heterogeneous price rigidities: the average price duration in Spain is 4.18 quarters, having relatively flexible prices, whereas in Germany prices are more rigid, lasting for 4.50 quarters on average.[40]

[37]Although the values of the upstreamness measure do not have a straightforward quantitative interpretation, the sales-weighted average of the sectors in the Spanish and German economies shows that, on average, Spanish sectors are 6.4% closer to final demand than German sectors.

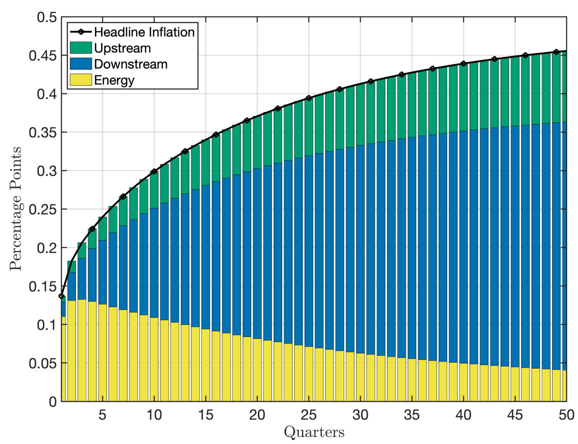
[38]This matrix assumes that, within EA economies, all sectors have the same productive structure. Therefore, within a given sector i, the weight of any sector j (υ_{kij} in our notation) as well as the different national varieties l ($\zeta_{kl\,ij}$ in our notation) is the same for all firms in the EA. Within each sector, these values are calibrated as the average of all EA countries. For consistency, this implies that the weight of a given sector in the GDP of its country is the same in all EA countries.

[39]In this case we set the different consumption shares by sector as well as by national varieties to be the equal to the mean for all households across the EA.

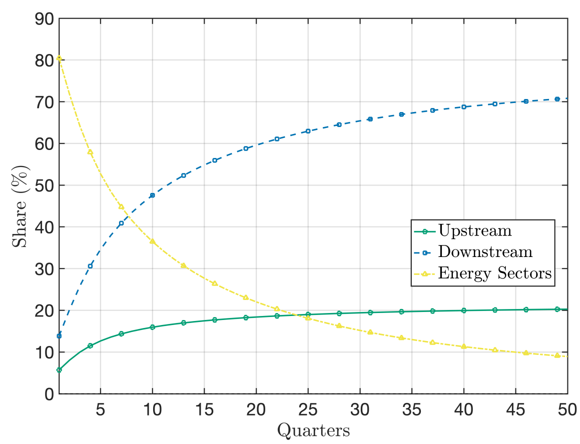
[40]The differences in sectoral price flexibility are particularly pronounced in energy and food sectors.The differences vanish when we consider only core CPI sectors (the average price duration is 6.45 quarters in Spain vs. 6.54 quarters in Germany), which explains the overlapping of core CPI inflation dynamics in the four countries in Panel C.2F.

C.3. Sectoral Decomposition

In order to study the sectoral composition of EA headline inflation, we catalog each of the 44 sectors in the economy into three categories: energy, upstream or downstream. We consider *Mining* (NACE B), *Coke and refined petroleoum* (NACE C.19), and *Electricity* (NACE D.35) as the energy sectors. Out of the remanining non-energy sectors, we follow the methodology proposed in Antràs *et al.* (2012) to rank sectors according to their relative proximity to the final consumer. According to their measure, a sector is more downstream (i.e. closer to the final consumer) when a larger share of its output is used as final consumption. Conversely, more upstream sectors are the ones that sell a larger fraction its output as intermediate input for other sectors.[41] We label a sector as upstream if its upstreamness measure is above the median, and downstream otherwise.[42]



A. CIRF of Headline Inflation



B. Contributions of Upstream, Downstream and Energy sectors

FIGURE C.3. Inflation Dynamics and its Sectoral Decomposition

Notes: Panel C.3A: CIRF of EA headline inflation and contributions of upstream and downstream sectors (Antràs *et al.* 2012). Panel C.3B: contributions of upstream and downstream sectors as a percent of total headline inflation.

We plot in Figure C.3 the decomposition of the cumulative impulse response function (CIRF) of the EA headline inflation after the energy price shock. In Panel C.3A, we document that the initial increase in headline inflation is entirely driven by the increase in energy sectors, directly transmitted to consumption prices. Over time, the energy price falls, reverting to its

[41]The upstreamness measure of an industry i, U_{ki}, is the solution to the system $U_{ki} = 1 + \sum_{l\in K}\sum_{j\in I} X_{klji}U_{kj}/Y_{ki}$, where X_{klji}/Y_{ki} denotes the share of industry i output sold to industry j in country l and U_j is the upstreamness measure of industry j. This measure takes into account the upstreamness of the sectors to which industry i supplies intermediate inputs.

[42]The three most downstream sectors are *Health and Education Services* (NACE P-Q), *Public Administration* (NACE O), and *Accommodation and food services* (NACE I). The three most upstream sectors (apart from energy-related sectors) are *Basic metals* (NACE C.24), *Chemical products* (NACE C.20) and *Warehousing* (NACE H.52-H.53).

initial level; and upstream and downstream sectors start contributing to headline inflation. Panel C.3B reports each component's share in total headline inflation. We find that the share of inflation coming from upstream sectors stabilizes after 30 quarters, whilst the share coming from downstream sectors is more persistent, increasing steadily over time.

These findings can be interpreted through the intuition developed in the price dynamics in equation (36): upstream sectors are less affected by the energy price increase since their intermediate input share in production is small, while downstream sectors depend directly on the intermediate input purchases on other sectors, including energy, and indirectly through their customers' IO network. Given that the pass-through is limited through price rigidities along the supply chain, this further increases the persistence of headline inflation.

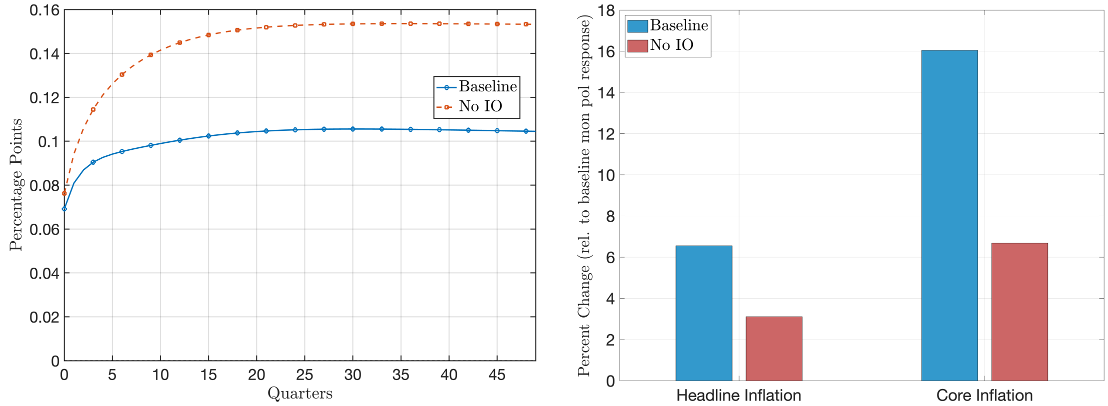
C.4. Systematic Monetary Policy and Production Networks

Previous research has shown that the presence of intermediate goods and IO links tends to increase the degree of monetary policy non-neutrality (see, for example, Nakamura and Steinsson 2010; Rubbo 2023). That is, upon a monetary policy shock, inflation tends to respond less and output more than in a counterfactual where these features are absent.

Our framework is consistent with this result. In Panel C.4A of Figure C.4, we show the CIRFs of headline inflation upon a monetary shock that increases EA interest rates. A monetary tightening leads to a larger drop in inflation when the IO structure is absent (red line), relative to our baseline calibration (blue line). Intuitively, the presence of intermediate goods with sticky prices reduces the volatility of marginal costs and reduces the pass-through of wages into prices (see expressio 36), leading to a muted inflation response.

We next explore the relevance of systematic monetary policy in the presence of production networks, being able to arrest the amplification on inflation. The Panel C.4B of Figure C.4 considers the following exercise. We draw a series of shocks to the international price of imported energy faced by European firms. We then simulate the model with and without production networks, subject to those shocks, and compute the volatility of headline and core inflation. When doing so, we consider two different inflation coefficients in the Taylor rule (46): the first with our baseline calibration $\phi^{\pi}_{MU} = 1.5$, and the second considering a weaker systematic response $\phi^{\pi}_{MU} = 1.1$. The blue bars in Panel C.4B show the increase in EA inflation volatility when we move from $\phi^{\pi}_{MU} = 1.5$ to $\phi^{\pi}_{MU} = 1.1$ in the economy with production networks. The red bars show the same statistic in the economy without IO links.

First, we observe that monetary policy has a greater impact on core inflation than on headline inflation, both with and without IO links. Specifically, the increase in inflation volatility from a weaker monetary policy response is more than twice as large for core inflation compared to headline inflation. This difference arises from cross-sector heterogeneity in price



A. CIRF of EA Headline Inflation to a Monetary Policy Shock in the EA

B. Percent Change in EA Inflation Volatility under Weaker Monetary Policy Response

FIGURE C.4. Monetary Policy and Production Networks

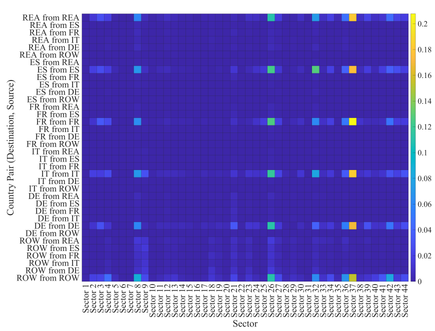
Notes: Panel C.4A: CIRF of headline inflation to a monetary policy shock (easing) in the baseline and without IO. Panel C.4B: percent change in inflation volatility (conditional on energy price shocks) with a lower coefficient on inflation in the Taylor rule.

flexibility and its interaction with import intensities. In our dataset, sectors contributing to core inflation, such as services and manufacturing, have stickier prices compared to energy- and food-producing sectors, which exhibit a higher pass-through from marginal costs to selling prices. In addition, the domestic energy and food sectors are heavily dependent on imported goods as key production inputs. Since domestic monetary policy has limited influence over the international prices of these imported goods, which strongly affect domestic prices, the monetary policy rule has a smaller impact on headline inflation than on core inflation.

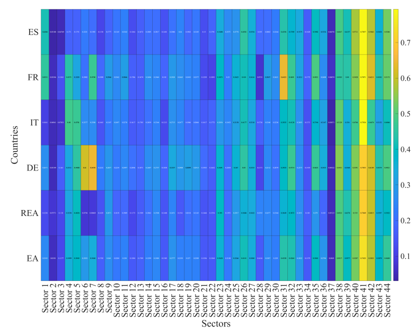
Second, the results of this simulation show that the systematic response of monetary policy becomes more significant in the presence of IO linkages, despite the higher degree of monetary non-neutrality following monetary shocks. Specifically, by comparing the red and blue bars, we observe that inflation volatility increases by more than twice as much when production networks are included. This finding aligns with our earlier results: IO linkages amplify the inflationary response to international energy price shocks. Consequently, even though a given change in interest rates has a smaller direct effect (as shown in Panel C.4A), a monetary policy response that fails to contain the propagation of such shocks—and allows the IO amplification to build up—will result in greater inflation volatility.

Appendix D. Additional Figures and Tables

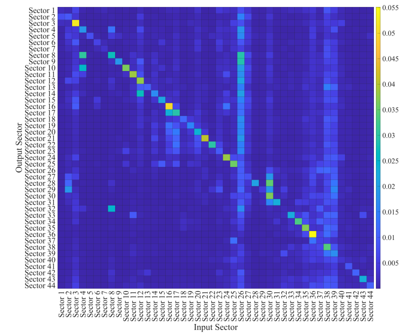
Figure D.1 presents a series of heatmaps that illustrate the structure of the consumption shares, labor shares, Calvo pricing rigidities, and EA production structure at a detailed, country-



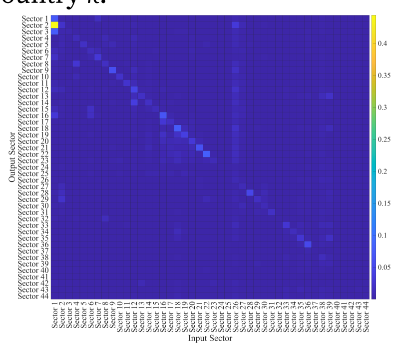
A. Heatmap of the Consumption matrix, where element $\beta_{kl\,i}$ denote the consumption share of sector i of country l in households' basket of country k.



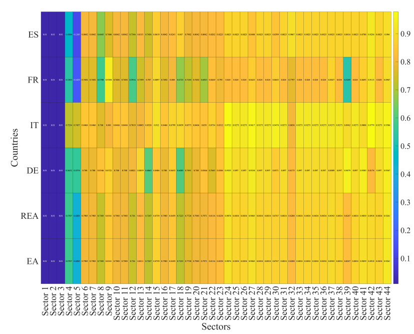
B. Heatmap of the Labor matrix, where element α_{ki} denotes the labor share of sector i in country k.



C. Heatmap of Home EA Input-Output matrix, where element ω_{ij} denote the input share of sector j for output sector i, both sectors inside the EA.



D. Heatmap of Foreign EA Input-Output matrix, where element ω_{ij} denotes the input share of sector j from ROW for output sector i inside EA.



E. Heatmap of the Calvo pricing rigidities matrix, where element θ^{p}_{ki} denotes the Calvo rigidity of sector i in country k.

FIGURE D.1. Sectoral Heterogeneity on Consumption Shares, Labor Shares, Input-Output Network, and Nominal Price Rigidities

Notes: Panel D.1A: heatmap of the consumption share. Panel D.1B: heatmap of the labor share. Panel D.1C: heatmap of the home input-output matrix of the EA. Panel D.1D: heatmap of the foreign input-output matrix of the EA. Panel D.1E: heatmap of Calvo rigidities.

specific level. Similarly, Figure D.2 illustrates the structure of the production networks at a detailed, country-specific level. In these visualizations, each cell represents the intensity of the input–output linkage between sectors—whether for domestic (home) or international (foreign) transactions. Lighter shades indicate higher input shares, revealing which sectors are more interconnected within a country and highlighting the key channels through which shocks can propagate. For instance, clusters of lighter cells in certain regions of the heatmaps point to sectors that rely heavily on inputs from specific domestic industries. The heatmaps provide a comparative view across different countries, capturing heterogeneities in both the domestic production structures and the degree of integration with foreign supply chains.

We obtain $\beta_{kl\,i}$ directly from the data. Using the empirical consumption shares, we compute $\zeta_{kl\,i} = \beta_{kl\,i}/(\sum_l \beta_{kl\,i})$, $\beta_k = \sum_{i\in I_E}\sum_l \beta_{kl\,i}$, $\nu_{ki} = \sum_{l=1}^{K} \beta_{kl\,i}/\beta_k$, and $\upsilon_{ki} = \sum_{l=1}^{K} \beta_{kl\,i}/(1-\beta_k)$. Similarly, we obtain $\omega_{kl\,ij}$ directly from the data. Using the empirical intermediate input shares, we compute $\zeta_{kl\,ij} = \omega_{kl\,ij}/(\sum_{m=1}^{K} \omega_{kmij})$, $\beta_{ki} = \sum_{j\in I_E}\left(\sum_l \omega_{kl\,ij}/\sum_l \sum_j \omega_{kl\,ij}\right)$, $\nu_{kij} = (\sum_l \omega_{kl\,ij}/\sum_{\sum_j} \omega_{kl\,ij})/\beta_{ki}$, and $\upsilon_{ki} = (\sum_l \omega_{kl\,ij}/\sum_{\sum_j} \omega_{kl\,ij})/(1-\beta_{ki})$.

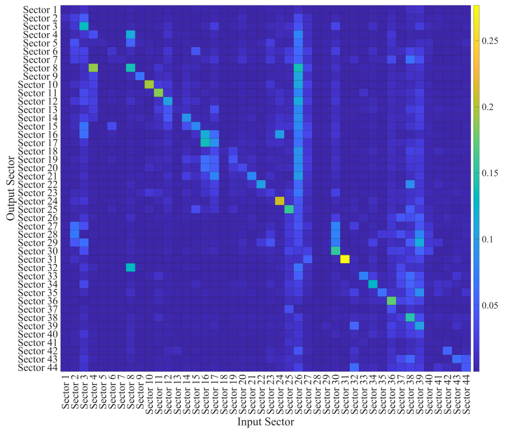
Table D.1 lists the 44 NACE sectors used in the analysis.

TABLE D.1. NACE Rev. 2 sectors used in the Analysis

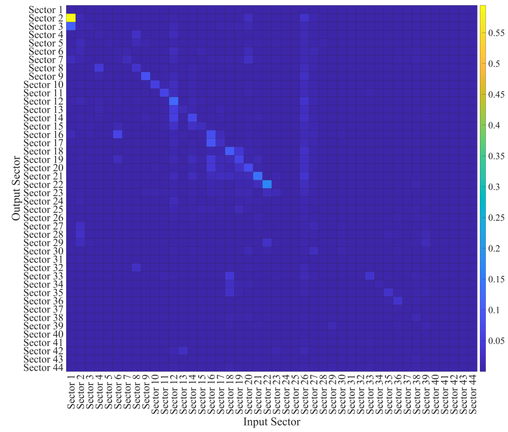
NACE Code	**Sector Name**
A.01–02	Agriculture, forestry and fishing
A.03	Fishing and aquaculture
B.05–06	Mining of coal, lignite, crude petroleum and natural gas
B.07–08	Mining of metal ores and other mining and quarrying
B.09	Mining support service activities
C.10–12	Manufacture of food, beverages and tobacco
C.13–15	Manufacture of textiles, wearing apparel and leather products
C.16	Manufacture of wood and products of wood and cork
C.17–18	Manufacture of paper, and printing and media reproduction
C.19	Manufacture of coke and refined petroleum products
C.20	Manufacture of chemicals and chemical products
C.21	Manufacture of basic pharmaceutical products
C.22	Manufacture of rubber and plastic products
C.23	Manufacture of other non-metallic mineral products
C.24	Manufacture of basic metals
C.25	Manufacture of fabricated metal products
C.26	Manufacture of computer, electronic and optical products
C.27	Manufacture of electrical equipment
C.28	Manufacture of machinery and equipment n.e.c.
C.29	Manufacture of motor vehicles, trailers and semi-trailers
C.30	Manufacture of other transport equipment
C.31–33	Other manufacturing and repair and installation of machinery
D.35	Electricity, gas, steam and air conditioning supply
E.36–39	Water supply; sewerage, waste management and remediation

Continued on next page

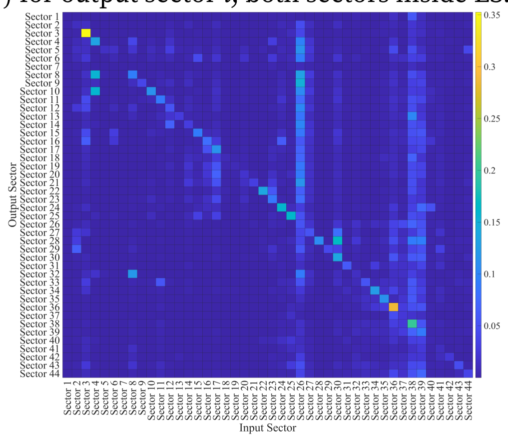
NACE Code	Sector Name
F.41–43	Construction
G.45–47	Wholesale and retail trade; repair of motor vehicles
H.49	Land transport and transport via pipelines
H.50	Water transport
H.51	Air transport
H.52	Warehousing and support activities for transportation
H.53	Postal and courier activities
I.55–56	Accommodation and food service activities
J.58–60	Publishing, audiovisual and broadcasting activities
J.61	Telecommunications
J.62–63	IT and other information services
K.64–66	Financial and insurance activities
L.68	Real estate activities
M.69–75	Professional, scientific and technical activities
N.77–82	Administrative and support service activities
O.84	Public administration and defence; compulsory social security
P.85	Education
Q.86–88	Human health and social work activities
R.90–93	Arts, entertainment and recreation
S.94–96	Other personal service activities



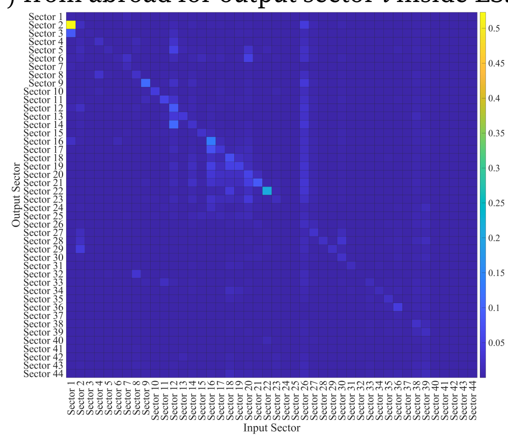
A. Heatmap of Home ES Input-Output matrix, where element ω_{ij} denotes the input share of sector j for output sector i, both sectors inside ES.



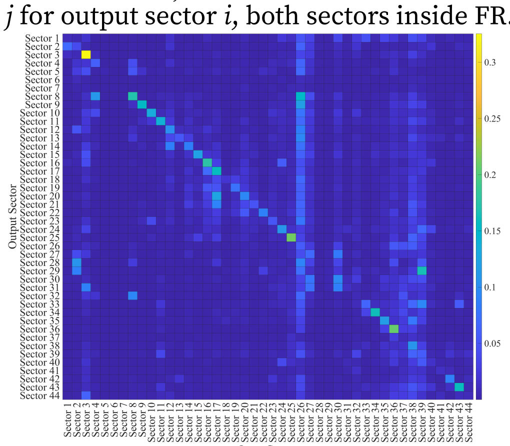
B. Heatmap of Foreign ES Input-Output matrix, where element ω_{ij} denotes the input share of sector j from abroad for output sector i inside ES.



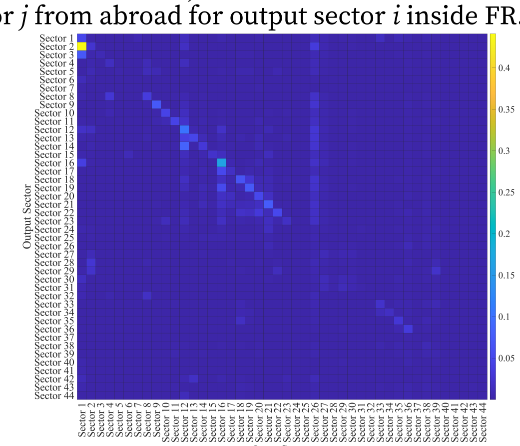
C. Heatmap of Home FR Input-Output matrix, where element ω_{ij} denotes the input share of sector j for output sector i, both sectors inside FR.



D. Heatmap of Foreign FR Input-Output matrix, where element ω_{ij} denotes the input share of sector j from abroad for output sector i inside FR.



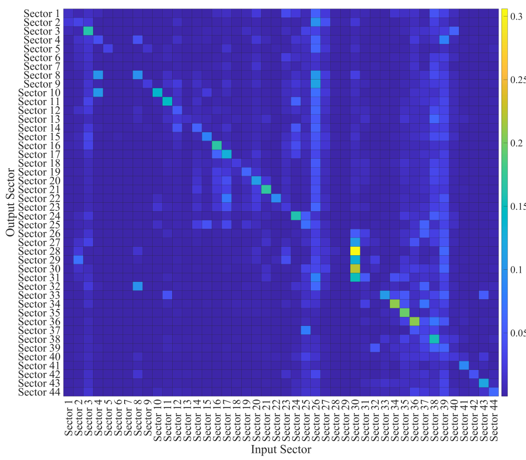
E. Heatmap of Home IT Input-Output matrix, where element ω_{ij} denotes the input share of sector j for output sector i, both sectors inside IT.



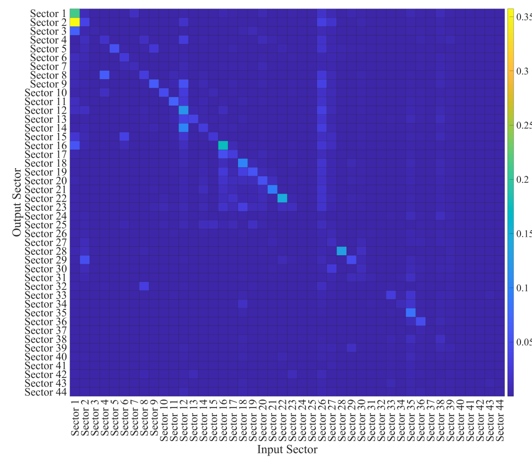
F. Heatmap of Foreign IT Input-Output matrix, where element ω_{ij} denotes the input share of sector j from abroad for output sector i inside IT.

FIGURE D.2. Heatmaps of the Input-Output Structure

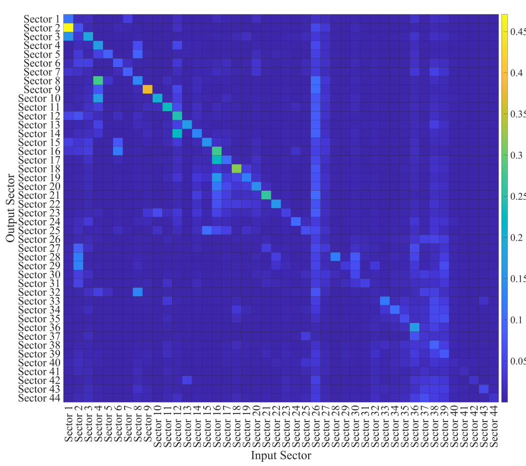
Notes: Heatmaps of the home (left column) and foreign (right column) input-output matrices of the Rest of the Euro Area, Spain, France, Italy, Germany, and the Rest of the world.



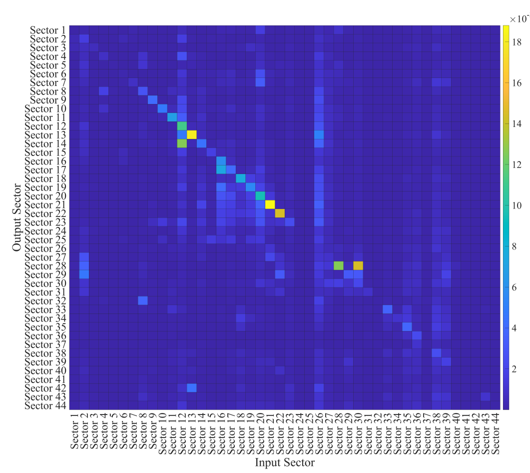
G. Heatmap of Home DE Input-Output matrix, where element ω_{ij} denotes the input share of sector j for output sector i, both sectors inside DE.



H. Heatmap of Foreign DE Input-Output matrix, where element ω_{ij} denotes the input share of sector j from abroad for output sector i inside DE.



I. Heatmap of Home ROW Input-Output matrix, where element ω_{ij} denotes the input share of sector j for output sector i, both sectors inside ROW.



J. Heatmap of Foreign ROW Input-Output matrix, where element ω_{ij} denotes the input share of sector j from abroad for output sector i inside ROW.

FIGURE D.2. (Continued)